Chapter 15. Gravity Separation

15. INTRODUCTION
Separation by density difference is a process that is as old as recorded history. Separation of gold by density difference dates back to at least 3,000 BC as depicted in writings from ancient Egypt. The principle employed in gravity separation goes back further in time to the formation and weathering of the rocks and the releasing of the minerals they contain and the transport of the mineral grains by heavy rains. It is the driving force for the formation of the alluvial deposits of precious metals and gemstones that have been worked since beyond recorded history as they still are today. Archaeological excavations have discovered mineral concentration activities such as the lead-silver concentrating plant in Attica, Greece, dating from 300-400 BC. So gravity separation has a long history as a mineral concentration process.

Not all mineral combinations are amenable to this type of concentration technique. To determine the suitability of gravity separation processes to a particular ore type, a concentration criterion is commonly used. A concentration criterion (CC) can be defined as [1]:

\[
\text{Concentration Criterion} = \frac{SG_{\text{of heavy mineral}} - SG_{\text{of fluid}}}{SG_{\text{of light mineral}} - SG_{\text{of fluid}}} \quad (15.1)
\]

where \( SG \) = specific gravity (or density), and the fluid is typically water or air.

Some concentration criterion ratios for minerals that are treated by gravity separation are given in Table 15.1.

<table>
<thead>
<tr>
<th>Mineral</th>
<th>Fluid</th>
<th>CC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold</td>
<td>water</td>
<td>10.3</td>
</tr>
<tr>
<td>Gold</td>
<td>air</td>
<td>6.8</td>
</tr>
<tr>
<td>Cassiterite</td>
<td>water</td>
<td>3.5</td>
</tr>
<tr>
<td>Coal</td>
<td>water</td>
<td>3.4</td>
</tr>
<tr>
<td>Hematite</td>
<td>water</td>
<td>2.5</td>
</tr>
</tbody>
</table>

A guideline for separability by gravity based on this concentration criterion is given in Table 15.2. Fig. 15.1 shows these limitations graphically over a separation curve described by Burt [2]. Separation is possible above the line and impossible for concentration criteria below the line.
Fig. 15.1. Size limit curve for gravity separation (solid line – Burt [2]) with data from Table 15.2 (points).

Table 15.2
Concentration criterion guide for gravity separation [1].

<table>
<thead>
<tr>
<th>Concentration criterion</th>
<th>Suitability to gravity separation</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC &gt; 2.5</td>
<td>easy down to 75 μm</td>
</tr>
<tr>
<td>1.75 &lt; CC &lt; 2.5</td>
<td>possible down to 150 μm</td>
</tr>
<tr>
<td>1.5 &lt; CC &lt; 1.75</td>
<td>possible down to 1.7 mm</td>
</tr>
<tr>
<td>1.25 &lt; CC &lt; 1.5</td>
<td>possible down to 6.35 mm</td>
</tr>
<tr>
<td>CC &lt; 1.25</td>
<td>impossible at any size</td>
</tr>
</tbody>
</table>

Table 15.1 shows that separation will be easier in a fluid of higher density.

The concentration ratio numbers in Fig. 15.1 and Table 15.2 are a guide only as the ratio is influenced by other factors such as particle shape. Particle shape can be taken into account by including a shape factor defined as the ratio of shape settling factors for the heavy and light minerals. The shape settling factor is the ratio of the terminal velocity of two particles of the same density, same size but different shape. That is:

\[
\text{Shape settling factor} = \frac{V_T(\text{particle})}{V_T(\text{sphere})} \quad (15.2)
\]

where \( v_T \) = terminal velocity.
15.1. Particle Settling Rates

The separability of minerals by gravity separation relies on a particle's settling rate in a fluid. The terminal velocity of solid spheres settling in a fluid is described by Stokes’ Law for fine particles (Eq. (15.3)) or Newton’s Law for coarse particles (Eq. (15.4)). Both these equations include particle density as well as particle size.

\[ v_T = \frac{g(\rho_s - \rho_F) d^2}{18 \mu} \quad \text{for viscous resistance (Stokes’ Law)} \quad (15.3) \]

\[ v_T = \sqrt{\frac{4g(\rho_s - \rho_F) d}{3C_D \rho_F}} \quad \text{for turbulent resistance (Newton’s Law)} \quad (15.4) \]

where \( d \) = particle diameter,
\( \rho_s, \rho_F \) = density of solid and fluid respectively,
\( C_D \) = the drag coefficient,
\( \mu \) = fluid viscosity and
\( g \) = gravitational acceleration.

Stokes’ equation is said to apply to conditions where the particle Reynolds number is less than 1 and Newton’s equation applies for Reynolds numbers > 1000. For particles of quartz in water, this represents an upper size limit of around 110 \( \mu \)m for Stokes’ Law and a lower limit of around 3.5 mm for Newton’s Law. Thus for particles of quartz between 110 microns and 3.5 mm neither equation accurately describes the settling rate of objects and this size range represents a major size range of interest in gravity separation. A number of researchers have developed empirical correlations to fill this size gap. Dietrich [3] derived a correlation from a data set of 252 values using dimensionless parameters, \( W^* \) and \( D^* \), and incorporating shape and angularity factors:

\[ W^* = \frac{v_T^2 \rho_F^2}{(\rho_s - \rho_F) g \mu} \quad (15.5) \]

and

\[ D^* = \left( \frac{\rho_s - \rho_F}{\rho_F} \right) g d_N^3 \frac{\rho_F}{\mu^2} \quad (15.6) \]

where \( d_N \) = nominal diameter of the largest projected area. An irregular particle will settle in a stable orientation when the largest projected area is perpendicular to the settling direction.

Dietrich’s dimensionless parameters are related by the expression:

\[ W^* = R_s 10^{R_1 + R_2} \quad (15.7) \]
where \( R_1 \) = a coefficient describing the effect of particle density and given by:

\[
R_1 = -3.76715 + 1.92944 \log D^* - 0.09815 \log D^* \cdot 2 - 0.00575 \log D^* \cdot 3 + 0.00056 \log D^* \cdot 4
\]

\( R_2 \) = a coefficient describing the effect of particle shape and given by:

\[
R_2 = \log \left( \frac{1 - \text{CSF}}{0.85} \right)
\]

\[
\text{CSF} = \text{Corey shape factor} = \frac{d_{\text{MIN}}}{\sqrt{d_{\text{MAX}} d_{\text{MID}}}}
\]

\( d_{\text{MIN}}, d_{\text{MAX}}, d_{\text{MID}} \) = minimum, maximum and mid-range particle dimension, and

\[
R_3 = \left( 0.65 \left( \frac{\text{CSF} \tanh (\log D^* - 4.6)}{2.83} \right)^{1-(\log D^*/2.5)} \right)
\]

\( P \) = a Powers roundness factor, equal to 6 for perfect rounded particles (spheres) and 2-3 for highly angular particles.

Jiménez and Madsen [4] simplified Dietrich's approach defining the dimensionless parameters:

\[
V^* = \frac{V}{\sqrt{\frac{P_S - P_F}{\rho_F}} g d}
\]

and

\[
S^* = \frac{d \rho_F}{4 \mu} \left( \frac{P_S - P_F}{\rho_F} g d \right)
\]

A linear regression between \( 1/V^* \) and \( 1/S^* \) gave the equation:

\[
\frac{1}{V^*} = A + \frac{B}{S^*}
\]

The coefficients \( A \) and \( B \) allowed a solution for the drag coefficient:

\[
C_D = \frac{1}{3} \left( \frac{A}{A^2 + \frac{16B}{\text{Re}_F}} \right)^2
\]

where \( \text{Re}_F \) = particle Reynolds's number.

Jiménez and Madsen obtained values of \( A \) and \( B \) from Dietrich's data for quartz spheres from \(-0.06 - 1 \) mm validating the equation for \( 0.2 < \text{Re}_F < 127 \). For Reynolds numbers less than 0.2, \( C_D \) approached a value of \( 24.5/\text{Re}_F \) in good agreement with Stokes' drag coefficient.
but for Reynolds numbers above 1000, $C_D$ approached a constant value of 0.83 compared with
a value of 0.44 for Newtonian turbulent resistance.

Table 15.3 shows values of $A$ and $B$ obtained by Jiménez and Madsen for different shaped
particles.

Example 15.1 explores the calculation of settling velocities using these equations.

Table 15.3
Coefficients $A$ and $B$ in equation 15.10 for particles of Corey shape factor 0.7 [4].

<table>
<thead>
<tr>
<th>Parameter $P$</th>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0 (crushed)</td>
<td>0.995</td>
<td>5.211</td>
</tr>
<tr>
<td>3.5 (natural)</td>
<td>0.954</td>
<td>5.121</td>
</tr>
<tr>
<td>6.0 (well rounded)</td>
<td>0.890</td>
<td>4.974</td>
</tr>
<tr>
<td>Spheres</td>
<td>0.794</td>
<td>4.606</td>
</tr>
</tbody>
</table>

**Example 15.1**

Determine the settling rates for spherical particles of quartz settling in water for particles of
size 38 $\mu$m to 16 mm. The density of quartz and water are 2650 and 1000 kg/m$^3$ respectively
and the viscosity of water is 0.001 Pa s.

**Solution**

Step 1: Calculate the terminal velocity for a 38 micron particle of quartz using Stokes' Law:

For a 38 micron particle, using Eq. (15.3),

$$v_T = \frac{(2650-1000) \times 9.81 \times (0.000038)^2}{18 \times 0.001} = 0.001299 \text{ m/s}$$

The Reynolds number for this size particle is:

$$Re_p = \frac{v_T d_p}{\mu} = \frac{0.001299 \times 0.000038 \times 1000}{0.001} = 0.0493$$

Step 2: Repeat Step 1 and calculate the terminal velocity of a 38 micron particle using
Newton's Law:

For the same particle, using Eq. (15.4):
\[ v_T = \sqrt{\frac{4 \times 9.81 \times (2650 - 1000) \times 0.000038}{3 \times 0.44 \times 1000}} = 0.0432 \text{ m/s} \]

and \[ \text{Re}_p = \frac{0.0432 \times 0.000038 \times 1000}{0.001} = 1.6406 \]

Step 3: Repeat Step 1 and calculate the terminal velocity of a 38 micron particle using Dietrich’s correlation:
Using Eq. (15.6),
\[ D^* = \frac{(2650 - 1000) \times 9.81 \times (0.000038)^3 \times 1000}{0.001^2} = 0.8882 \]
and from Eq. (15.7), for CSF = 1.0 for a sphere and P = 6 for a perfect round object,
\[ R_1 = -3.76715 + 1.92944(\log 0.8882) - 0.09815(\log 0.8882)^2 - 0.00575(\log 0.8882)^3 + 0.00056(\log 0.8882)^4 \]
\[ = -3.8668 \]
\[ R_2 = \log \left( \frac{1 - 1.0}{0.85} \right) = 0 \]
\[ R_3 = \left( 0.65 - \frac{1.0}{2.83} \tanh(\log 0.8882 - 4.6) \right)^{\frac{0.35 - 0.09215}{0.25}} = 1.0 \]

Then from Eq. (15.7);
\[ W^* = 1.0 \times 10^{-3.8668 - 0} = 0.000136 \]
and from Eq. (15.5);
\[ v_T = 3 \sqrt{\frac{0.000136 \times (2650 - 1000) \times 9.81 \times 0.001}{1000^2}} = 0.0013 \text{ m/s} \]

and \[ \text{Re}_p = \frac{0.0013 \times 0.000038 \times 1000}{0.001} = 0.0494 \]

Step 4: Repeat Step 1 and calculate the terminal velocity of a 38 micron particle using the Jiménez and Madsen correlation:
From Eq. (15.9) and Table 15.3, A = 0.794 and B = 4.606;
500

\[ S^* = \frac{0.000038 \times 1000}{4 \times 0.001} \sqrt{\frac{(2650 - 1000) \times 9.81 \times 0.000038}{1000}} = 0.2356 \]

and substituting into Eq. (15.10)

\[ V^* = \frac{S^*}{(AS^* + B)} = \frac{0.2356}{(0.2356 \times 0.794) + 4.606} = 0.04916 \]

Then from Eq. (15.8);

\[ \nu_T = 0.04916 \times \sqrt{\frac{(2650 - 1000)}{1000} \times 9.81 \times 0.000038} = 0.00122 \text{ m/s} \]

and \( \text{Re}_p = \frac{0.00122 \times 0.000038 \times 1000}{0.001} = 0.0463 \)

Step 5: Repeat calculations in steps 1-4 for other sizes up to 16 mm and compare.

By changing the particle size from 38 microns to 16 mm, the following table is compiled.

<table>
<thead>
<tr>
<th>( d_N ) (m)</th>
<th>( \nu_T ) Stokes m/s</th>
<th>( \nu_T ) Newton m/s</th>
<th>( \nu_T ) Dietrich m/s</th>
<th>( \nu_T ) Jiménez m/s</th>
<th>Re</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000038</td>
<td>0.001</td>
<td>0.0432</td>
<td>0.0013</td>
<td>0.0012</td>
<td>0.05</td>
</tr>
<tr>
<td>0.000050</td>
<td>0.002</td>
<td>0.0495</td>
<td>0.0022</td>
<td>0.0021</td>
<td>0.10</td>
</tr>
<tr>
<td>0.000075</td>
<td>0.005</td>
<td>0.0607</td>
<td>0.0046</td>
<td>0.0044</td>
<td>0.33</td>
</tr>
<tr>
<td>0.000106</td>
<td>0.010</td>
<td>0.0721</td>
<td>0.0082</td>
<td>0.0083</td>
<td>0.87</td>
</tr>
<tr>
<td>0.000175</td>
<td>0.028</td>
<td>0.0926</td>
<td>0.0182</td>
<td>0.0192</td>
<td>3.00</td>
</tr>
<tr>
<td>0.00050</td>
<td>0.225</td>
<td>0.1566</td>
<td>0.0740</td>
<td>0.0747</td>
<td>37.00</td>
</tr>
<tr>
<td>0.00075</td>
<td>0.506</td>
<td>0.1918</td>
<td>0.1161</td>
<td>0.1083</td>
<td>81.00</td>
</tr>
<tr>
<td>0.0020</td>
<td>0.899</td>
<td>0.2215</td>
<td>0.1550</td>
<td>0.1355</td>
<td>135.00</td>
</tr>
<tr>
<td>0.0040</td>
<td>3.597</td>
<td>0.3132</td>
<td>0.2829</td>
<td>0.2129</td>
<td>425.00</td>
</tr>
<tr>
<td>0.0060</td>
<td>14.388</td>
<td>0.4429</td>
<td>0.4565</td>
<td>0.3133</td>
<td>1253.00</td>
</tr>
<tr>
<td>0.0080</td>
<td>32.373</td>
<td>0.5425</td>
<td>0.5757</td>
<td>0.3877</td>
<td>2326.00</td>
</tr>
<tr>
<td>0.0100</td>
<td>57.552</td>
<td>0.6264</td>
<td>0.6666</td>
<td>0.4496</td>
<td>3596.00</td>
</tr>
<tr>
<td>0.0160</td>
<td>89.925</td>
<td>0.7004</td>
<td>0.7406</td>
<td>0.5038</td>
<td>5038.00</td>
</tr>
<tr>
<td>0.0160</td>
<td>230.208</td>
<td>0.8859</td>
<td>0.9073</td>
<td>0.6391</td>
<td>10225.00</td>
</tr>
</tbody>
</table>

The following graphs compares the calculated settling velocities.

In the fine particle range (< 100 microns) the Stokes settling equation and Dietrich and Jiménez correlations are similar. Above 150 microns the Stokes equation starts to deviate from the Dietrich and Jiménez plots. The Newton settling line is significantly different from the other plots at this size range. At the course end of the size range, as seen below, the Jiménez correlation also deviates from the Newton and Dietrich plots.
At particle sizes above about 3 mm, the Newton and Dietrich plots are still close. At this point, the Reynolds number is around 1000, the region above which Newton’s Law is valid. The Dietrich correlation seems to adequately describe the transition region between the Stokes and Newtonian regimes. The Jiménez correlation deviates from the Dietrich correlation above a size of around 1 mm or a Reynolds number around 135.
The separation by gravity is based on the difference in settling rates or terminal velocities of particles of different density and size. However, with short distances of travel in some separation processes, particles may not have a chance to reach their terminal velocity. How long it takes particles to reach their terminal velocity and what are the displacement distances between particles when they attain their terminal velocity could be a determining factor in the concentration of particles by gravity separation.

The forces acting on a particle settling in a fluid under free settling conditions are gravity, buoyancy in the fluid and drag. Thus:

\[ M_s a_p = F_g - F_B - F_D = M_s g - M_s \left( \frac{\rho_f}{\rho_s} \right) g - F_D \]  
\[ (15.12) \]

where \( M_s \) = solid mass,
\( M_s \left( \frac{\rho_f}{\rho_s} \right) \) = mass of fluid displaced by the particle,
\( a_p \) = the particle acceleration, and
\( F_D, F_B, F_G \) = the drag, gravitation and buoyancy forces respectively.

Dividing Eq. (15.12) through by the solid mass gives:

\[ a_p = \frac{d^2}{dt^2} \left( \frac{1}{M_s} \right) \left( \rho_f - \rho_s \right) \left( \frac{\rho_s}{\rho_s} \right) g - \frac{F_D}{M_s} \]  
\[ (15.13) \]

The drag force increases with increasing particle velocity and eventually balances the other forces acting on the particle to yield a constant falling velocity, the terminal velocity. The drag force is given by:

\[ \frac{F_D}{M_s} = \frac{C_D v^2 A_c \rho_f}{2M_s} \]  
\[ (15.14) \]

where \( A_c \) = the cross-sectional area of the particle, and \( v \) = particle velocity.

For a spherical particle:

\[ M_s = \frac{\pi d^3 \rho_s}{6} \quad \text{and} \quad A_c = \frac{\pi d^3}{4} \quad \text{thus:} \]
\[ \frac{F_D}{M_s} = \frac{3C_D v^2 \rho_f}{4d \rho_s} \]  
\[ (15.15) \]

At low particle velocities, for a spherical particle, \( C_D \) is given by:

\[ \frac{C_D}{Re_f} = \frac{24}{\mu v d \rho_f} \]  
\[ (15.16) \]
where $\mu$ = fluid viscosity and 
\[d = \text{particle diameter.}\]

Thus:

\[
\frac{F_D}{M_s} = \frac{18 \mu v}{d^3 \rho_s}
\]

and substituting into Eq. (15.13) gives:

\[
\frac{dv}{dt} = \frac{g(\rho_p - \rho_r)}{\rho_r} - \frac{18 \mu v}{d^3 \rho_s}
\]

for $Re_p < 1$  \hspace{1cm} (15.17)

At higher velocities, $C_D$ is taken as 0.44 for spherical particles and:

\[
\frac{dv}{dt} = \frac{g(\rho_p - \rho_r)}{\rho_r} - \frac{(0.44 \times 3) v^3 \rho_r}{4d \rho_s}
\]

for $Re_p > 1000$  \hspace{1cm} (15.18)

Fig. 15.2 is constructed using Eq. (15.17). It shows the increase in particle velocity with time for particles of the same size but different densities. Fig. 15.3 shows a similar plot for larger particles settling as described by Eq. (15.18).

![Fig. 15.2. Settling velocity of 50 μm particles of different density in water for different concentration criteria.](image-url)
Fig. 15.3. Settling velocity of 6.35 mm particles of different density in water for different concentration criteria.

Table 15.4 summarises the settling velocities and distances travelled for a combination of different concentration criteria and particle sizes corresponding to the size limits shown in Table 15.2. The calculations indicate that the time required for a particle to reach its terminal velocity is quite short, ranging from 0.001 to 0.4 seconds. The lighter particles reach their terminal velocity marginally ahead of the heavier particles.

Table 15.4
Time and distance apart when particles reach their terminal velocity, based on Eqs. (15.17) and (15.18).

<table>
<thead>
<tr>
<th>Particle size (mm)</th>
<th>Concentration criterion</th>
<th>Difference in $v_T$ (m/s)</th>
<th>Separating distance (mm)</th>
<th>Time for heavy particle to reach $v_T$ (seconds)</th>
<th>Distance apart after 1 second (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.050</td>
<td>1.10</td>
<td>0.0002</td>
<td>0.0009</td>
<td>0.0018</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>1.25</td>
<td>0.0006</td>
<td>0.0036</td>
<td>0.0025</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>1.50</td>
<td>0.0012</td>
<td>0.0050</td>
<td>0.0025</td>
<td>1.17</td>
</tr>
<tr>
<td></td>
<td>1.75</td>
<td>0.0018</td>
<td>0.0064</td>
<td>0.0025</td>
<td>1.75</td>
</tr>
<tr>
<td></td>
<td>2.50</td>
<td>0.0035</td>
<td>0.0195</td>
<td>0.0040</td>
<td>3.49</td>
</tr>
<tr>
<td>6.35</td>
<td>1.10</td>
<td>0.028</td>
<td>8.80</td>
<td>0.32</td>
<td>27.6</td>
</tr>
<tr>
<td></td>
<td>1.25</td>
<td>0.068</td>
<td>21.60</td>
<td>0.32</td>
<td>67.6</td>
</tr>
<tr>
<td></td>
<td>1.50</td>
<td>0.127</td>
<td>54.60</td>
<td>0.34</td>
<td>127.3</td>
</tr>
<tr>
<td></td>
<td>1.75</td>
<td>0.182</td>
<td>88.20</td>
<td>0.36</td>
<td>182.3</td>
</tr>
<tr>
<td></td>
<td>2.50</td>
<td>0.329</td>
<td>177.00</td>
<td>0.40</td>
<td>329.2</td>
</tr>
</tbody>
</table>
For particles of 6.35 mm size, the terminal velocity is reached after about 0.4 seconds and particles of concentration criterion (CC) 2.5 are separated by several hundred millimetres and separation of particles should be easy. The separation distance between particles decreases as the concentration criterion decreases and thus separation should become more difficult with decreasing concentration criterion.

If the particles don’t reach their terminal velocities then the separation distance between particles is reduced. For example, for a particle of CC = 1.25, after 0.1 seconds of settling, the settling velocities are very close and the separation distance between the particles is 3.2 mm which for particles of 6.35 mm diameter is insufficient to segregate into separate layers.

For 50 μm particles, the terminal velocity is reached after a very short time and the distance separating particles at this point is as much as 20 μm for particles with a concentration criterion of 2.5. For 50 μm particles, this is obviously insufficient for separation to occur. As settling time is increased up to 1 second, particle separation, in free settling conditions, will increase to 3.5 mm for particles with a concentration criterion of 2.5. With this separation distance between heavy and light particles, segregation, and ultimately separation, may not be possible for these sized particles. Increasing the settling time further will increase the separation distance and make separation easier. However, the fact that particles of this size are not easily separated even at this value of concentration criterion, indicates that other factors such as particle shape and separator characteristics come into play.

15.1.1. The Effect of Particle Size and Shape
One of the complicating factors is that particles are not a single size. In any feed there is going to be a size variation, even in a closely sized sample. Consider an elutriation column with a prepared feed of -150 +125 μm containing a mixture of pyrite (S.G. 5.0) and arsenopyrite (S.G. 6.1). If a rising column of water is flowing at a velocity between the settling velocity of the two minerals then the heavier mineral will be able to sink and the lighter mineral will be lifted by the water and hence a separation of the two minerals can be made. Fig. 15.4 shows the effect of particle size on the settling velocity and hence the water velocity required to bring about separation. If the water velocity is above the curve, the particle will be lifted and if the water velocity is below the curve, the particle will sink. Thus the region between the curves represents the range of possible water flows that will separate the particles. For the -150 +125 μm size fraction, point D, where the lower size intersects the lower particle density curve represents the water flow rate below which all particles will sink and point C, where the large size intersects the higher particle density curve represents the water flow rate above which all particles will be lifted. Flowrates between C and D will achieve a separation of some sort with some particles being lifted and some particles sinking. Point A at the intersection of the lower size and the higher density curve and point B, the intersection of the higher size and the lower density curve, will be the boundary of the region where a complete separation is possible, but only if point A lies higher than point B. In the case of Fig. 15.4, where the concentration criterion is 1.275, this is not the case and there will always be some contamination of the light mineral and the heavy mineral in each product. For a smaller size fraction such as -106 +90 μm, the window of possible separation flow rates decreases. That is, the separation would become more difficult.

For a higher concentration criterion, such as in the separation of arsenopyrite (S.G. 6.1) from gold (S.G. 18.0), Fig. 15.5 shows the settling curves as a function of particle size. From this plot, for the particle size fraction of -150 +125 μm, point A is above point B and any flowrate within this range should produce a clean separation. But how does it work in practice?
Fig. 15.4. Settling curves for separation of pyrite from arsenopyrite using the Dietrich correlation. The concentration criterion is 1.275.

Fig. 15.5. Settling curves for separation of gold from arsenopyrite using the Dietrich correlation. The concentration criterion is 3.33.
A sample of sulphide concentrate, predominantly arsenopyrite and some pyrrhotite and free gold was sized to $-150 \, +125 \, \mu m$ and placed in an elutriation column at a water flowrate of 9 L/min. From Fig. 15.5, this flowrate is at the top limit of the separation zone at which all particles should have risen with the water flow. However a very small number of particles were recovered as a sinks product. If any particles were recovered it would be expected to be predominantly the gold particles. The gold was estimated to have a density of $18000 \, kg/m^3$ as some impurities such as silver and copper occur with native gold. However, inspecting the sinks product under a microscope revealed that it contained only about 5% gold with the bulk of the product being sulphides and iron oxides. The size of the particles were towards the top end of the size range.

The float fraction from the elutriation column was re-tested at a water flowrate of 7 L/min which should also have produced a high grade gold product. The mass recovery was also low and contained about 50% gold and 50% sulphides and iron oxides. The gold particles tended to be flatter in shape compared to the sulphides because of the malleability of the gold metal. The flatter shape of the particles would thus give the gold the appearance of similar sized particles but would actually have a lower mass than a more rounded particle of gold of the same size. Thus, the particle shape is allowing a lighter gold particle to fall into the same size fraction as a similarly weighted sulphide particle and the equally settling particles would be extremely difficult to separated by gravity.

Increasing the acceleration on the particles by using some centrifugal device will not change the ease of separation for particles with a low concentration criterion though it will improve the separation for high concentration criteria. For example, for a concentration criterion of 1.275, the difference between point A and point B in Fig. 15.4 is -0.2 L/min for an acceleration of 1G. The minus sign indicating that point A is below point B. By increasing the acceleration to 100G the difference becomes -2.0 L/min, still negative and still a poor separation. By contrast, for a concentration criterion of 3.33, an increase in acceleration from 1 to 100G increases the difference between point A and point B in Fig. 15.5 from +3.6 to +84 L/min indicating a broader range a separating velocities and a greater ease of separation, particularly at finer sizes.

At fine particle sizes, separation by density difference under gravity, becomes less efficient. At this size range flotation is the dominant separation process, though the application of centrifugal acceleration has extended the useful separation size by gravity processes down to 5-10 \, \mu m, provided the concentration criterion is favourable.

The advantages of gravity separation are:

1. a lower installed cost per tonne of throughput than flotation,
2. a lower installed power requirement per tonne of throughput,
3. gravity separation does not use expensive reagents and
4. the environmental impact of gravity plant effluent is considerably less than for flotation, due to the absence of organic chemicals.

15.2. Gravity Separation Operations

Gravity concentrating operations are characterised by processes that allow particles to be held slightly apart so that they can move relative to each other and therefore to separate into layers of dense and light minerals. The mechanisms by which this interparticular spacing is accomplished may be used as a convenient means of classifying gravity concentrators.

1. **Jigging** - uses an essentially vertical expansion and contraction of a bed of particles by a pulse of fluid.
2. **Shaking Concentrators** - employ a horizontal motion to the solids-fluid stream to effectively fluidise the particles causing segregation of light and heavy particles.

3. **Flowing Film Concentrators** - initiates particle separation by a layer of slurry flowing down an inclined surface under the influence of gravity. Some of the oldest known concentrators, such as sluices and troughs are flowing film concentrators.

### 15.3. Jigs

Stratification in a bed of particles results from the repeated pulsation of a current of fluid up through the bed. The particles in the bed are expanded so that when pulsation ceases, the particles are allowed to consolidate under the influence of gravity. Fig. 15.6 illustrates the expansion and contraction of the bed with the heavier, larger particles falling under hindered settling conditions.

The expansion and contraction of the bed is repeated in a cyclic operation until the heavy and light particles have stratified according to their specific gravity. The frequency of pulsations usually varies from 50 - 300 cycles per minute.

![Fig. 15.6. Expansion and contraction of a bed of particles due to jigg action.](image)

A particle settling in a viscous fluid is described by Eq. (15.12). As a particle just starts to move from rest, the particle velocity is small and hence the drag force acting on the particle, \( F_D \), is negligible since the drag force increases with particle velocity relative to the fluid. Thus:

\[
a_p = g \left( \frac{\rho_s - \rho_f}{\rho_s} \right)
\]

15.19

That is, the initial acceleration of the particles depend only on the specific gravity of the solid and fluid and is independent of the particle size. Once the particles reach an appreciable velocity the fluid drag force becomes significant and it opposes the particle's further acceleration to the extent that eventually the particle acceleration drops to zero and a constant terminal velocity is reached which will dependent on the particle diameter as well as density. If the duration of the particle movement under gravity is kept short by having a high cycle frequency then the total distance travelled by the particles will be governed more by the difference in the initial acceleration between particles due to their density difference rather than by their terminal velocities which is also influenced by the particle size. That is, for
particles with a similar terminal velocity, such as would be experienced by small heavy particles and large light minerals, a short jigging cycle would be necessary for separation.

However, for coarser particles, longer strokes with decreased speed is found to give better stratification and hence it may be preferable to split the feed into closely sized fractions and have a jig optimised for each size fraction.

With a long stroke cycle particles will reach their terminal velocities which will depend on the particle density and size. Hindered settling conditions will prevail. By adjusting the upward flow of fluid the settling velocity of the fine light particles can be overcome and the fine particles will be carried upwards and away from the denser heavier particles. A stronger pulsation stroke will then allow only the large heavy particle to settle against the rising force of fluid. For particles having a similar terminal velocity such as the small heavy particles and the large light particles, separation by this means would not be possible.

Hindered settling is more significant for coarse particle separations where a slower stroke cycle is used, although with coarser feeds, the larger particles may not have time to reach their terminal velocities. The parameters that will effect hindered settling during jigging are particle size, density and shape, the fluid density and viscosity, the percent solids and the separator characteristics.

As the pulsation approaches the top of the stroke the upward velocity of water slows and particles will start to settle again starting with the particles of higher terminal velocity. The particles will then begin to compact down as they settle against the jig screen. The large particles pack together leaving large voids between them into which the smaller particles can continue to settle under gravity. This consolidation trickling will help to bring the fine heavy particles down into the heavy layer (Fig. 15.6) and if allowed to go for too long, will also draw the fine light particles down into the heavy layer and thus contaminate the heavy fraction.

Stratification during the dilation stage is controlled by hindered-settling classification with some modification by differential acceleration, and during the stage that the bed is compacted, it is controlled by consolidation trickling. The frequency of the jig cycle and the control of events within each cycle is critical in determining the behaviour of particles within the jig bed. A minimum cycle time is required to allow each phase of the cycle to be optimum for a given feed. Any further increase in cycle time would not be optimum or the bed would be in a compacted state and no further separation would occur during this interval hence affecting the capacity. Cycle speed adjustment is therefore the most important operating variable.

Parameters which determine the cycle frequency include the feed rate, feed size and density and the jig design. A jig pulsation is a case of simple harmonic motion. The period of pulsation can be given by the basic formula for simple harmonic motion based on a compound pendulum [5]:

\[
T = 2t = 2\pi \sqrt{\frac{L}{g}}
\]  

(15.20)

where \( T \) = period of pulsation or time for a complete pulsation cycle, 
\( t \) = duration of the stroke (half cycle), and 
\( L \) = distance between the centre of suspension of the compound pendulum and its centre of oscillation.
For a large jig, the tremendous mass of water in oscillating motion (up to 20 tonnes) must be allowed to follow its natural pulsation motion as expressed by Eq. (15.20) which says that the pulsation cycle must vary as the square root of the stroke length as shown in example 15.2.

**Example 15.2**

If the pulsation frequency is 60 strokes/minute, then:

\[ T = 1 \text{ s} = \frac{1}{\text{frequency}} \text{ and from Eq. (15.20):} \]

\[
L_1 = \frac{T_1^2 g}{4\pi^2} = \frac{1^2 \times 9.81}{4 \times (3.1415)^2} = 0.248 \text{ m}
\]

If the pulsation rate is to be halved, then \( T = 2 \text{ s} \) (30 pulsations/minute) and the stroke length should be increased according to:

\[
\frac{T_1}{\sqrt{L_1}} = \frac{T_2}{\sqrt{L_2}}
\]

\[
L_2 = \left( \frac{T_2}{T_1} \right) L_1 = \left( \frac{2}{1} \right)^2 0.248 = 4 \times 0.248 = 1.0 \text{ m}
\]

If this principle were not followed the result would be excessive stress on the walls of the jig and turbulence within the bed that would disrupt the separation and lower power efficiency.

15.3.1. Length of Pulsation Stroke

The length of the pulsation stroke can be calculated by [5]:

\[
v = \frac{Na\pi}{60} \quad (15.21)
\]

where \( v \) = velocity of water required to suspend the mineral bed,
\( a \) = amplitude of the stroke, and
\( N \) = number of strokes per minute.

Use of this equation is given in example 15.3.
Example 15.3
For a jig treating 5 mm coal and shale of density 1800 and 2500 kg/m$^3$ respectively, calculate the jig stroke amplitude required to expand the bed. The fluid is water at a density of 1000 kg/m$^3$ and viscosity 0.001 Pa s and the Newtonian drag force is 0.44.

Solution
The pulsation must produce a water velocity capable of raising the largest pieces of 5 mm shale. From Eq. (15.4), the terminal velocity of the largest particle is:

$$v_T = \sqrt{\frac{4 \times 9.81 \times (2500-1000) \times 0.005}{3 \times 0.44 \times 1000}} = 0.4722 \text{ m/s}$$

If we assume the pulsation rate $N = 60$ pulses/min, then by rearranging Eq. (15.21):

$$a = \frac{60v}{N \pi} = \frac{60 \times 0.4722}{60 \times 3.1415} = 0.15 \text{ m}$$

Thus a surge of about 150 mm would be required in the jig.

Table 15.5 shows some typical amplitudes and pulsation rates for some jig types.

Table 15.5
Some operating data for various jigs [2].

<table>
<thead>
<tr>
<th>Jig Type</th>
<th>Particle Size, mm</th>
<th>Amplitude, mm</th>
<th>Frequency, Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baum</td>
<td>5-200</td>
<td>30-40</td>
<td>30-60</td>
</tr>
<tr>
<td>Batac</td>
<td>0.5-100</td>
<td>30-60</td>
<td>40-60</td>
</tr>
<tr>
<td>Diaphragm</td>
<td>0.25-25</td>
<td>20-30</td>
<td>125-150</td>
</tr>
<tr>
<td>Diaphragm</td>
<td>0.2-10</td>
<td>10-15</td>
<td>150-200</td>
</tr>
</tbody>
</table>

In jigging it is desirable to have a high system density while the bed is fluidised. Lovell and Luckie [6] compared the settling velocities of two particles, to illustrate the effect of relative density with increasing medium density. For two particles of coal having densities of 1650 and 1550 kg/m$^3$ in air (density 1.239 kg/m$^3$), the concentration criterion (ratio of relative densities) is calculated as:

$$CC_{(air)} = \frac{1650-1.239}{1550-1.239} = 1.065$$

(15.22)

In water it becomes:
512

\[
CC_{\text{(water)}} = \frac{1650-1000}{1550-1000} = 1.182
\]

\{15.23\}

and in a pulp media of density 1540 it is:

\[
CC_{\text{(media)}} = \frac{1650-1540}{1550-1540} = 11.0
\]

\{15.24\}

Thus as the medium density approaches the density of the lighter mineral, the settling ratio of these particles will approach infinite. Such high media densities can only be maintained by generating a pseudo heavy liquid from contamination of the water with a build up of small heavy particles from the feed. Though this will benefit the separation, media viscosities will increase under these conditions which will retard particle flow and hinder separation.

### 15.3.2. Types of Jigs

Jigs are commonly used to clean coal but are also used in heavy mineral separations including alluvial gold. When treating coal, the light fraction is the concentrate and in the mineral industry, the heavy fraction is the concentrate. For this reason, gravity separation products will be referred to as light or heavy rather than the concentrate or tailing.

The jig is commonly an open tank filled with water, with a horizontal screen near the top. Some early jigs were designed where the screen surface, in the form of a basket, moved up and down in a barrel or tank of water hence producing the vertical flow of fluid through the bed of particles. This manual operation is reported in the 16th century work by Agricola [7]. Modern prospectors may still use this simple manual device in water drums or streams. Some movable screen jigs are still designed today, though most modern jigs employ a stationary screen and pulse the water through it. The differences between the various types of jigs available relate to the methods used to generate the pulsation and the manner in which the heavy fraction is removed from the jig. The screen is there to support the bed of particles and the area underneath the screen is called the **hutch**. The tank is usually divided into two main sections; one containing the support screen with the bed of ore and another section which generates the fluid pulse.

**Heavy mineral discharge**

The heavy discharge from the jig may be either *through the screen* or *over the screen*. In jigging through the screen, all particles in the feed are smaller than the screen aperture and thus have the potential to drop through the screen and collect in the hutch. To stop the light fraction falling through the screen, a false support is provided in the form of a layer of coarse heavy particles called **ragging** which when contacting the screen surface pack down to effectively close off the screen apertures to the feed particles. During the pulsation cycle, the ragging is also dilated and will allow the particles that have formed on top of the ragging, by segregation, to work their way through the ragging and the screen into the hutch. The ragging is usually, but not always, a material of density between the light and heavy fractions of the feed. An example of ragging material is lead shot for gold jigs, steel balls for cassiterite separation and natural materials such as feldspar for coal jigs and hematite for cassiterite and scheelite.

Large jigs are divided into separate compartments with different operating conditions for each compartment, such as roughing and scavenging duties. In large compartments, barriers
are provided on the screen to keep the ragging in place on the screen surface to prevent migration of the ragging to one side of the jig (Fig. 15.7). Up to four successive compartments are placed in series in the hutch. A high-grade heavy fraction may be produced in the first compartment with successive compartments producing higher recoveries but lower grades so that a final light fraction overflowing the final compartment can be discarded.

For feed particles larger than the apertures of the supporting screen, *jigging over the screen* may be practiced, and the heavy product grade is partly controlled by the thickness of the bottom layer which in turn is controlled by the rate of withdrawal through the heavies discharge port. Gates are operated to allow the heavy fraction to drop into a bucket elevator for removal. For coal separation, the light fraction is a *clean coal* product and the heavy fraction is a *reject* or shale product.

Positioning of the gate opening is controlled by the location of the boundary between the light and heavy layers and this is determined by a weighted float positioned in the bed or monitoring the pressure fluctuations in the pulsating water.

**Pulsation**

The pulsation of water or air through the mineral bed may be generated by rubber diaphragms, pistons or compressed air chambers. Examples of air actuated water pulsation units are the Baum and Batac jigs used extensively for coal separation. Here the settings of air and water are critical for efficient separation with large stroke amplitudes at the feed end for rapid stratification of the coarse shale and short stroke settings at the discharge end for precise stratification of *near gravity* material and fine coal.

Fig. 15.7. Jigging through the screen.
**InLine Pressure Jig**

The Inline Pressure Jig (IPJ) is a new application of the jigging principle with a completely enclosed and pressurised jig with a moveable screen action in a circular bed. The pressurisation of the unit, up to 200 kPa, allows the Inline Pressure jig to be completely filled with slurry and water which slows the slurry velocity and eliminates the air-water surface tension for potentially improved recovery. A hydraulic ram pulses the screen in the water with a jigging through the screen operation [8,9].

The advantages of the IPJ are a low water consumption, allowing operation in the recirculating stream of a grinding circuit, a high mass pull of up to 30% to the heavy fraction, feed capacity up to 110 tph, feed sizes up to 30 mm and low power consumption. When used for treating alluvial deposits of precious metals or gemstones, the completely sealed unit offers security.

**Centrifugal Jig**

In 1990, Kelsey introduced the first commercial unit that incorporated a centrifugal force to jigging. The Kelsey Centrifugal Jig (KCJ) operates at up to 40 times gravitation acceleration in order to extend the separation range of gravity separation down to less than 40 microns.

The Kelsey Centrifugal Jig utilises the Harz design which is divided into two parts, the top section of heavy mineral (ragging) above a screen, and the jig chamber filled with water and pulsed by a diaphragm plunger. The screen and hutch arrangement is turned 90° from the horizontal to the vertical and spun about a vertical axis. Gains achieved with the KCJ include:

1. Operating and maintenance cost savings
2. Improved recovery
3. Improved final concentrate grade, and
4. Simplifying the processing circuit.

**15.3.3. Operations**

The control of a jig separation is determined by the water addition, stroke frequency and amplitude, the feed rate and the ragging layer. Water is added to the jig as either top water (water added above the screen) or back water (water added beneath the screen or hutch water). The total water flowing across the top of the jig bed is the cross water. This cross water controls the horizontal flow of particles across the top of the bed. The back water reduces the effect of the suction part of the cycle and hence affects the falling water velocity relative to the rising water velocity during the pulsation part of the stroke.

The feed rate must be matched with the discharge rate of the heavy fraction so that a steady state operation can be maintained. If the discharge of the heavy fraction does not keep up with the heavy particles reaching the separated layer then this layer will build up until ultimately some heavy minerals will be lost to the light fraction. Conversely if the discharge rate of heavies through the ragging or through the discharge gate is greater than the rate of segregation of heavy particles into the separation layer then some light particles will eventually be drawn into the heavy fraction, lowering the grade.

The stroke length and frequency are linked according to Eq. (15.21) and the stroke length must be sufficient to produce the amount of bed dilation required for separation.

The size, size distribution, shape and density of the ragging are all important factors that will affect the separation. The deeper and heavier the ragging layer and shorter the pulsation stroke the more difficult it will be for particles to penetrate the ragging and hence the slower will be the heavy fraction discharge. The ragging size is about 3-4 times the maximum
particle size in the feed. Table 15.6 summarises the effect of a number of these variables on the jig operation.

Table 15.6
Effect and operating range of some operational variables on jig separation.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Effect on Jig operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ragging density</td>
<td>increasing</td>
<td>decreases heavy fraction flow</td>
</tr>
<tr>
<td>Ragging size</td>
<td>increasing</td>
<td>increases heavy fraction flow</td>
</tr>
<tr>
<td>Ragging depth</td>
<td>increasing</td>
<td>decreases heavy fraction flow</td>
</tr>
<tr>
<td>Ragging contamination</td>
<td>increasing</td>
<td>decreases heavy fraction flow</td>
</tr>
<tr>
<td>Feed size</td>
<td>50 mm – 20 mm</td>
<td>normal range for heavy mineral separation</td>
</tr>
<tr>
<td>Feed size</td>
<td>0.5 – 200 mm</td>
<td>normal operation for coal</td>
</tr>
<tr>
<td>Capacity</td>
<td>17 – 25 t/h/m²</td>
<td>normal for tin</td>
</tr>
<tr>
<td>Capacity</td>
<td>30 – 60 t/h/m²</td>
<td>normal for coal</td>
</tr>
<tr>
<td>% solids</td>
<td>30 – 50%</td>
<td>normal operation</td>
</tr>
<tr>
<td>Hutch water</td>
<td>increasing</td>
<td>increases recovery to a maximum</td>
</tr>
<tr>
<td>Hutch water</td>
<td>increasing</td>
<td>increases enrichment ratio</td>
</tr>
</tbody>
</table>

Taggart [1] gives an estimate of the power consumption of a jig as:

\[ P = 7310.16 A \sqrt{d} \]

where

- \( P \) = power in watts,
- \( A \) = screen area in \( m^2 \) and
- \( d \) = feed size in \( m \).

15.4. Differential Motion Table Separators

Wet concentrating tables developed from continuous belt concentrators which utilised a flowing film of water to effect a separation. The ore moved up an incline slope on an endless belt where the lighter minerals were washed away from the heavy minerals by a film of water flowing down the belt, similar to the \( Strake \) tables. The \( Vanner \), a vibrating continuous belt was developed in the 1860’s and bumping tables followed before the modern differential shaking table was developed by Wilfrey in 1896.

A bed of particles which experiences a horizontal shaking motion will undergo segregation on the basis of size and density, for example a gold pan and particles of a conveyor belt. If the particles are of the same density then particles will segregate according to size with the fine particles sinking and the coarse particles rising to the top (Fig. 15.8). If particles of different density exist in the mixture then particles of higher density will sink to a lower level than similarly sized but lighter density particles. To achieve this stratification, the shaking motion must be strong enough to expand the bed to the extend that allows particles to penetrate. The shaking motion however must still maintain a particle to particle contact. The fact that small particles of light mineral and large particles of heavy mineral segregate to the same position in the bed suggests that density is not the sole separating force.

15.4.1. Shaking Tables

In the shaking table concentrator, differential motion and a riffled deck with cross flowing water is used to create a particle separation. The shaking motion is asymmetrical, being slow
in the forward direction and rapid in the reverse direction. This differential motion imparts a conveying action to the table moving those particles which are in contact with the table deck, through friction, in the direction of the motion.

The Wilfley table was designed for ore concentration and since the early days of its development has been used extensively for this purpose. When the table is equipped with decks specially designed for coal washing it was known as a Massco table. Since the introduction of the Wilfley table many different makes of tables have been developed for use with minerals and coal.

The table consists of a slightly inclined flat surface or deck with a series of parallel ridges or riffles along the direction of motion (Fig. 15.9). The riffles are tapered towards the opposite end to the reciprocating drive. Feed is introduced at the corner of the table at about 25% solids (by mass) and with the shaking motion, the particles spread out over the table. Wash or dressing water is introduced along the top edge of the deck to assist in segregation and transport of particles on the table. The net effect is that the particles move diagonally across the deck from the feed end.
As the feed material spreads out over the table the particles stratified in layers behind the riffles. The riffles help to transmit the shaking motion to the particles and prevent the particle washing directly off the table. Successive layers of particles are removed from the top of the riffles by the cross-flowing water as they become exposed by the shortening riffle height as the bed moves away from the feed end of the table. When the remaining particles reach the end of the riffles only a thin layer remains on the table surface. If the table has a smooth unriffled end, then the flowing film of water further cleans the heavy particles before discharging them off the end of the table.

The separating action on a shaking table combines the selective action of the cross-flowing water film (flowing-film) and stratification and hindered settling behind the riffles.

15.4.2. Stratification and Hindered Settling
Stratification due to the nearly horizontal action of the table deck and the flow of water is not the only mechanism at work on the table. There is some suggestion that hindered settling may also assist in the separation in some minor way. The stratification due to the shaking motion of the deck and flow of water is referred to as table stratification. Under this process, the small particles will segregate towards the bottom of the bed, behind the riffles, while the large particles collect towards the top. For a mixture of mineral densities in the feed, there will be a mid-layer of particles where the large heavy and small light particles will overlap as indicated in Fig. 15.8. As the cross flowing water flows over the riffles it can cause eddy currents to penetrate the mobile bed before rising to flow over the next riffle. This rising current of water can lift the finer particles to higher positions in the bed by a hindered settling type action and this can assist in the segregation of heavy and light minerals. This effect of hindered settling along any individual riffle is likely to be small but the cumulative effect along the entire series of riffles on the deck might be sufficient to effect the separation of the fine light particles away from the large heavy particles in the bed.

Hindered settling on a table is more effective if the particles in the feed are closely sized. Classification of the table feed improves the performance of the table and increase the capacity.

15.4.3. Operating parameters
Factors which affect the operation of the shaking concentrator are particle size and density, particle shape, the riffle design, deck shape, water and feed flow, stroke and speed of the table and the deck slope. The effect of these variables are summarised in Table 15.7. The correct operation of the table has the middling fraction discharged at the diagonally opposite corner of the table to the feed. For any feed variation, the operating variables are adjusted to maintain this separation point.

The particle shape is not a major factor in the overall tabling process however flat particles do not roll easily across the deck and would tend to be carried along to the heavy mineral discharge end of the table.

Of considerable more importance is the particle size. In both table stratification and hindered settling, the separation of particles becomes more difficult as the range of sizes in the feed increases. If a table feed contains too wide a range of sizes, some size fractions will be separated inefficiently. For efficient table operation, a normal feed size for coal treatment ranges from 0.3 – 9.5 mm [10]. The lower size limit for an effective separation on a table is about 50 μm even if the density difference is high.

For optimum table operation, the feed flow of solids and water onto the table must be uniform and constant. Approximately 90% of the water reports to the light fraction. The
dressing water represents approximately 25% of the total water on the table. The table capacity varies according to the size of the feed particles. Tables can handle up to 2 t/h of 1.5 mm sand and 1 t/h of fine sand. Capacities can be as low as 0.5 t/h for a slimes feed.

Table 15.7
Effect of variables on table performance [2].

<table>
<thead>
<tr>
<th>Variable</th>
<th>value</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deck shape</td>
<td>diagonal</td>
<td>increased capacity</td>
</tr>
<tr>
<td></td>
<td></td>
<td>increased grade</td>
</tr>
<tr>
<td></td>
<td></td>
<td>lower middling flow</td>
</tr>
<tr>
<td></td>
<td></td>
<td>finer size separation</td>
</tr>
<tr>
<td>Riffling</td>
<td>partial deck</td>
<td>cleaning duty</td>
</tr>
<tr>
<td></td>
<td></td>
<td>treats unsized feed</td>
</tr>
<tr>
<td></td>
<td>full deck</td>
<td>roughing duty</td>
</tr>
<tr>
<td></td>
<td></td>
<td>treats sized feed</td>
</tr>
<tr>
<td>Feed rate</td>
<td>2 t/h</td>
<td>for 1.5 mm sand</td>
</tr>
<tr>
<td></td>
<td>0.5 t/h</td>
<td>for -150 μm slimes</td>
</tr>
<tr>
<td></td>
<td>15 t/h</td>
<td>for up to 15 mm coal</td>
</tr>
<tr>
<td>Speed &amp; Stroke</td>
<td>260-300 strokes/min</td>
<td>for coarse ore</td>
</tr>
<tr>
<td></td>
<td>12-25 mm</td>
<td></td>
</tr>
<tr>
<td></td>
<td>280-320 strokes/min</td>
<td>for fine ore</td>
</tr>
<tr>
<td></td>
<td>8-20 mm</td>
<td></td>
</tr>
<tr>
<td></td>
<td>260-285 strokes/min</td>
<td>for coal</td>
</tr>
<tr>
<td></td>
<td>20-35 mm</td>
<td></td>
</tr>
<tr>
<td>slope (length) &amp;</td>
<td>11-25 mm/m</td>
<td>coarse sand</td>
</tr>
<tr>
<td>tilt (cross)</td>
<td>20-25 mm/m</td>
<td></td>
</tr>
<tr>
<td></td>
<td>9-15 mm/m</td>
<td>medium sand</td>
</tr>
<tr>
<td></td>
<td>15-30 mm/m</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2-9 mm/m</td>
<td>fine sand</td>
</tr>
<tr>
<td></td>
<td>8-20 mm/m</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1-7 mm/m</td>
<td>slimes</td>
</tr>
<tr>
<td></td>
<td>4-12 mm/m</td>
<td></td>
</tr>
<tr>
<td>water/solid ratio</td>
<td>20-25% (mass)</td>
<td>mineral separation</td>
</tr>
<tr>
<td></td>
<td>33-40% (mass)</td>
<td>coal separation</td>
</tr>
</tbody>
</table>

The stroke rate for normal operation is between 250-300 strokes per minute. The stroke length required for coal separation can ranges from 10-25 mm. A longer stroke moves the reject (heavy) to the heavy discharge end of the table more rapidly, but requires more water. The amplitude and stroke frequency are interdependent. That is, an increase in stroke length requires a decrease in the stroke frequency to maintain the same transportation speed of the heavy fraction to the discharge point. A fine feed will generally require a higher speed and shorter stroke than a coarse feed. For difficult separations, where the density difference between the two fractions is small or the feed size is small, a short stroke length must be used.

Early tables were generally covered with linoleum with wooden riffles. Modern tables use either rubber riffles cemented to a rubber covered deck or the whole deck is moulded in fibreglass.
Table 15.8 gives a general guide to table operation.

### Table 15.8

Table settings for various duties.

<table>
<thead>
<tr>
<th>Duty</th>
<th>Operating conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>roughing</td>
<td>increase water flow&lt;br&gt;increase feed rate&lt;br&gt;increase tilt&lt;br&gt;increase stroke&lt;br&gt;use fully riffled deck</td>
</tr>
<tr>
<td>cleaning</td>
<td>decrease water flow&lt;br&gt;decrease feed rate&lt;br&gt;decrease tilt&lt;br&gt;decrease stroke&lt;br&gt;use partially riffled deck</td>
</tr>
<tr>
<td>fine feed</td>
<td>decrease water flow&lt;br&gt;decrease feed rate&lt;br&gt;increase speed&lt;br&gt;decrease stroke&lt;br&gt;use low profile ruffles</td>
</tr>
<tr>
<td>coarse feed</td>
<td>increase water flow&lt;br&gt;increase feed rate&lt;br&gt;decrease speed&lt;br&gt;increase stroke&lt;br&gt;use high profile ruffles</td>
</tr>
</tbody>
</table>

#### 15.4.4. Types of Tables

A number of different types of tables are available for different applications, and these vary mainly in relation to the type of head motion used.

**Sand Tables**

A sand table is used for treatment of coarse particles, greater than 100 μm. Some types of sand tables are the Diester table used extensively in the coal preparation industry is capable of a longer stroke than a standard Wilfley table, which is required for the concentration of the coal particles. The head motion of the Holman or James table is applied to the corner of the table rather than to the centre as in a normal Wilfley table. In cases where floor space is at a premium, tables can be mounted in vertical stacks of two or three high.

**Slimes Tables**

The treatment of slime particles (< 100 μm) on any gravity separation device is difficult. The separations achieved are not efficient but before the introduction of flotation or centrifugal devices, slime tables were used. The basic principles of slimes tabling are:

1. the deck area required varies inversely as the feed size,
2. the finer the feed, the gentler and slower the table action must be,
3. the feed size distribution must be even and channelling avoided,
4. the flocculation characteristics of the feed affect its response to gentle flowing action.
The concentration criterion for a mixture of quartz and cassiterite is 3.5 and at less than 50 microns particle size, this is not sufficient to give sharp separation. The concentration criterion for a quartz-gold mixture is nearly 9, and good separation is possible at this size.

Slime tables have a gentler slope and a series of plane surfaces rather than ripples on the deck. The concentrate and/or middlings produced by a slime table often require further treatment. A number of units have been designed to use the principle of a flowing film of water on a flat deck with a shaking motion to concentrate these slimes. Units such as the Bartles-Mozley table and the Bartles Crossbelt concentrator were first introduced for tin processing around 1967. Cassiterite is a friable mineral which readily produces slime size particles.

The Bartles-Mozley separator consists of two stacks of twenty lightweight fibreglass decks. Each deck surface is smooth with dimensions of 1.1 m by 1.5 m long and separated by a gap of 12 mm. The stacks are suspended within a steel framework at an angle of 1-3 degrees. A motor and out of balance weight drives the stack with an orbital motion. The operation is a batch process with feed on for up to 35 minutes then the feed is shut off and the stack assembly is tilted with flushing water to remove the heavy fraction before the cycle starts again. The Bartles-Mozley concentrator is used as a roughing device and the concentrate is cleaned on a Crossbelt concentrator. It is claimed to recover particles down to 5 µm at a rate up to 5 t/h [11].

The heavy fraction from the Bartles-Mozley separator is treated on the Bartles Crossbelt concentrator, introduced in the mid-1970's [12]. The crossbelt is a 2.5 m wide endless belt which is raised along a central ridge to provide a gentle slope towards the two edges of the belt. The belt moves slowly and is also shaken with an orbital motion between two end pulleys approximately 3 m apart. The unit feed is introduced along the central ridge and the light particles which are suspended in the water flow by the orbital motion, flow down the slope and discharge off the edge of the belt. The clean heavy fraction is discharged over the head pulley.

![Graph](image)

Fig. 15.10. Comparison of the size recovery for a Bartles Crossbelt concentrator and a standard slime table, treating cassiterite slimes [12].
A comparison of the performance of the Bartles Crossbelt performance with a standard slimes table, treating a feed of 0.7% Sn grade and 97% -40 μm, showed a similar overall recovery of just 55% but the Crossbelt produced a grade of 32% Sn compared with just 13% Sn for the slime table [12]. Fig. 15.10 compares the size by size recovery of the two units and indicates the superior performance of the Bartles Crossbelt in the 5-10 micron range but has a worse performance above 20 microns.

**Gemeni Gold Table**

The Gemeni table has grooves on the table deck instead of riffles. It thus behaves like a mechanised gold pan in that the heavy gold is trapped in the grooves and the light gangue is washed over the groove and off the table. The gold migrates from groove to groove working its way along the table by the action of the shaking mechanism. The wash water is introduced along the centre of the table, dividing the table into two sloping surfaces. Fig. 15.11 shows the shape of the table and movement of the particles on the deck.

The Gemeni table is a low capacity cleaning device. The table is used in the secure gold room for the upgrading of gold concentrates from other gravity concentration devices. The advantage is that a very high grade at high recovery is achieved and this can be directly smelted to produce gold bullion. The feed size is less than 850 microns for a top feed rate of 0.12 t/h for the Gemeni 250 which is 1.32 m wide by 2.0 m long.

15.5. **Flowing Film Concentrators**

Settling and separation of particles in a flowing film of fluid form the third classification of gravity separation processes. When a film of water flows down a smooth surface under laminar flow conditions, the velocity gradient across the film thickness is approximately parabolic. That is, the velocity decreases at positions close to the slope surface due to friction with the surface and also decrease at positions close to the air/water interface, also due to friction with the air (Fig. 15.12). For turbulent flow conditions, the velocity profile across the thin film is flatter but still decreases towards the deck surface due to friction or drag.

Particles of different density and size that are dropped into the flowing film will reach the slope surface at different points because of the difference in settling rate. Particles that take longer to settle such as the fine light particles will be carried further down the slope than particles that are faster settling (Fig. 15.13A). For equi-settling particles such as large light particles and small heavy particles, these will contact the slope surface at the same position.

Once the particles start to roll down the slope they will be influenced by their size as smaller particles will be closer to the surface and experience a slower water velocity than large particles which extend further into the water film where the water velocity is greater (Fig. 15.13B). Thus large particles will be pushed down the slope faster than the smaller particles and a new arrangement will eventually form as shown in Fig. 15.13C.

If the particle shape is flat then it will have less tendency to roll and will complicate the separation.

15.5.1. **Simple Sluice**

The sluice is the simplest implementation of the flowing film principle. This consists of an inclined trough open at both ends. Solid and water are fed in at the top and a flowing film separation occurs on the sloping surface. The completeness of the separation depends on the density and size of the particles and the amount of water and length of the sluice. Tin sluices may be up to 100 m in length. The sluice carries a bed of material many particles thick and
Fig. 15.11. Gemeni gold table.

Fig. 15.12. Velocity profile in a flowing film of water

Fluidisation of the bed, which allows segregation of the heavy and light particles in the bed, is produced by the flow of water. This fluidisation is enhanced by having a rough textured surface on the slope such as a layer of pebbles or a cross-riffle section. These obstructions introduce turbulent flow and eddies and provide obstacles against which particles in the bottom layer of the bed can be trapped while the upper layers flow over with the water (Fig. 15.14).
Fig. 15.13. Effect of particle size, density and water velocity on particle segregation in a flowing film of water (● heavy particles, ○ light particles). A – particles of different settling rate; B – different size particles pushed with different force; C – final segregated layers in a flowing film.

Feed sizing is an important pre-treatment to efficient gravity separation by a film flow. When this is satisfied, the smallest and heaviest particles will work their way to the bottom behind any obstruction as the water flowing over the obstruction lifts the lighter particles towards the top and eventually over the obstacle.

Fig. 15.14. Effect of a riffle on segregation on a sluice.
The main control factors in sluicing are the slope of bed, the water film thickness, the roughness of the surface, the density difference between the valuable mineral and gangue and the pulp density.

The simple sluice or sluice box is a batch process. After a period of time, some heavy mineral will start to wash over the riffle and be lost in the light tail. Thus, after a certain time the flow is stopped and the heavy minerals collected. The clean-up frequency depends on the feed grade, the feed rate and the size of the sluice and can vary from once a day to once every several weeks.

15.5.2. Strake Table
The gold stroke consists of a stationary flat sloping surface. The light mineral particles roll down, while the heavier ones are held on a roughened surface such as a removable cloth covering. For example, corduroy cloth was used to cover the strake surface, where the ribs of the corduroy were laid perpendicular to the direction of flow. The variables affecting the separation are the slope, feed size, pulp density and the feed rate.

The cloth covering was subsequently replaced with a rubber belt which had ribs moulded into the surface lying across the flow direction to act as riffles. The rubber belt is continuous and driven at a slow rate up the inclined surface. The water film washed the light particles down the slope and the heavy particles caught behind the ribs on the belt are moved up to discharge off the top of the table. The feed % solids of the pulp is 20-50%.

15.5.3. Spiral Concentrator
The spiral concentrator first appeared as a production unit in 1943 in the form of the Humphrey Spiral, for the separation of chrome-bearing sands in Oregon. By the 1950's, spirals were the standard primary wet gravity separation unit in the Australian mineral sands industry.

In the spiral concentrator the length of the sluicing surface required to bring about segregation of light from heavy minerals is compressed into a smaller floor space by taking a curved trough and forming into a spiral about a vertical axis. The slurry is fed into the trough at the top of the spiral and allowed to flow down under gravity. The spiralling flow of pulp down the unit introduces a mild centrifugal force to the particles and fluid. This creates a flow of pulp from the centre of the spiral outwards to the edge. The heaviest and coarsest particles remain near the centre on the flattest part of the cross-section, while the lightest and finest material is washed outwards and up the sides of the launder (Fig. 15.15). This separation may be assisted by the introduction of additional water flowing out from the centre of the spiral either continuously or at various locations down the length of the spiral. This wash water may be distributed through tubes or by deflection from a water channel that runs down the centre of the spiral. Some present designs have overcome the need for this wash water. Once the particle stream has separated into the various fractions, the heavy fraction can be separated by means of splitters at appropriate positions down the spiral. A concentrate, middlings and tailing fraction can be recovered.

In practice spirals are arranged in stacks or modules of roughers, scavengers and cleaners, where the initial concentrate is retreated to upgrade the fraction to its final grade. Spiral length is usually five or more turns for roughing duty and three turns in some cleaning units. For coal concentration, 6 turns providing a gentler slope with longer residence time for the more difficult separation.

The performance of spirals is dependent on a number of operating parameters, summarised in Table 15.9. Spirals generally achieve an upgrade ratio of 3:1 (heavy fraction:feed grade)
and hence multiply treatments are required [13]. The presence of slimes adversely affects the spiral performance. More than 5% of -45 µm slimes will affect the separation efficiency.

With the steep pitch of a spiral, two or three spirals can be wound around the same common column and these types of spirals have been used in Australia for more than 20 years. The multistart spirals conserve floor space and launder requirements. These triple-start spirals are built into a twelve spiral module and for these modules, the design of the distributor is critical to ensure that each spiral has a uniform feed.

The splitter blades on these spirals are all adjustable to direct the heavy fraction into pipes or a collecting launder. The current range of spirals available consist of a number of different profiles which have individual separation characteristics. The dimensions of some of the available spirals range from 270 – 406 mm pitch, 590 - 700 mm diameter and 2.1 – 2.4 m high.

The advantages that modern spirals offer are simple construction requiring little maintenance, low capital cost and low operating cost - no reagents required, no dense media losses occur, low operating personnel required.

Table 15.9
Summary of spiral operating parameters.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>feed size</td>
<td>0.075 – 3 mm</td>
<td>optimum size for coal</td>
</tr>
<tr>
<td></td>
<td>0.045 – 0.85 mm</td>
<td>size range for minerals</td>
</tr>
<tr>
<td>feed rate</td>
<td>1 – 5 t/h</td>
<td>design capacity</td>
</tr>
<tr>
<td>feed % solids</td>
<td>20 – 30% solids (mass)</td>
<td>less for fine sands</td>
</tr>
<tr>
<td></td>
<td>up to 50%</td>
<td>for coarse feed</td>
</tr>
<tr>
<td></td>
<td>high % solids</td>
<td>high recovery (heavy) low grade</td>
</tr>
<tr>
<td></td>
<td>low % solids</td>
<td>high heavy fraction grade</td>
</tr>
<tr>
<td>splitter position</td>
<td>depends on feed properties and required duty</td>
<td></td>
</tr>
</tbody>
</table>
15.5.4. Cone-Separators or Reichert Cone

The Reichert Cone concentrator was developed in Australia in the 1960's to treat mineral sands. In principle the cone is similar to the sluice in that a slurry stream flows over a sloping surface causing the heavy particles to sink to the bottom of the bed. The separating surface in the Reichert cone is in the form of an upward sloping truncated cone. The heavy minerals, in close contact with the cone surface pass through an annular slot near the bottom of the cone while the light fraction flows over the slot to be treated in another stage or permanently removed. The separation efficiency of the cone is relatively low therefore the fractions (light or heavy fraction or both) are re-treated over several cone sections arranged vertically to allow several stages of cleaning.

The cones are made from fibreglass ranging from 2–3.5 m in diameter. The cones are mounted in frames up to 6 m high. Reichert cones are used in the mineral sand industry, in the treatment of alluvial and placer sands to recover gold, tin or other heavy minerals. The advantages of the cones are:

1. capacity in the range 40–300 t/h with a feed density of 55–70% solids by mass. The cones can handle feed sizes from 30 μm to 3 mm but are most efficient in the range 100–600 μm.
2. low installation cost per unit capacity
3. very low operating cost per tonne treated (cf. spirals and tables)
4. on stream adjustable inserts allow for variations in feed
5. high density streams ensure minimum energy consumption
6. environmentally acceptable, since no reagents used.

15.5.5. Centrifugal Separators

Poor separation of heavy minerals occurs at fine particles sizes. This poor performance can be overcome to some extent by increasing the settling rate of the fine particles by using centrifugal acceleration rather than gravitational acceleration. This lead to the development of the centrifugal separator.

The Knelson Concentrator was one of the first of this type of concentrator to find commercial success and is a highly efficient centrifugal separator for free gold or other heavy mineral recovery. The concentrator is a compact high-capacity centrifugal separator with an active fluidised bed to capture heavy minerals. A centrifugal force equal to or exceeding fifty times the gravitational force acts on the particles enhancing the specific gravity difference between the heavy particles and the gangue. The strong centrifugal force traps the heavy mineral in a series of rings located in the spinning drum while the gangue overflows the rings and is flushed out.

The concentrator has the capability to treat 40 t/h of minus 6 mm feed material and will recover in excess of 95% of the precious metal values. Gold particles from 6 mm to minus 30 μm are more efficiently recovered than with other known gravity method at high capacity. Even fine, platy or flour gold particles can be recovered.

The hydraulic section forms a self-cleaning fluidised bed that efficiently entraps high-density minerals while pushing out lighter material. This eliminates problems previously encountered with other centrifugal concentrators, including frequent shutdowns to remove black sands, low throughput and low concentrate ratios in the concentrate. The concentrate grade can be as high as 1,000 times the feed grade.

The original design was a batch operation but the latest designs offer semi-continuous and continuous extraction of the heavy fraction. These concentrators are popular units in the
grinding circuit of gold plants that contain free gold. The cyclone underflow, or a percentage of the cyclone underflow is passed through these concentrators with the concentrate going to the gold room for further upgrading or smelting and the tailings going back to the ball mill. This removes the coarse gold which would remain in the mill circulating load due to the high density of gold. These devices are suitable to capture recirculating gold in the grinding circuit because of their high % solids feed capability allowing a minimum of additional water into the high % solids grinding streams.

Similar devices are the Falcon Concentrator which uses a smooth sided bowl and the Gekko InLine Spinner which uses a riffled bowl and a cutter bar to create turbulence near the bowl surface to enhance the displacement of light particles by heavy particles.

### 15.5.6. Mazley Multi-Gravity Separator (MGS)

The MGS is designed for fine and ultra fine mineral separation. It is essentially an enhanced gravity separation device using the flowing film and shaking table principle. It can be looked upon as essentially wrapping the horizontal concentrating surface of a conventional shaking table into a cylindrical drum and rotating it. A force of many times greater than normal gravity is exerted on the particles in the flowing film thus greatly increasing the fine particle separation process. An additional shaking action adds shear to the particles increasing the separation process. These actions enable the MGS to recover particles down to 1 μm in diameter with at least 1.0 S.G. difference. The capacities are up to 30 t/h for mineral separation and 50 t/h for coal. Advantages of the Multi-Gravity Separator are quoted as:

1. high recovery: Typically better than 95% of free gold is recovered in a single pass (better than 99% for high grade)
2. high capacity
3. compact size
4. simple design, only one moving part
5. low cost per unit throughput
6. easy cleanup, typically five minutes
7. low maintenance, all wear parts are easily replaced
8. low labour requirement, no specific skills required
9. insensitive to feed variations.

### 15.6. Dense (or Heavy) Media Separation

#### 15.6.1. Introduction

The previous sections have explored the separation of minerals of different density in a fluid medium where the density of the fluid is less than the density of either mineral. If the density of the medium lies between the densities of the two minerals, then separation becomes much simpler since the lighter mineral will float in the fluid medium and the heavy mineral will sink. Hence separations of this kind are referred to as sink-float separation.

The medium used for the separation depends on the specific gravity of the minerals and may be made up of dissolved salts such as calcium chloride in water, where densities around 1350 kg/m³ may be produced or zinc chloride up to 1800 kg/m³. This medium density is low but has been used for the separation of coal. Heavy organic liquids have been proposed for industrial use but for reasons of toxicity and cost these are restricted to small scale laboratory use and even here they are becoming less utilised because of the toxic or carcinogenic nature of the organics. Table 15.10 summarises the properties of the heavy liquids used.
The tungstate based inorganic heavy liquids are regenerated by driving off some of the water. In this case the LST is preferred as it is more thermally stable than SPT or LMT and has a lower viscosity. The LST liquid has a pH of around 4 which may have to be considered when treating some minerals such as carbonates and sulphides [14].

Densities up to 12.0 can be achieved for separation of non-magnetic minerals by the use of magnetohydrostatics. This is produced in a paramagnetic salt solution or a ferrofluid by the application of a magnetic field gradient. A ferrofluid is a suspension of a ferromagnetic material, such as magnetite, in a fluid such that the slurry behaves like a magnetic fluid. The particles have an average size of about 10 nm so that they form a stable suspension. The particles are coated with a stabilising dispersing agent or surfactant which helps to prevent particle agglomeration when a strong magnetic field gradient is applied to the fluid.

In a magnetic field the whole fluid responds as a homogeneous magnetic liquid. In magnetohydrostatic separation, the ferrofluid is placed in a non-uniform magnetic field where it experiences an increasing attraction force in the direction of the higher field intensity. If the field strength is in the direction of the gravitational force then the apparent density of the fluid is the sum of the actual density and the magnetic field. This means that the apparent density of the fluid can be adjusted simply by adjusting the magnetic field strength. This type of high density medium is applicable to the separation of non-magnetic particles down to about 50 μm.

15.6.2. Pseudo Heavy Liquids
More commonly where high medium densities are required, pseudo liquids are used which consist of a suspension of finely divided high density particles in water. One of the earliest dense mediums used for the cleaning of coal was -75 μm magnetite in 1922. This was not successful at that time since there was no simple method of keeping the medium clean. In the 1930's the Barvoys process was developed in Holland, which used a mixture of clay (S.G. 2.3) and finely ground (75 μm) barytes (S.G. 4.2) in a ratio of 2:1 which gave specific gravities up to 1.8. The barytes-clay mixture formed an almost stable pulp which gave an accurate cut between sink and float fractions. Froth flotation was used to regenerate the fouled dense-media by removing fine coal. Another Dutch process, the Tromp process used a finely ground magnetite or specially treated pyrite, to produce a less stable fluid. The settling characteristics of the medium caused the bath density to be lower at the top than at the bottom which tended to aid the separation.

In the treatment of ores other than coal, the lightest mineral is usually quartz or a silicate at a density between 2600 and 3500 kg/m³. Therefore the substances used to form the dense media must be far denser than used for coal treatment, as the working density of the bath is twice as high as that used for coal cleaning. For example, in coal separation the media fluid needs a density of around 1500 kg/m³ while separation of galena/sphalerite from gangue silicates requires a fluid density around 2850 kg/m³, diamonds from kimberlite rock around 2780 kg/m³ and hematite from silicate gangue around 3000 kg/m³.

Any substance used for media must have the following characteristics:

1. **Hardness:** It must not easily break down or abrade into a slime under working conditions.
2. **Chemical Stability:** It must not be chemically corrosive or liable to react with the ore minerals undergoing treatment.
3. **Slow settlement at reasonable viscosity:** It must form a fairly stable pulp without having to be ground very fine, otherwise the medium will be too viscous.
4. **Specific gravity**: It must have high enough specific gravity to give the required bath density at low % solids, again to minimise the viscosity.

5. **Regeneration**: The dense media must be easy to clean for recycling.

6. **Price and availability**: The solid should be readily available and cheap. Losses of up to 0.5 kg/t could lead to high operating costs.

<table>
<thead>
<tr>
<th>Heavy liquid</th>
<th>Formula</th>
<th>S.G.</th>
<th>Dilution</th>
<th>Health</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tri-chloro-ethylene</td>
<td>CCl2CHCl</td>
<td>1.46</td>
<td>-</td>
<td>group 2A carcinogen</td>
</tr>
<tr>
<td>Carbon-tetrachloride</td>
<td>CCl4</td>
<td>1.5</td>
<td>Most</td>
<td>group 2B carcinogen</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>organic</td>
<td>liquids</td>
</tr>
<tr>
<td>Bromoform, Tribromomethane</td>
<td>CHBr3</td>
<td>2.87</td>
<td>Alcohol,</td>
<td>liver damage, group 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>CCl4</td>
<td></td>
</tr>
<tr>
<td>Tetrabromoethane (TBE)</td>
<td>C2H3Br4</td>
<td>2.95</td>
<td>Alcohol,</td>
<td>suspected carcinogen</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>CCl4</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Chloroform</td>
<td></td>
</tr>
<tr>
<td>Di-iodo methane (Methylene iodide)</td>
<td>CH2I2</td>
<td>3.31</td>
<td>CCl4,</td>
<td>moderate toxicity</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Benzene</td>
<td>central nervous system</td>
</tr>
<tr>
<td>Clerici solution (thallium malonate/thallium formate)</td>
<td>(TlCOOH)2C/TlCOOH</td>
<td>4.2</td>
<td>Water</td>
<td>highly toxic, cumulative poison.</td>
</tr>
<tr>
<td>lithium heteropolytungstate (LST)</td>
<td>LiₙXₙ(W₁₂O₄₀)</td>
<td>2.95</td>
<td>Water</td>
<td>Low to moderate toxicity</td>
</tr>
<tr>
<td>sodium polytungstate (SPT)</td>
<td>Naₙ(H₂W₁₂O₄₀)</td>
<td>3.1</td>
<td>Water</td>
<td>Low to moderate toxicity</td>
</tr>
<tr>
<td>lithium metatungstate (LMT)</td>
<td>Liₙ(H₂W₁₂O₄₀)</td>
<td>3.0</td>
<td>Water</td>
<td>Low to moderate toxicity</td>
</tr>
</tbody>
</table>

Material suitable for use as media are listed in Table 15.11.

<table>
<thead>
<tr>
<th>Material</th>
<th>SG</th>
<th>Regeneration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Galena</td>
<td>7.4–7.6</td>
<td>Froth-flotation</td>
</tr>
<tr>
<td>Magnetite</td>
<td>5.0–5.2</td>
<td>Magnetic separation</td>
</tr>
<tr>
<td>Mill-scale</td>
<td>~5</td>
<td>Magnetic separation</td>
</tr>
<tr>
<td>Ferro-silicon</td>
<td>6.3–7.0</td>
<td>Magnetic separation</td>
</tr>
<tr>
<td>Pyrite</td>
<td>5.02</td>
<td>-</td>
</tr>
<tr>
<td>Quartz sand</td>
<td>2.65</td>
<td>-</td>
</tr>
</tbody>
</table>
An important consideration in any heavy liquid used for gravity separation is the viscosity. The viscosity must be low enough to allow the particles to separate quickly and not become misplaced. For example, when pure galena is used, a specific gravity of 4.3 is obtainable but the practical limit is about 3.3. Above this S.G. the viscosity became too great to effectively deal with small particle sizes particularly those particles having a density close to that of the medium (near gravity particles). This will place a restriction on the maximum working density of any solid suspension.

Galena has a very high density but is little used in practice because it is soft and brittle and readily breaks down to a slime which is difficult to recover by flotation. In addition, the galena is an expensive commodity and flotation is an expensive process. In recent years the trend is towards the use of magnetic materials to constitute the dense medium because of the ease of recovery by magnetic separation.

Magnetite is low cost and cheap but because of its low density is used only for media densities up to 2.5. This is too low for ore separations but is suitable for coal cleaning.

For treating ores, ferro-silicon is the most widely used material. It is available in a range of silicon compositions but only a certain composition range is suitable for dense medium applications. For example, if the silicon content is greater than 22% it is only weakly magnetic, whereas if the silicon content is less than 15% the corrosion resistance is unsuitable. The specific gravity of ferro-silicon containing 10% Si is 7.0 and decreases to 6.3 at 25% Si. The media densities achievable with different solids is summarised in Table 15.12. As ferro-silicon is expensive some blending with cheaper magnetite is possible for some densities. Coarser ferro-silicon is used for the higher densities as the pseudo liquid will have a lower viscosity at the higher % solids needed to achieve the higher media densities.

The 15% ferro-silicon is non-rusting and has good magnetic properties. Sometimes lime is added to the suspension and a dispersing agent such as tri-sodium phosphate can be used to prevent the slimes from coagulating.

Ferro-silicon media may be prepared by grinding in a ball-mill or by atomisation of the molten material. The atomised variety has a more spherical shape which allows a higher medium density to be obtained (up to 3.4) and a lower apparent viscosity than the more angular particles. The spherical particle are also more chemically stable. The better chemical and physical properties of the atomised variety of ferro-silicon must be balanced against the higher cost.

For efficient separation, the media viscosity should be low. This allows rapid movement of particles and high capacity. Therefore the volume concentration of solids in the medium

<table>
<thead>
<tr>
<th>Solid</th>
<th>limiting media S.G.</th>
</tr>
</thead>
<tbody>
<tr>
<td>galena</td>
<td>3.3</td>
</tr>
<tr>
<td>magnetite</td>
<td>2.5</td>
</tr>
<tr>
<td>15% Si - ferrosilicon</td>
<td>2.5 - 3.5</td>
</tr>
<tr>
<td>-150 μm Fe-Si + 10-20% magnetite</td>
<td>2.65 - 2.9</td>
</tr>
<tr>
<td>-150 μm Fe-Si only or</td>
<td>2.8 - 3.0</td>
</tr>
<tr>
<td>-212 μm Fe-Si + magnetite</td>
<td></td>
</tr>
<tr>
<td>-212 μm Fe-Si</td>
<td>&gt; 3.0</td>
</tr>
</tbody>
</table>
Fig. 15.16. Viscosity of the heavy media produced from selected solids [16].

should be low since the viscosity of a suspension of solids is dependant on the solids concentration and the particle size. For efficient separation of a feed containing a wide size range, the media volume concentration should be kept to 35-38%. For a narrow size feed, the media can be up to 45-48% solids by volume. Fig. 15.16 shows the viscosity versus medium density for a number of common media solids. The dotted line in Fig. 15.16 represents an approximate critical concentration of solids in water above which the media viscosity is too high for efficient separation.

**Example 15.4**

How much 10% FeSi needs to be added to 1L of water to make a heavy liquid of S.G. 2.8? What is the maximum media S.G. that can be used with this solid?

Density of FeSi = 7000 kg/m$^3$  
Density of water = 1000 kg/m$^3$

**Solution**

Step 1: Calculate the mass of solid
Volume of water = 1 L = 0.001 m³
Mass of water = 0.001 x 1000 = 1.0 kg

Density of heavy medium = \( \frac{\text{mass of medium}}{\text{volume of medium}} = \frac{\text{mass of solid} + \text{mass of water}}{\text{volume of solid} + \text{volume of water}} \)

Let the Mass of solid = \( X \) kg
then volume of solid = \( \frac{X}{7000} \) m³

Therefore, density of medium = 2800 = \( \frac{X + 1.0}{\frac{X}{7000} + 0.001} \)

and solving for \( X \):

\[
X = \frac{(2800 \times 0.001) - 1}{1 - \left(\frac{2800}{7000}\right)} = 3.0 \text{ kg}
\]

Step 2: Calculate the maximum medium density.

Assume that the viscosity limit of the medium occurs at a volume % solids of 35%.

Let the volume of the liquid medium = 100
then the volume of solid = 35
and the volume of water = 100 - 35 = 65

The density of the medium then is given by:

\[
\text{media density} = \frac{\text{mass of medium}}{\text{volume of medium}} = \frac{(\text{vol. x density})_{\text{solid}} + (\text{vol. x density})_{\text{water}}}{100}
\]

\[
= \frac{(35 \times 7000) + (65 \times 1000)}{100} = \frac{245000 + 65000}{100} = 3100 \text{ kg/m}^3
\]

This calculation assumes that the solid is immiscible in the liquid and hence no volume change will occur. In the case of a soluble solid or two soluble liquids, the volumes will not be additive.

15.6.3 Types of Dense Medium Separators
The aim of the dense medium separation is to produce a float product of lower density and a sink product of higher density than the medium. In some instances a third, middlings product is also produced. A variety of equipment is used to bring about this separation and they are
usually classified into bath or trough separators and centrifugal separators, depending on the separation forces employed.

**Gravity Dense Medium Separators**

In gravity dense medium separators the minerals and dense medium are fed into a large quiescent pool or tank or trough of the medium. Particles denser the medium will sink and the low specific gravity particles float. The floating material overflows or is removed from the bath by scrapers while the sink material falls to the bottom of the tank and is removed by some means. The many types of static bath separators include those used for coal separation and mineral separation. Since coal separation has a very high floats content in the feed (up to 95%), the separator will need a high floats capacity whereas separators for the mineral industry may require a high sinks capacity depending on the ores being treated (up to 80% for iron ore). Operating requirements therefore differ depending on the type of ore being treated. The types of separators include:

1. **Drum Separator** consisting of a cylindrical rotating drum, used for mineral and coal separation. The floating product overflows from a weir at the opposite end to the drum feed. The size of the drums range up to about 4.6 m diameter by 7.0 m long, with capacities up to 800 t/h. The drums may consist of a single compartment, producing two products from a single dense medium suspension or consist of two compartments with two baths of different density to produce three or four products. Drum feed size ranges from 6 mm to 30 cm.

2. **Drewboy** bath separator is used widely in the cleaning of coal. The coal is fed into the bath at one end and the floats scraped from the opposite end while the sinks are lifted out from the bottom of the bath by the vanes of a slowly revolving inclined wheel. The Drewboy bath has a high floats capacity and handles a feed from 12.7-600 mm at up to 820 t/h for a 4 m diameter bath.

3. **Cone Separator** is used for ore treatment since it has a relatively high sinks capacity. The feed is dropped into a gently agitated media bath. The floats overflow a weir while the sinks product is removed directly from the bottom of the cone shaped vessel by pump or by an air lift. Cone separators are available in diameters from 0.9 to 6.1 m, with capacities up to 450 t/h. The method of sink discharge limits the maximum feed particle size to about 10 cm. The Chance cone is a similarly shaped vessel to the cone separator but differs from the normal dense medium methods in that it uses a rising flow of water to fluidise a bed of sand to simulate a dense fluid. The sand used is sized between 150-600 μm and when fluidised by water rising at 6-12 mm/s the density is in the range 1.4-1.7 and hence is suitable only for coal/shale separation. Gravity control is achieved by varying the water flowrate. Shale or refuse is discharged periodically through a double gate arrangement at the bottom of the cone.

**Centrifugal Dense Medium Separators**

The buoyant forces acting on the light particles in a dense medium cause them to rise to the surface but the dense particles, being heavier than the liquid they displace, sink to the bottom. The magnitude of the gravitational and buoyant forces that separate the particles is a primary consideration because it governs the velocity with which the particles separate, which it turn determines the capacity of the separating vessel. In a static bath the net gravitational force minus buoyant force may be written as follows:
\[ F_g = (M_s - M_f)g \]  \hspace{1cm} (15.25)

where  \( F_g \) = gravitational force,  
\( M_s \) = mass of solid and  
\( M_f \) = mass of fluid displaced by the particle.

For particles which float, \( F_g \) will have a negative value and for sink particles, it will be positive. In a centrifugal separator, specific gravity separations result from application and utilisation of similar forces except that the acceleration of gravity is substituted by a centrifugal acceleration. The equation then becomes:

\[ F_c = (M_s - M_f)\frac{v^2}{R} \]  \hspace{1cm} (15.26)

where  \( F_c \) = centrifugal force,  
\( v \) = tangential velocity and  
\( R \) = radius of the centrifugal separator.

The centrifugal force will be balanced by the resistance of the liquid when the terminal velocity is reached. For small forces, as experienced by particles with a specific gravity near that of the medium, the particles fall in the Stokes range where the fluid resistance is essentially due to viscosity. However, for large forces, the particles will fall in Newton's range where the fluid resistance is primarily inertial and substantially independent of viscosity. It is, therefore, not possible to write an exact equation for the terminal velocity that would be applicable for all particles. Nevertheless, it is apparent that the forces causing the particles to separate in a static bath are proportional to \( g \), whereas in a centrifugal separator they are proportional to \( \frac{v^2}{R} \) which is much larger.

Cyclones can be used to develop this centrifugal force. In a typical cyclone the centrifugal force acting on a particle in the inlet region is 20 times greater than the gravitational force in a static bath. In the conical section of the cyclone, \( v \) is further increased according to the relationship:

\[ v\sqrt{R} = \text{constant} \]  \hspace{1cm} (15.27)

At the apex of the cyclone where \( R \) decreases, the acceleration increases to over 200 times greater than gravity. Thus, the forces tending to separate the light and heavy particles are much greater in a cyclone than in a static bath. This offers two advantages:

1. a relatively high capacity and
2. Because the forces acting on the small particles are also much larger than static separations, the cyclone is much more applicable to the separation of small particles.

The general flow pattern of the medium in the cyclone is similar to that existing in a classifying cyclone. The centrifugal forces within the cyclone not only acts on the fine mineral particles but also acts to some extent on the fine medium particles. The effect is to
progressively increase the specific gravity of the medium as it descends towards the apex since the concentration of medium particles at the apex will be higher than in the feed. Conversely the specific gravity of the medium passing through the overflow orifice is less.

This thickening of the dense medium towards the apex of the cyclone has been postulated as a cause for the observed fact that separations in a cyclone always occurs at a specific gravity higher than the specific gravity of the feed medium. For example a medium made up to a density of 2.6 would produce a separation equivalent to a specific gravity of 2.8 in the cyclone. However it has been observed that a similar increase in separation density is found when heavy organic liquids are used in a cyclone. Therefore it is apparent that there is some other factor contributing to the effect of centrifugal forces on the gravity of separation. Nevertheless the zone near the apex of the cyclone is important when using an unstable suspension as the medium. This is indicated by the influence of the size of the medium particles on cyclone performance. If the medium particle are too coarse, performance is adversely affected. Also, the geometry of the lower part of the cyclone becomes very critical, especially with regard to the underflow diameter. Conversely for medium particles below a limiting size, performance is relatively unaffected and the geometry of the lower part of the cyclone becomes less important.

Dense-medium cyclones are available in several sizes with diameters of 0.5–1.0 m being common in the coal industry. The 0.5 m cyclones have a capacity of about 50 tph of raw coal. A cone angle of about 20° is pretty standard and they are generally mounted with just sufficient angle to the horizontal to allow drainage after a shutdown. For example a 20° cyclone would be mounted with the longitudinal axis about 10° from the horizontal. At very low inlet pressures this gives a superior performance. At normal inlet pressures the cyclone orientation has no influence on performance. Dense medium cyclones will operate with inlet pressures as low as 42–56 kPa, but pressures of 140 kPa and higher are used in practice. At very low pressures the separation is much less effective than at higher pressures, especially for the finer sizes. The pressure can be developed by two methods. The simplest is to simply feed the ore directly into the medium sump and pump the mixture to the cyclone. Alternatively, the medium is pumped to a head tank, which is 5-6 m above the cyclone inlet, where the ore is added. This second method is particularly applicable where the solid feed is friable such as coal, because degradation of the coal in the medium pump is eliminated.

In coal processing, a medium-to-coal ratio of about 5:1 is recommended. A ratio as low as 3:1 can be used, but with some sacrifice in performance.

The Dense Medium Cyclone or DSM cyclone (developed by the Dutch State Mines) is used to treat ores and coal in the size range 0.5 - 40 mm. Other centrifugal dense media devices include the Swirl Cyclone, the Dyna Whirlpool and the Vorsyl Separator. The hydrocyclone, Water-Washing or Water-only cyclone is used in the coal industry for cleaning of the -0.6 mm coal. It does not employ a dense medium and is not generally regarded as a dense medium separation device, although some researchers believe that separation occurs by some autogenous dense medium developed from the coal feed. It was developed by the Dutch State Mines and differs from the conventional dense-medium cyclone by having a much larger cone angle, up to 120° and a longer vortex finder. The hydrocyclone is effective on coal up to 50 mm and down to 150 μm. The essentially unseparated minus 150 μm material is separated from the cleaned coal (overflow) by screening (sieve bend).
Two theories have been put forward to explain the separation in the water only cyclone:

1. Autogenous dense medium
   This theory proposes that the hydrocyclone behaves as a dense-medium cyclone as particles of intermediate or high specific gravity collect and recirculate in the conical section of the cyclone. These particles then form an autogenous dense-medium through which the sinks have to penetrate to enter the underflow, but the light coal particles cannot penetrate and report to the overflow.

2. Acceleration theory
   Fontein & Dijksman [17] considered the separation to be occurring in the ascending vortex of the cyclone where both light and heavy particles are subjected to the centrifugal forces. For particles at the same position in the vortex, the initial movement of a particle outwards towards the wall of the cyclone will be dominated by the particle acceleration and the drag force will be low. Eq. (15.26) describes the centrifugal force acting on the particles with negligible drag and thus the acceleration of a particle is given by:

   \[
   a_p = \frac{F}{M} = \frac{(M_s - M_f) \nu^2}{M_s} \frac{1}{R}
   \]

   For particles occupying the same radial position in the cyclone, having R and \( \nu \) the same and since \( M_f \) is the mass of an equal volume of fluid then this simplifies to:

   \[
   a_p = \left(1 - \frac{M_f}{M_s}\right) C = \left(1 - \frac{\rho_f V_s}{\rho_s V_s}\right) C = \left(1 - \frac{1}{SG_s}\right) C
   \]

   where \( C = \nu^2/R \) which is constant and equal for all particles and \( SG_s = \) specific gravity of the particle.

   Now for a shale particle of S.G. 2.4 and a coal particle of S.G. 1.3 the initial acceleration of the particle outwards will be given by:

   \[
   a_{\text{shale}} = \left(1 - \frac{1}{2.4}\right) C = 0.583 C \quad \text{and}
   \]

   \[
   a_{\text{coal}} = \left(1 - \frac{1}{1.3}\right) C = 0.231 C
   \]

   That is, the acceleration on the shale particle is about 2.5 times that on the coal particle. The shale particle is therefore more likely to move out of the ascending vortex to the descending vortex and thus report to the cyclone spigot. The coal particle then is more likely to remain in the ascending vortex and report to the overflow independent of its size.

   As the particle size decreases, the fluid resistance becomes a significant factor. For large particles, the residence time in the cyclone is not long enough for the fluid resistance to become significant. This is not the case for the finer particles and hence the very fine particles tend to report to the light fraction regardless of their density.
The density of separation of a water only cyclone, and hence the grade of the coal product, is determined by the diameters of the discharge orifices (U/F or O/F) or by varying the length of the vortex finder. For example, the washed coal ash content can be reduced by decreasing the diameter of the vortex finder, increasing the diameter of the underflow orifice or decreasing the length of the vortex finder [18].

The separations obtained in a water only cyclone are not as sharp (not as efficient) as those characteristic of the dense-medium cyclone. Therefore the water only cyclone is not applicable to difficult feeds (that is, where the feed contains a large percentage of near-gravity material) or at low separation densities. The hydrocyclone cleaning stage is therefore usually a two-stage recirculation circuit in order to improve the efficiency.

15.6.4. Comparison of Dense Medium Separators and Jigs in Coal Processing
Table 15.13 summarises the performance comparisons between jigs and other gravity and centrifugal separators.

15.7. Gravity Separation Performance
The evaluation of the separation method or performance of a gravity separation device is usually based on a sink-float analysis and washability curves. A great many applications of the washability curves are applied to cleaning operations in the coal preparation field (coal washing) so that most reference is made to coal washability curves, however, it must be remembered that the principles will also apply to separations of heavy minerals.

An ideal separation process would be one in which all particles of density lower than the separating density would be recovered in the light or clean product (coal) and all material denser than the separating density would be rejected as the heavy or refuse fraction. This is not achievable in practice. The type of separation between the light and heavy components that might actually occur is illustrated in Fig. 15.17. Material that is much heavier or lighter than the separating density tend to report to their proper fraction but as the density of the particle approaches the density of separation of the unit, the amount of misplaced material increases rapidly. At the density of separation, the amount of misplaced material peaks at 50% as this is how we have defined the density of separation.

The imperfect separation of materials is characteristic of all gravity separation processes. The shape of the curve is determined largely by the inherent difficulty of stratifying materials of only slight density difference and will depend on the feed particles themselves, the feed rate, the media viscosity and separator characteristics. The more efficient the processes the sharper the separation and the narrower the peak. With low efficiency separation processes, the two arms of the peak are more widely separated. The peak will not necessarily be symmetrical.

15.7.1. Sink-Float analysis
The characteristics needed for evaluating a unit performance are normally derived from a float-sink analysis of samples of the clean coal and the reject, followed by ash determinations of the various fractions. Usually a sample of feed is also collected to determine its ash content to determine the yield. No single characteristic describes adequately the efficiency of a separation unit. The most significant is the error curve (Fig. 15.17) and the associated separating gravity, probable error, and imperfection.

A sink-float analysis is akin to a size analysis in that the sample is broken down into its various density fractions rather than size fractions. The sink-float test involves placing the
sample progressively into baths of increasing density and recording the mass of sample within any given density fraction. Normally test work includes determining floats from 5-8 gravity

<table>
<thead>
<tr>
<th>Table 15.13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Performance comparison of gravity separation units.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Jig</th>
<th>Shaking Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>can adjust to fluctuating feed rates</td>
<td>lower power consumption</td>
</tr>
<tr>
<td>can treat feed with high % reject</td>
<td>nominal unit capacity 8 – 12 t/h</td>
</tr>
<tr>
<td>can process reasonable capacity in a single unit</td>
<td>nominal size range 0.15 – 12.5 mm</td>
</tr>
<tr>
<td>lower operator involvement</td>
<td>Ep 0.10</td>
</tr>
<tr>
<td>lower maintenance costs</td>
<td>nominal separation density 1.6 – 1.8</td>
</tr>
<tr>
<td>lower floor space</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Jig</th>
<th>Dense Media cyclone</th>
</tr>
</thead>
<tbody>
<tr>
<td>lower capital cost</td>
<td>higher sharpness of separation</td>
</tr>
<tr>
<td>lower operating cost</td>
<td>suitable for large % of near gravity material (&gt; 10-15%)</td>
</tr>
<tr>
<td>can adjust to fluctuating feed rates</td>
<td>nominal unit capacity 10 – 120 t/h</td>
</tr>
<tr>
<td>can handle wide size range in feed</td>
<td>Ep 0.04</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Jig</th>
<th>Dense media bath</th>
</tr>
</thead>
<tbody>
<tr>
<td>minimum water consumption</td>
<td>precise control of separation SG</td>
</tr>
<tr>
<td>treat &lt; 5% of near gravity material</td>
<td>easily vary S.G. of separation over large range</td>
</tr>
<tr>
<td>lower operating costs – no media</td>
<td>treat &gt; 10% of near gravity material</td>
</tr>
<tr>
<td>can treat sizes to 0.5 mm</td>
<td>can separate at S.G. &lt; 1.45</td>
</tr>
<tr>
<td>simpler plant control</td>
<td>lower size limit is 5 mm</td>
</tr>
<tr>
<td>easier to operate at high cut-points</td>
<td>can treat sizes &gt; 200 mm</td>
</tr>
<tr>
<td></td>
<td>can separate at an accurate cut-point</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Jig</th>
<th>Water only cyclone</th>
</tr>
</thead>
<tbody>
<tr>
<td>higher feed rate</td>
<td>suitable for feed rates &lt; 200 t/h</td>
</tr>
<tr>
<td>change washing gravity easily</td>
<td>Ep 0.25 (single stage), 0.15 (2 stage)</td>
</tr>
<tr>
<td>handle new feed size easily</td>
<td>nominal cut-point 1.6 – 2.0</td>
</tr>
<tr>
<td>can adjust to wide swings in feed grade</td>
<td>size range 0.1 – 12.5 mm</td>
</tr>
<tr>
<td>wash a wide size range</td>
<td>nominal capacity 150 t/h</td>
</tr>
<tr>
<td>Ep 0.08</td>
<td></td>
</tr>
<tr>
<td>nominal cut-point, 1.5 – 2.0 SG</td>
<td></td>
</tr>
<tr>
<td>size range 0.15 – 19 mm</td>
<td></td>
</tr>
<tr>
<td>nominal capacity 200-800 t/h</td>
<td></td>
</tr>
</tbody>
</table>

increments between the density of the light and heavy minerals. The steps may include S.G.’s of 1.30, 1.40, 1.50, 1.60, 1.70, 1.80 up to a S.G. of 2.0. Additional densities between those listed may be required depending on the feed sample. The procedures are described in various standards such as AS 4156.1 [20].

After being rinsed, dried and weighed, all the gravity fractions are analysed for ash. The observed data are presented in table form for each particle size over a range of specific gravities (Table 15.14). Column 1 lists the density fractions with F1.35 representing the material floating at a S.G. of 1.35 and S1.30 representing material that has sunk at a S.G. of
Fig. 15.17. Actual gravity separation performance [19].

1.30. Column 2 contains the mass% of the sample that lies within the corresponding density range and column 3 is the grade of the particles in the density fraction as ash%. Column 4 has the amount of ash (mass) in the density fraction obtained by multiplying columns 2 and 3. Column 5 lists the nominal density of the fraction, taken as the floating density. Columns 6 – 11 are the cumulative values of both floats and sinks and the associated ash content summed from the top down for floats and from the bottom of the table up for the sinks. Column 8 is the cumulative grade (weighted average) of the combined floats up to the nominated density and hence the value of 27.5% at 100% cumulative floats corresponds to the total grade of the coal sample being tested. Similarly, column 11 is the cumulative grade of the combined sinks and the value of 27.5% ash at 100% sinks will also equal the sample grade. Columns 12 and 13 calculate the instantaneous ash which represents the highest ash content of any individual particle in the floats at that density. The instantaneous ash is found as follows: Consider an increase in median density from 1.55 to 1.60. At the higher density, the cumulative floats ash is 15.7% and the average ash in this new incremental density range (1.55-1.60) is 33.2% (column 3, Table 15.14). Although the average ash in this density increment is 33.2%, there will be some particles which have a higher ash and some which have a lower ash. This average ash content corresponds, for want of a more precise estimate, to a particle having a density half way between 1.55 and 1.60 (Fig. 15.19). The cumulative floats corresponding to this average density is given by the average of the cumulative floats at 1.55 (73.6%, column 6, Table 15.14) and the average cumulative floats at density 1.60 (77.0%). That is:

\[
\text{Cumulative floats at 33.2% ash} = \frac{73.6 + 77.0}{2} = 75.3
\]
Thus if the coal sample was placed in a heavy liquid to float off 75.3% of the mass, then the maximum ash content of any one particle in the floats would have an ash content of 33.2%.

This adjusted cumulative % floats value of 75.3 could also have been obtained by:

\[ \text{Cumulative floats at 1.55} = \frac{\text{mass\% in range 1.55-1.60}}{2} \]

or

\[ \text{Cumulative floats at 1.60} = \frac{\text{mass\% in range 1.55-1.60}}{2} \]

Column 13 is a repeat of column 3.

15.7.2 Washability Curves

The information from Table 15.14 is presented graphically (Fig. 15.19) in the form of a family of 5 washability curves which indicate the ease or difficulty of the proposed separation.

Cumulative float curve

This curve is obtained by plotting the cumulative mass percent of floats at each relative density increment against the cumulative ash at that point (columns 6 and 8). Both scales are usually arithmetic although a logarithmic scale may be used for the ash to accentuate
differences in the lower ash ranges. The curve may be used to indicate the yield obtainable for any set ash required. It is plotted on a y-axis with increasing values from top to bottom.

**Cumulative sinks curve**

This curve is obtained by plotting the cumulative mass percent sinks at each relative density increment against the cumulative ash of the sinks for that separation (column 9 and 11). The ash point for 100% sinks must equal the ash point for 100% floats. This curve indicates the ash content of the rejects when a certain yield of clean coal is required.

**Instantaneous ash curve**

This is sometimes referred to as the *elementary ash curve*, *characteristic ash curve* or *observed curve* and describes the rate of change of ash at different yields (column 12 and 13). It is a derivation of the cumulative percent ash in the floats and shows the rate of change of the ash content at different specific gravities or yields. The instantaneous ash is the highest ash content of any individual particle in the floats at any density.

**Relative density curve**

This is obtained by plotting the cumulative percent of floats against the relative density for that separation (column 5 and 6). It indicates the yield of clean coal for a perfect separation at a selected relative density.

**Distribution or ±0.1 S.G. curve**

A density interval, usually ± 0.1 (or 0.05) for specific gravity is specified and then the difference in yield (cumulative floats) between two relative densities, 0.2 apart, is plotted against the mean of those densities. For example, for a S.G. of 1.6, calculate the difference in cumulative floats from a S.G. of 1.50 and 1.70. This represents the percentage of the feed that is close to a separating density of 1.60 (within ± 0.1) or the percentage of near gravity material in the feed. Thus the curve indicates the difficulty of separation. A low value, less than 10%, is satisfactory, whereas a figure in excess of 20% indicates a very difficult separation (see Table 15.15).

The combined curves are shown in Fig. 15.19 and can be used to indicate the parameters of separation. For example, in the washability curves, for a yield of 75%, the clean coal would contain 15.1% ash, rejects 64% ash, separation would be made at 1.575 S.G. and separation would be very difficult (distribution curve reading 24). Table 15.15 is to be used as a guide only and in some cases the distribution curve may have to be extended to ±0.02 or ±0.05 to increase the sensitivity.

In Fig. 15.19 the specific gravity scale increases right to left. This is simply to reduce clutter in the graph as the relative density curve would overlap with the instantaneous and cumulative floats curves. The scale can equally be plotted in ascending order left to right.

15.7.3. Tromp Curves

The data from the float-sink analysis indicates what should be obtained under ideal operating conditions since in the laboratory, time is given for particles to find their correct product. Such conditions do not exist in plant practice and some material will be misplaced by degradation of the coal during processing and the imperfection of the separator unit in that particles of density near that of the medium may not have enough time in the separating vessel to report to the sinks or the floats.
Table 15.14
Sink-Float test Data; QASH = Quantity of Ash; Sep SG = separating density; DistpisT, = Distribution of Ash.

<table>
<thead>
<tr>
<th>S.G.</th>
<th>Mass%</th>
<th>Ash%</th>
<th>QASH</th>
<th>Sep S.G.</th>
<th>Cumulative float</th>
<th>Cumulative Sink</th>
<th>Instantaneous ± 0.1 Dist</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
<td>(12)</td>
</tr>
<tr>
<td></td>
<td>(2)x(3)</td>
<td>Sum(2)</td>
<td>Sum(4)</td>
<td>(7)/(6)</td>
<td>Sum(2)</td>
<td>Sum(4)</td>
<td>(10)/(9)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F1.30</td>
<td>9.5</td>
<td>5.0</td>
<td>47.5</td>
<td>1.30</td>
<td>9.5</td>
<td>47.5</td>
<td>90.5</td>
</tr>
<tr>
<td>S1.30-F1.35</td>
<td>9.7</td>
<td>8.1</td>
<td>78.6</td>
<td>1.35</td>
<td>19.2</td>
<td>126.1</td>
<td>80.8</td>
</tr>
<tr>
<td>S1.35-F1.40</td>
<td>13.4</td>
<td>9.9</td>
<td>132.7</td>
<td>1.40</td>
<td>32.6</td>
<td>258.8</td>
<td>67.4</td>
</tr>
<tr>
<td>S1.40-F1.45</td>
<td>17.8</td>
<td>17.2</td>
<td>306.2</td>
<td>1.45</td>
<td>50.4</td>
<td>564.9</td>
<td>49.6</td>
</tr>
<tr>
<td>S1.45-F1.50</td>
<td>17.2</td>
<td>21.9</td>
<td>376.7</td>
<td>1.50</td>
<td>67.6</td>
<td>941.6</td>
<td>32.4</td>
</tr>
<tr>
<td>S1.50-F1.55</td>
<td>6.0</td>
<td>26.0</td>
<td>156.0</td>
<td>1.55</td>
<td>73.6</td>
<td>1097.6</td>
<td>26.4</td>
</tr>
<tr>
<td>S1.55-F1.60</td>
<td>3.4</td>
<td>33.2</td>
<td>112.9</td>
<td>1.60</td>
<td>77.0</td>
<td>1210.5</td>
<td>23.0</td>
</tr>
<tr>
<td>S1.60-F1.70</td>
<td>3.3</td>
<td>40.1</td>
<td>132.3</td>
<td>1.70</td>
<td>80.3</td>
<td>1342.8</td>
<td>19.7</td>
</tr>
<tr>
<td>S1.70-F1.80</td>
<td>2.8</td>
<td>45.9</td>
<td>128.5</td>
<td>1.80</td>
<td>83.1</td>
<td>1471.3</td>
<td>16.9</td>
</tr>
<tr>
<td>S1.80-F1.90</td>
<td>1.5</td>
<td>53.2</td>
<td>79.8</td>
<td>1.90</td>
<td>84.6</td>
<td>1551.1</td>
<td>15.4</td>
</tr>
<tr>
<td>S1.90-F2.00</td>
<td>2.0</td>
<td>65.1</td>
<td>130.2</td>
<td>2.00</td>
<td>86.6</td>
<td>1681.3</td>
<td>13.4</td>
</tr>
<tr>
<td>S2.00</td>
<td>13.4</td>
<td>79.5</td>
<td>1065.3</td>
<td>-</td>
<td>100.0</td>
<td>2746.6</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mass %</th>
<th>Ash %</th>
<th>(12)</th>
<th>(13)</th>
<th>(14)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2)x(3)</td>
<td>Sum(2)</td>
<td>Sum(4)</td>
<td>(7)/(6)</td>
<td>Sum(2)</td>
</tr>
<tr>
<td>47.5</td>
<td>78.6</td>
<td>132.7</td>
<td>258.8</td>
<td>67.4</td>
</tr>
<tr>
<td>14.4</td>
<td>8.1</td>
<td>58.1</td>
<td>41.5</td>
<td>17.2</td>
</tr>
<tr>
<td>70.6</td>
<td>26.0</td>
<td>12.7</td>
<td>75.3</td>
<td>33.2</td>
</tr>
<tr>
<td>78.7</td>
<td>40.1</td>
<td>6.1</td>
<td>81.7</td>
<td>45.9</td>
</tr>
<tr>
<td>83.9</td>
<td>53.2</td>
<td>3.5</td>
<td>85.6</td>
<td>65.1</td>
</tr>
</tbody>
</table>

---

Table 15.14
Sink-Float test Data; QASH = Quantity of Ash; Sep SG = separating density; DistpisT, = Distribution of Ash.
Fig. 15.19. Washability curves based on the sink-float analysis date in Table 15.14.

Table 15.15
Ease of gravity separation based on the % of near gravity material.

<table>
<thead>
<tr>
<th>mass% within ±0.1 S.G.</th>
<th>Burt [2]; Herbst &amp; Sepulveda [21]</th>
<th>Mills [22], Bird [23]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 7</td>
<td>simple</td>
<td>simple</td>
</tr>
<tr>
<td>7 – 10</td>
<td>relatively simple</td>
<td>moderately difficult</td>
</tr>
<tr>
<td>10 – 15</td>
<td>moderately difficult</td>
<td>difficult</td>
</tr>
<tr>
<td>15 – 20</td>
<td>difficult</td>
<td>very difficult</td>
</tr>
<tr>
<td>20 – 25</td>
<td>very difficult</td>
<td>exceedingly difficult</td>
</tr>
<tr>
<td>&gt; 25</td>
<td>very difficult</td>
<td>formidable</td>
</tr>
</tbody>
</table>

The partition or distribution curve or Tromp curve is useful in assessing the efficiency or sharpness of the separation, or to predict the performance of a plant treating a particular coal feed. Normally it is relatively independent of the float and sink properties of the coal being dependent upon the particle size distribution and the type of separating unit. Data from float and sink analysis on the raw coal, the clean coal and the reject is used to determine the partition coefficients, which is defined as:

\[
\text{Partition Coefficient} = \frac{\text{mass of coal reporting in any SG range}}{\text{mass of feed coal present in that SG range}} \times 100
\]  

(15.34)
Table 15.16 shows the sink-float data obtained from a sample of the light fraction and the heavy fraction from a gravity separation process such as a jig or dense medium bath. As in the determination of the Tromp Curve of a classifier, the mass split between the floats (M<sub>FT</sub>) and sinks (M<sub>SK</sub>) produced by the separator is required. Thus:

\[
M_{FT} = \text{Floats yield} = \frac{\text{mass of floats (clean coal)}}{\text{mass of raw feed}} \times 100
\]

Table 15.16
Data for determination of a separator efficiency. M<sub>FT</sub> = floats mass, M<sub>SK</sub> = sinks mass.

<table>
<thead>
<tr>
<th>Specific Gravity</th>
<th>Clean Coal analysis (%)</th>
<th>Reject analysis (%)</th>
<th>Floats as % of feed</th>
<th>Sinks as % of feed</th>
<th>Reconstituted feed (%)</th>
<th>Partition coeff</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1.30</td>
<td>72.75</td>
<td>10.10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.30/1.35</td>
<td>19.50</td>
<td>7.03</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.35/1.40</td>
<td>2.67</td>
<td>5.92</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.40/1.45</td>
<td>1.83</td>
<td>5.45</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.45/1.50</td>
<td>0.76</td>
<td>4.19</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.50/1.55</td>
<td>0.55</td>
<td>4.68</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.55/1.60</td>
<td>0.43</td>
<td>9.85</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.60/1.70</td>
<td>0.30</td>
<td>9.06</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.70/1.80</td>
<td>0.12</td>
<td>8.37</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.80/1.90</td>
<td>0.09</td>
<td>5.85</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.90/2.00</td>
<td>0.05</td>
<td>4.67</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.00</td>
<td>0.95</td>
<td>24.83</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

100.00  100.00  100M<sub>FT</sub>  100M<sub>SK</sub>

\[
M_{SK} = \text{Sinks yield} = \frac{\text{mass of sinks (reject coal)}}{\text{mass of raw feed}} \times 100 = 100 - M_{FT}
\]

The quantities represented in columns 2 – 7 are given as:

\[
\text{column 2} = \frac{\text{mass of clean coal that floats at SG range}}{\text{total mass of clean coal}} \times 100 \quad \{15.35\}
\]

\[
\text{column 3} = \frac{\text{mass of rejects that float at SG range}}{\text{total mass of reject}} \times 100 \quad \{15.36\}
\]
column 4 = \frac{\text{mass of clean coal (floats) that float at SG}}{\text{total mass of feed}} \times 100

= \frac{\text{mass of clean coal that float at SG}}{\text{total mass of clean coal}} \times \frac{\text{total mass of clean coal}}{\text{total mass of feed}} \times 100

= \text{column 2} \times M_{\text{PT}} \quad \{15.37\}

column 5 = \frac{\text{mass of reject (sinks) that float at SG}}{\text{total mass of feed}} \times 100

= \frac{\text{mass of reject that floats at SG}}{\text{total mass of reject}} \times \frac{\text{total mass of reject}}{\text{total mass of feed}} \times 100

= \text{column 3} \times M_{\text{SK}} = \text{column 3} \times (100 - M_{\text{PT}}) \quad \{15.38\}

column 6 = \frac{\text{mass of feed that floats at SG}}{\text{total mass of feed}} \times 100

= \frac{\text{mass of clean coal that floats at SG}}{\text{total mass of feed}} + \frac{\text{mass of rejects that float at SG}}{\text{total mass of feed}}

= \text{column 4} + \text{column 5} \quad \{15.39\}

Now since it is only the clean coal fraction that floats in the separator:

column 7 = \frac{\text{mass of feed that floats (or sinks) at SG}}{\text{mass of feed at that SG}} \times 100

= \frac{\text{mass of clean coal (or reject) that floats at SG}}{\text{mass of feed}} \times \frac{\text{mass of feed}}{\text{mass of feed at that SG}} \times 100

= \frac{\text{column 4}}{\text{column 6}} \times 100 \quad \text{or} \quad \frac{\text{column 5}}{\text{column 6}} \times 100 \quad \{15.40\}

The partition curve or Tromp curve (introduced by Tromp in 1937) is then obtained by plotting the partition coefficient against the mean of its density range. From the partition curve the separation density or cut point or \(d_{50}\) is determined (Fig. 15.20).

The ideal partition curve would indicate that all particles having a density higher than the separating density report to the sinks and particles lighter than the separating gravity report to the floats. The partition curve for the real situation shows that the efficiency is greatest for particles of density furtherest from the separating density and decreases for particles
Fig. 15.20. Partition curve for gravity separation of coal indicating the $d_{50}$.

approaching the separating density. The gradient of the curve is a measure of the sharpness of separation and to indicate the accuracy of the separation, the slope of the partition curve between the 25 and 75 percent partition coefficients is used.

The *probable error of separation* ($E_p$) is defined as half the difference between the S.G. where 75% is recovered in the sinks and the S.G. at which 25% is recovered in the sinks:

$$E_p = \frac{S_{G_{75}} - S_{G_{25}}}{2}$$

(15.41)

The density at which 50% of the feed reports to the floats or to the sinks is known as the effective density of separation, which may not be exactly the same as the operating density.

The probable error is relatively characteristic of a process and units which have a low $E_p$, such as heavy medium processes ($E_p = 0.02$ to $0.03$) are considered efficient separators. Table 15.16 indicates the probable error for different gravity separation processes. The higher the difficulty of separation as indicated in Table 15.15 the greater is the need to use a separator with a low probable error. The value of the probable error depends on the specific gravity of separation, the $d_{50}$. The probable error will increase as the specific gravity of separation increases, unjustifiably indicating a less sharp separation. Therefore the Imperfection is used as a further method of comparing separating processes independent of the separating density. The Imperfection is defined as:

$$I = \frac{\text{probable error}}{\text{partition density} - 1} = \frac{E_p}{d_{50} - 1}$$

(15.42)
Table 15.17
Probable errors for some gravity separation processes.

<table>
<thead>
<tr>
<th>Process</th>
<th>$E_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table</td>
<td>0.03-0.11</td>
</tr>
<tr>
<td>DM Bath</td>
<td>0.01-0.02</td>
</tr>
<tr>
<td>DM Cyclone</td>
<td>0.01-0.04</td>
</tr>
<tr>
<td>Hydrocyclone</td>
<td>0.08-0.14</td>
</tr>
<tr>
<td>Baum Jig</td>
<td>0.03-0.12</td>
</tr>
</tbody>
</table>

However, these refer to only a portion of the partition curve and to assess the overall operation it is preferable to consider the full series of partition coefficients. In the case of a new coal deposit, if the float-sink analysis data is available and the partition curve for the separation process is known, then it is possible to calculate the potential percentage recovery and product grade for the processing of the coal by the preparation method selected.

Table 15.18 shows the sink-float analysis of a ROM coal sample with the partition coefficient of a jig separator. In this table, column 6 is the actual mass of floats to be expected from the separator and column 7 is the mass of ash expected at that density. These are expressed as:

column 6 = column 2 x column 5

\[
= \frac{[\text{total mass of floats at SG}] \times 100}{\text{mass of feed}} \times \frac{\text{mass that actually floats at SG} \times 100}{[\text{total mass of floats at SG}]}
\]

\[
= \frac{\text{mass that actually floats at SG}}{\text{mass of feed}} \times 10000
\]

(15.43)

column 7 = column 6 \times \frac{\text{mass of floats ash}}{\text{mass of floats at SG}} \times 100

\[
= \frac{\text{mass of actual floats at SG}}{\text{mass of feed}} \times 10000 \times \frac{\text{mass of floats ash}}{[\text{mass of floats at SG}]} \times 100
\]

\[
= \frac{\text{mass of floats ash at SG}}{\text{mass of feed}} \times 10^4
\]

(15.44)

From Table 15.18, the yield and grade of the floats and sinks can be estimated.

The predicted yield is given by:
Table 15.18
Sink-float analysis of a raw coal sample with the partition coefficient of a gravity separator.

<table>
<thead>
<tr>
<th>Specific Gravity</th>
<th>Mass %</th>
<th>Ash %</th>
<th>Mass x Ash</th>
<th>Partition Coeffic.</th>
<th>Floats Mass</th>
<th>Mass of Ash</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
</tr>
<tr>
<td>F1.30</td>
<td>2.2</td>
<td>3.5</td>
<td>7.7</td>
<td>100</td>
<td>220</td>
<td>770</td>
</tr>
<tr>
<td>S1.30/F1.35</td>
<td>8.2</td>
<td>5.8</td>
<td>47.6</td>
<td>99</td>
<td>812</td>
<td>4712</td>
</tr>
<tr>
<td>S1.35/F1.40</td>
<td>14.3</td>
<td>8.2</td>
<td>117.3</td>
<td>95</td>
<td>1358</td>
<td>11144</td>
</tr>
<tr>
<td>S1.40/F1.45</td>
<td>15.6</td>
<td>10.4</td>
<td>162.2</td>
<td>82</td>
<td>1279</td>
<td>13300</td>
</tr>
<tr>
<td>S1.45/F1.50</td>
<td>9.6</td>
<td>21.3</td>
<td>204.5</td>
<td>63</td>
<td>605</td>
<td>12884</td>
</tr>
<tr>
<td>S1.50/F1.55</td>
<td>6.4</td>
<td>28.2</td>
<td>180.5</td>
<td>38</td>
<td>243</td>
<td>6859</td>
</tr>
<tr>
<td>S1.55/F1.60</td>
<td>3.8</td>
<td>38.9</td>
<td>147.8</td>
<td>20</td>
<td>76</td>
<td>2956</td>
</tr>
<tr>
<td>S1.60/F1.70</td>
<td>3.1</td>
<td>43.1</td>
<td>133.6</td>
<td>5</td>
<td>16</td>
<td>668</td>
</tr>
<tr>
<td>S1.70/F1.80</td>
<td>2.3</td>
<td>55.8</td>
<td>128.3</td>
<td>1</td>
<td>2</td>
<td>128</td>
</tr>
<tr>
<td>S1.80/F1.90</td>
<td>2.6</td>
<td>56.6</td>
<td>147.2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>S1.90/F2.00</td>
<td>3.0</td>
<td>68.8</td>
<td>206.4</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>S2.00</td>
<td>28.9</td>
<td>83.6</td>
<td>2416.0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \Sigma )</td>
<td>4611</td>
<td>53421</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\text{predicted yield} = \frac{\text{total mass of floats}}{\text{mass of feed}} \times 100 \quad (15.45)
\]

and since the sum of column 6 = \( \frac{\text{total mass of floats}}{\text{mass of feed}} \times 10000 \)

then \[
\text{Predicted yield} = \frac{\Sigma (6)}{100} = 46.11\% \quad (15.46)
\]

and \[
\text{Predicted ash} = \frac{\text{total mass of floats ash}}{\text{total mass of floats}} \times 100
\]

\[
= \frac{\Sigma (7)}{\Sigma (6)} = \frac{53421}{4611} = 11.6\% \quad (15.47)
\]

The theoretical yield is given by the cumulative sum of the floats up to the specific gravity of separation. In the above example, this is at 1.50 S.G. Therefore:

Theoretical Yield = \( \Sigma (2) \) to S.G. 1.50 = 49.9 % \quad (15.48)

The Theoretical Ash is given by the sum of the floats mass x ash up to the specific gravity of separation. In the above example:
Theoretical Ash \[ \frac{\sum (4 \text{to SG} 1.50)}{\sum (2 \text{to SG} 1.50)} = \frac{539.3}{49.9} = 10.8\% \] (15.49)

The actual yield, \( M_{FT} \), is found from the expression:

\[ M_{FT} = \frac{A_R - A_F}{A_R - A_C} \times 100 \] (15.50)

where \( A_R \) = ash of total refuse, \( A_F \) = ash of total feed and \( A_C \) = ash of clean coal.

### 15.7.4. Sink-Float Alternatives

Because of the importance of washability and sink-float analysis to the coal industry and the health hazards associated with organic liquids, considerable effort is being aimed at alternatives to the organic liquid method. To determine the partition curve of a gravity separation unit, density tracers may be used. These are plastic particles manufactured to precise density such as 0.005 SG units [24]. These tracers are available in cubic shape from 1 to 64 mm or as crusher particles to simulate real ore with sizes from 0.125 to 32 mm or more. Density ranges are from 1.24 to 4.5 S.G. and can be colour coded or made magnetic or fluorescent for ease of recovery. A range of tracers of different density and size are added to the unit feed and retrieved from the floats and sinks fractions. The ratio of numbers in the floats and feed will give the partition coefficient.

For an alternative to the sink-float analysis, the Julius Krutschnitt Mineral Research Centre (JKMRC) have developed an automatic gas pycnometer in which the dry density of individual particles is determined by separate mass and volume measurements [25]. The instrument is capable of analysing 30 particles a minute. A sink-float data analysis requiring about 3000 particles can be obtained in 100 minutes.

### 15.8. Problems

#### 15.1

Calculate the terminal velocity difference between hematite and quartz particles at a particle size of 150 microns and 50 microns. Comment on the differences and hence discuss the limitations of normal gravity separation devices and how these might be overcome.

Density of hematite \( = 5000 \text{ kg/m}^3 \)
Density of quartz \( = 2600 \text{ kg/m}^3 \)
Density of water \( = 1000 \text{ kg/m}^3 \)

#### 15.2

Plastic tracers of 2 mm size were added to the feed of a dense medium cyclone. The number and density of each tracer added and retrieved in the underflow are given below. Determine the separating density of the cyclone by plotting the Tromp curve.
Cost studies have shown a favourable financial return from using dense media separation to pre-concentrate gold ore waste dumps comprising +1mm material. The following laboratory results have been obtained.

### Tracer density

<table>
<thead>
<tr>
<th>Tracer density</th>
<th>number of tracers in the feed</th>
<th>number of tracers in the sinks</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.7</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>2.8</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>2.9</td>
<td>50</td>
<td>1</td>
</tr>
<tr>
<td>3.0</td>
<td>100</td>
<td>16</td>
</tr>
<tr>
<td>3.1</td>
<td>300</td>
<td>241</td>
</tr>
<tr>
<td>3.2</td>
<td>100</td>
<td>98</td>
</tr>
<tr>
<td>3.3</td>
<td>100</td>
<td>99</td>
</tr>
<tr>
<td>3.53</td>
<td>300</td>
<td>299</td>
</tr>
</tbody>
</table>

1. What would be the separation density required to obtain a gold recovery of at least 95%.
2. What type of medium could be used to achieve this density.
3. What is the percentage of near gravity material at the cut density and hence the difficulty of separation.

### Specific gravity analysis

<table>
<thead>
<tr>
<th>Head Grade</th>
<th>S.G.</th>
<th>Cum. mass %</th>
<th>Cum. Au (g/t)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Sinks</td>
<td></td>
</tr>
<tr>
<td>1.15</td>
<td>2.875</td>
<td>21.99</td>
<td>5.04</td>
</tr>
<tr>
<td></td>
<td>2.900</td>
<td>16.33</td>
<td>6.63</td>
</tr>
<tr>
<td></td>
<td>2.925</td>
<td>11.66</td>
<td>9.15</td>
</tr>
<tr>
<td></td>
<td>2.950</td>
<td>9.11</td>
<td>11.51</td>
</tr>
</tbody>
</table>

12 x 9.5 mm: 15.2%
<table>
<thead>
<tr>
<th>Specific gravity</th>
<th>Percent</th>
<th>Cumulative percent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mass</td>
<td>Ash</td>
</tr>
<tr>
<td>Float-1.28</td>
<td>5.9</td>
<td>1.8</td>
</tr>
<tr>
<td>1.28-1.30</td>
<td>30.8</td>
<td>3.5</td>
</tr>
<tr>
<td>1.30-1.35</td>
<td>32.1</td>
<td>7.6</td>
</tr>
<tr>
<td>1.35-1.40</td>
<td>8.5</td>
<td>13.5</td>
</tr>
<tr>
<td>1.40-1.45</td>
<td>3.3</td>
<td>18.9</td>
</tr>
<tr>
<td>1.45-1.50</td>
<td>1.4</td>
<td>24.1</td>
</tr>
<tr>
<td>1.50-1.60</td>
<td>1.6</td>
<td>30.4</td>
</tr>
<tr>
<td>1.60-1.70</td>
<td>0.9</td>
<td>38.9</td>
</tr>
<tr>
<td>1.70-1.80</td>
<td>0.8</td>
<td>45.1</td>
</tr>
<tr>
<td>1.80-Sink</td>
<td>14.7</td>
<td>84.9</td>
</tr>
</tbody>
</table>

6.3 x 2.3 mm: 41.3 %

<table>
<thead>
<tr>
<th>Specific gravity</th>
<th>Percent</th>
<th>Cumulative percent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mass</td>
<td>Ash</td>
</tr>
<tr>
<td>Float-1.28</td>
<td>11.1</td>
<td>1.3</td>
</tr>
<tr>
<td>1.28-1.30</td>
<td>33.0</td>
<td>2.8</td>
</tr>
<tr>
<td>1.30-1.35</td>
<td>27.6</td>
<td>6.8</td>
</tr>
<tr>
<td>1.35-1.40</td>
<td>9.0</td>
<td>12.2</td>
</tr>
<tr>
<td>1.40-1.45</td>
<td>3.6</td>
<td>17.3</td>
</tr>
<tr>
<td>1.45-1.50</td>
<td>1.2</td>
<td>22.0</td>
</tr>
<tr>
<td>1.50-1.60</td>
<td>1.5</td>
<td>28.3</td>
</tr>
<tr>
<td>1.60-1.70</td>
<td>0.8</td>
<td>37.1</td>
</tr>
<tr>
<td>1.70-1.80</td>
<td>0.6</td>
<td>42.0</td>
</tr>
<tr>
<td>1.80-Sink</td>
<td>11.6</td>
<td>83.7</td>
</tr>
</tbody>
</table>

1.18 x 0.6 mm: 1.2 %

<table>
<thead>
<tr>
<th>Specific gravity</th>
<th>Percent</th>
<th>Cumulative percent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mass</td>
<td>Ash</td>
</tr>
<tr>
<td>Float-1.28</td>
<td>3.3</td>
<td>1.2</td>
</tr>
<tr>
<td>1.28-1.30</td>
<td>40.1</td>
<td>1.7</td>
</tr>
<tr>
<td>1.30-1.35</td>
<td>24.3</td>
<td>5.7</td>
</tr>
<tr>
<td>1.35-1.40</td>
<td>8.3</td>
<td>11.3</td>
</tr>
<tr>
<td>1.40-1.45</td>
<td>4.2</td>
<td>15.7</td>
</tr>
<tr>
<td>1.45-1.50</td>
<td>1.8</td>
<td>19.5</td>
</tr>
<tr>
<td>1.50-1.60</td>
<td>1.9</td>
<td>25.2</td>
</tr>
<tr>
<td>1.60-1.70</td>
<td>0.9</td>
<td>34.3</td>
</tr>
<tr>
<td>1.70-1.80</td>
<td>0.5</td>
<td>39.1</td>
</tr>
<tr>
<td>1.80-Sink</td>
<td>14.7</td>
<td>82.3</td>
</tr>
</tbody>
</table>
Distribution, percent to washed coal
(specific gravity fraction):

<table>
<thead>
<tr>
<th>Specific Gravity Fraction</th>
<th>12 x 9.5 mm</th>
<th>6.3 x 2.3 mm</th>
<th>1.18 x 0.6 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Float-1.28</td>
<td>99.8</td>
<td>99.4</td>
<td>98.8</td>
</tr>
<tr>
<td>1.28-1.30</td>
<td>99.5</td>
<td>99.4</td>
<td>98.7</td>
</tr>
<tr>
<td>1.30-1.35</td>
<td>98.1</td>
<td>98.5</td>
<td>96.9</td>
</tr>
<tr>
<td>1.35-1.40</td>
<td>79.8</td>
<td>92.1</td>
<td>92.7</td>
</tr>
<tr>
<td>1.40-1.45</td>
<td>12.8</td>
<td>57.1</td>
<td>81.7</td>
</tr>
<tr>
<td>1.45-1.50</td>
<td>2.2</td>
<td>16.6</td>
<td>49.1</td>
</tr>
<tr>
<td>1.50-1.60</td>
<td>0</td>
<td>3.2</td>
<td>12.9</td>
</tr>
<tr>
<td>1.60-1.70</td>
<td>0</td>
<td>2.1</td>
<td>4.7</td>
</tr>
<tr>
<td>1.70-1.80</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1.80-Sink.</td>
<td>0</td>
<td>0</td>
<td>2.9</td>
</tr>
</tbody>
</table>

15.5
Calculate the mass ratio of solid to liquid to make up a dense media of 3.1 S.G. using galena (S.G. 7.4) as the solid and bore water (S.G. 1.1) as the liquid.

15.6
What material could be used, from the following list (material/S.G.), to achieve a medium density of 2800 kg/m³? (and avoiding viscosity problems)

- magnetite (5.2)
- ferrosilicon (7.1)
- lead shot (11.4)
- galena (7.5)
- quartz (2.6)
- pyrite (5.0)
- mill scale (4.95)
- barites (4.5)

15.7
A sink-float analysis is performed on a sample of drill core from a coal deposit. The results are given below.
What is the best possible yield and grade that could be expected from this deposit using a jig washer? Compare this with the theoretical yield and grade

<table>
<thead>
<tr>
<th>SG</th>
<th>Mass %</th>
<th>Assay %</th>
<th>SG</th>
<th>Partition coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.30</td>
<td>6.9</td>
<td>5.0</td>
<td>1.30</td>
<td>99.2</td>
</tr>
<tr>
<td>1.35</td>
<td>9.7</td>
<td>7.1</td>
<td>1.35</td>
<td>97.7</td>
</tr>
<tr>
<td>1.40</td>
<td>10.4</td>
<td>8.2</td>
<td>1.40</td>
<td>89.3</td>
</tr>
<tr>
<td>1.45</td>
<td>15.7</td>
<td>9.6</td>
<td>1.45</td>
<td>79.6</td>
</tr>
<tr>
<td>1.50</td>
<td>17.2</td>
<td>10.8</td>
<td>1.50</td>
<td>32.1</td>
</tr>
<tr>
<td>1.55</td>
<td>14.1</td>
<td>18.7</td>
<td>1.55</td>
<td>18.5</td>
</tr>
<tr>
<td>1.60</td>
<td>9.8</td>
<td>33.2</td>
<td>1.60</td>
<td>10.9</td>
</tr>
<tr>
<td>1.70</td>
<td>5.1</td>
<td>40.1</td>
<td>1.70</td>
<td>8.5</td>
</tr>
<tr>
<td>1.80</td>
<td>2.1</td>
<td>45.9</td>
<td>1.80</td>
<td>3.9</td>
</tr>
<tr>
<td>1.90</td>
<td>1.5</td>
<td>53.2</td>
<td>1.9</td>
<td>2.3</td>
</tr>
<tr>
<td>2.0</td>
<td>2.0</td>
<td>65.1</td>
<td>2.0</td>
<td>0.9</td>
</tr>
<tr>
<td>+2.0</td>
<td>5.5</td>
<td>79.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
A number of plastic tracers of different density are added to the feed to a JIG separator. The FLOATS and SINKS are sampled and as many of the tracers as possible are recovered. From the data below determine the partition coefficient for the jig and then determine the density of separation for the jig by plotting a Tromp Curve.

<table>
<thead>
<tr>
<th>S.G. of tracer</th>
<th>Number of tracers in feed</th>
<th>in heavy fraction</th>
<th>in light fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>10</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>2.0</td>
<td>10</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2.5</td>
<td>10</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3.0</td>
<td>10</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>3.5</td>
<td>10</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>4.5</td>
<td>10</td>
<td>9</td>
<td>0</td>
</tr>
<tr>
<td>5.0</td>
<td>10</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>6.0</td>
<td>10</td>
<td>10</td>
<td>0</td>
</tr>
</tbody>
</table>

A representative sample of a crushed hard rock cassiterite (SnO₂) deposit is subjected to a sink-float analysis using magnetohydrostatics. The results are given below. From this information and the partition coefficients of the Jig from problem 15.8, determine the actual yield of concentrate (SINKS) and the grade (% SnO₂) expected for this ore and a jig concentrator.

Compare this ACTUAL yield with the THEORETICAL yield (obtained with a perfect separator).

<table>
<thead>
<tr>
<th>SG</th>
<th>Mass %</th>
<th>Assay % SnO₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>F2.00</td>
<td>9.5</td>
<td>0.001</td>
</tr>
<tr>
<td>S2.00-F2.50</td>
<td>9.7</td>
<td>0.01</td>
</tr>
<tr>
<td>S2.50-F3.00</td>
<td>13.4</td>
<td>0.01</td>
</tr>
<tr>
<td>S3.00-F3.50</td>
<td>17.8</td>
<td>18.8</td>
</tr>
<tr>
<td>S3.50-F4.00</td>
<td>18.8</td>
<td>39.1</td>
</tr>
<tr>
<td>S4.00-F4.50</td>
<td>9.8</td>
<td>54.3</td>
</tr>
<tr>
<td>S4.50-F5.00</td>
<td>6.4</td>
<td>66.2</td>
</tr>
<tr>
<td>S5.00-F5.50</td>
<td>3.3</td>
<td>75.6</td>
</tr>
<tr>
<td>S5.50-F6.00</td>
<td>2.8</td>
<td>83.4</td>
</tr>
<tr>
<td>S6.00-F6.50</td>
<td>1.5</td>
<td>86.9</td>
</tr>
<tr>
<td>S6.50-F7.00</td>
<td>2.0</td>
<td>95.3</td>
</tr>
<tr>
<td>S7.00</td>
<td>5.0</td>
<td>99.5</td>
</tr>
</tbody>
</table>

If an ore contains a valuable mineral of density 4500 kg/m³ and a gangue of density 2800 kg/m³ discuss the likely success of separation using gravity separation in potable water (density 998 kg/m³) and bore water (density 1290 kg/m³)? The mean particle size is 100 microns.
REFERENCES

Chapter 16. Flotation

16. INTRODUCTION

In 2005 flotation celebrated its 100th anniversary since the first commercial introduction in Broken Hill in 1905. Since then flotation has grown to be the prime mineral processing operation for both hard rock and coal separations. Flotation is a separation process that has found prominence because of the need to treat complex or low grade ores where the average particle size for liberation is too small for efficient gravity separation or where the gravity difference between minerals is too small. Flotation relies on the surface of the valuable mineral being hydrophobic while the surface of the gangue minerals is hydrophilic. When the three phases, solid, liquid and gas are in contact, an equilibrium is established between the solid-air, solid-liquid and liquid-air interfacial tensions, $\gamma_{SA}$, $\gamma_{SL}$, and $\gamma_{LA}$ as shown in Fig. 16.1.

The angle between the tangent to the curve at the three phase contact and the solid surface, at equilibrium, is the contact angle $\theta$, and is taken as the angle through the fluid phase of higher density. In a flotation system this will be the water phase.

At equilibrium, the balance of the surface forces is given by the Young equation:

$$\gamma_{SA} = \gamma_{SL} + \gamma_{LA} \cos \theta \quad (16.1)$$

The balance of forces can be altered by any factor which changes any of the interfacial tensions. A new equilibrium position is established and a new contact angle formed. The contact angle is a measure of how well the air bubble spreads or wets the solid surface. A low contact angle (nominally less than 90°) indicates a hydrophilic surface while an angle greater than 90° represents a hydrophobic surface. A hydrophobic surface is one which will favour

---

Fig. 16.1. Three phase contact between solid, liquid and air.
contact with air over water due to a lower free energy and hence will readily stick to an air interface if one is available. However in the flotation of mineral particles, a contact angle much less than 90° is still capable of sticking to an air bubble and hence floating.

The interfacial tension, $\gamma_{LA}$, is the surface tension of the liquid and is readily measured. The solid interfacial tensions or surface energies is mainly determined by the chemical bonds within the solid and are not easily measured. For example, the two forms of carbon, graphite and diamond, although having the same composition, have vastly different surface energies as a result of their different lattice structure (graphite 110 and diamond 5600 mN\text{m}^{-1}). For high surface energy solids (i.e. high $\gamma_{SA}$) such as metals and ionic solids such as sulphides and oxides, according to Eq. (16.1), the contact angle will tend to be small and the contact area between solid and air will be small and the liquid will wet the solid (hydrophilic or aerophobic).

If the solid surface energy is low, for example graphite, hydrocarbon solids such as coal, kerogen (hydrocarbon component of oil shale) or sulphur then the contact angle and contact area can be large and the solid will be hydrophobic or aerophilic. (In addition a hydrophobic surface needs to be non-polar or at least not capable of producing a polar surface by association with water molecules).

Since the measurement of the solid surface energy in air or water is difficult, the usefulness of Eq. (16.1) to make any prediction as to the floatability of a mineral is limited. If we consider a three phase contact as shown in Fig. 16.1 where the air bubble detaches from the solid surface then the solid/air interface is replaced by a solid/water and an air/water interface. The change in free energy accompanying this replacement of a unit area of the solid/air interface by a solid/liquid interface is given as:

\[
\Delta G = (\gamma_{SL} + \gamma_{LA}) - \gamma_{SA}
\]  

Combining with Eq. (16.1) gives:

\[
\Delta G = \gamma_{LA}(1 - \cos \theta)
\]  

where $\Delta G = \text{the free energy change.}$

This equation contains the terms $\gamma_{LA}$ and $\theta$ which are both easily measured. If the free energy change is negative then the separation of bubble and solid will occur and if the free energy change is positive then conditions are not favourable for separation to occur.

The free energy is often referred to as the work of adhesion between a bubble and the solid surface. An increase in contact angle would then indicate an increase in $\Delta G$ and hence the forces tending to hold the bubble and solid together is greater. But it must be remembered that this is a thermodynamic function expressing the maximum possible increase in free energy of the system resulting from the bubble-particle detachment, which is realised only when there are no other energy consuming effects such as deformation of the bubble surface, and where all gravitation effects are absent and the system is at equilibrium.

This hydrophobic characteristic occurs naturally on a small number of common minerals including graphite, sulphur, talc, molybdenite and coal. For solids which are naturally hydrophilic (which covers most minerals) the contact angle can be modified by changing the surface energy of one or more of the phases. If able to do so, surfaces will tend to contract to reduce surface area and hence reduce the surface energy. Alternatively if another component
is introduced into the system it will adsorb or concentrate at an interface if it reduces the surface energy of that interface.

The term adsorption refers to the existence of a higher concentration of any particular component at the surface of a liquid or solid phase than is present in the bulk. Adsorption is important in relation to the chemical reagents in flocculation and frothers and collectors in flotation. For flotation to be successful, the adsorption of chemicals onto the mineral surface must be performed selectively so that only the valuable mineral surface becomes hydrophobic while the gangue mineral surfaces become or remain hydrophilic.

16.1. Flotation Reagents

The chemicals used in flotation are divided into three classes:

1. **Collectors**: organic chemicals which make the surface hydrophobic and hence the mineral is capable of being collected in the process.
2. **Frothers**: organic chemicals which reduce the surface tension of the water to stabilize the bubbles into a froth layer at the top of the flotation cell to make concentrate removal easier.
3. **Modifiers**: organic or inorganic chemicals used to modify the slurry conditions to enhance the difference in surface chemistry between the valuable and gangue minerals.

16.1.1. Collectors

Collectors are generally heteropolar organics with a charged polar group and an uncharged non-polar group. The non-polar group is typically a hydrocarbon chain and in the beginning of the flotation process this hydrocarbon chain was provided in the form of oil. This oil flotation was phased out in the early 1920's by the development of more selective organics such as the xanthates and dithiophosphates. Oil in the form of kerosene or diesel is still used as the principal collecting agent in the coal and other industries. The process of collection can be illustrated by taking a clean glass plate and placing a few drops of water onto the surface as shown in Fig. 16.2.

The water spreads out over the plate because glass is hydrophilic. In Fig. 16.3, a thin film of oil or grease is smeared over the plate surface and a few drops of water are again placed on the plate. In this case the water shrinks up into droplets as the oil film is hydrophobic. The thin film of oil has converted the hydrophilic glass surface to an hydrophobic surface.

Minimisation of the system free energy is the driving force for the reduction of the amount of high energy oil/water interface. Thus the water will shrink to minimize the contact area between the water and oil. When an air interface is available as shown in Fig. 16.4, the oil molecule, represented by a stick figure will concentrate at the air interface and extend into

![Fig. 16.2. Spreading of water over a hydrophilic clean glass surface.](image-url)
the air phase as much as possible. The molecule is represented as aligning perpendicular to the air/water interface. The collectors used in flotation contain a non-polar hydrocarbon chair with a polar group on the chain. The polar group may be ionised and hydrophilic so that the organic may be water soluble. The polar group is also modified to be specifically attracted to certain minerals. At an air/water interface the hydrocarbon chain of the collector will stick out into the air phase at right angles to the interface while the hydrophilic polar group remains in the water phase (Fig. 16.3). If a suitable mineral surface is present and the polar group is attracted to the solid surface, a raft of collector ions will be adsorbed onto the mineral surface, effectively forming a thin film of oil on the mineral surface and hence making the surface hydrophobic, through adsorption rather than a physical smearing (Fig. 16.4B). If an air interface is now provided, in the form of an air bubble, the hydrocarbon chain will extend into the air phase and if the bond strength between the polar group and the mineral surface is strong enough, the particle will be lifted to the surface by the buoyancy of the air bubble (Fig. 16.4C).

Collectors may be non-ionic (hydrocarbon oils) or ionised. The ionised collectors are either cationic, (amines) or anionic (fatty acids or sulphydril compounds such as xanthates or dithiophosphates).

![Fig. 16.3. Beading of water on an oiled hydrophilic surface.](image)

![Fig. 16.4. A: Collector dissolved in the aqueous phase, B: Adsorption onto a mineral surface and C: attachment of an air bubble onto the hydrophobic surface.](image)
16.1.2. Frothers
Frothers are surfactants, usually organic heteropolar compounds such as alcohols or polyglycol ethers. Due to the heteropolar nature, the frother adsorbs at the air/water interface and as a result, lowers the water surface tension. This has the effect of producing smaller bubbles and more importantly it stabilises the froth when it reaches the top of the slurry. If the bubbles burst when they reach the air/water surface then any minerals they carry will drop back into the slurry forming a scum at the surface. The prime role of the frother is to stabilise the froth formed at the interface long enough for the concentrate to be removed from the flotation cell. Once removed from the cell, the froth must break to allow the mineral particles to be re-pulped for further processing.

The effect of frother concentration on the surface tension of water is an initial rapid drop in surface tension before leveling out to a constant minimum value. The optimum frother concentration is not the amount which gives the minimum surface tension but in the region where the surface tension is capable of rapid change with concentration. This is the region of greatest froth stability.

When the thin liquid film at the bubble wall experiences an external stress and stretches locally, the increase in surface area produces a local decrease in surfactant concentration. This in turn increases the surface tension momentarily to allow the thin film to recover without rupturing. If there is no change in surface tension with local film thinning, for example if the liquid is pure or the frother concentration is in excess then the froth will be unstable. For excess frother concentration, all that will happen is the formation of very small bubbles which burst on reaching the pulp surface.

Frothers commonly used include natural chemicals like pine oil, cresylic acid and synthetic reagents such as methyl isobutyl carbinol (MIBC) and polyglycol ethers. Ideally frothers should possess little or no collecting properties and vice versa so that both functions can be controlled separately.

16.1.3. Modifiers
This class of reagents covers all chemicals whose principal function is neither collecting nor frothing. These may be further divided into depressants, activators and pH regulators.

A depressant is any chemical which inhibits or prevents the adsorption of a collector by a mineral particle and thereby prevents its flotation. An activator prepares the mineral surface to enhance the adsorption of the collector. pH regulators adjust the pulp pH to give optimum performance for a particular reagent and mineral ore. The common pH regulators are lime, soda ash and sulphuric acid.

Other reagents that may be used in specific cases include dispersants for removing clay slimes from mineral surfaces and precipitants for removing interfering ions from solution.

The main use of these modifying agents is in the differential flotation of a mixed ore. That is the successive removal of two or more valuable minerals from each other by flotation. For example the separation of each copper, lead, zinc and iron sulphides from a single ore is selective flotation whereas the flotation of the combined sulphides from the gangue is referred to as bulk flotation.

16.2. Flotation Equipment
The equipment used in flotation must provide an air interface for hydrophobic particles to become attach. The air interface is provided in the form of air bubbles introduced into a tank of slurry with agitation to provide an environment for maximum contact between particles and
bubbles. The flotation equipment can be divided into several categories according to the method of introducing the air into the cell. These include:

1. Mechanical (Sub Aeration)
2. Pneumatic
3. Vacuum
4. Electroflotation
5. Dissolved air flotation

The first two types of flotation machines are by far the most widely used in industry. Vacuum and dissolved air flotation relies on the precipitation of air dissolved in the water onto the hydrophobic particles. Electroflotation involves the electrolysis of water into fine bubbles of hydrogen and oxygen.

16.2.1. Mechanical Flotation Cells
These types of machines or cells consist of a highly turbulent region produced by an impeller, to provide the necessary agitation to keep the particles in suspension, disperse the air bubbles and bring about particle-bubble contact. In addition to this, the cell must contain a quiescent zone where the mineral-ladened bubbles can rise to the surface of the cell without loss of particles due to disruptive turbulence. A number of different cell designs have been developed to meet these needs. Removal of froth from the cell can either be by unassisted overflow or mechanical scraping by the use of paddles.

Continuous flotation cannot be performed in a single cell because of losses due to short circuiting of pulp between feed inlet and pulp outlet. It is therefore usual to use 4 to 12 cells in series. This also increases the residence time of the particles in the cells, giving the slow floating particles a chance to report to the froth layer. Cells in series can be interconnected with an overflow weir between cells, a partial baffle or no baffle at all. When no baffle is present it is known as a hog trough or open flow machine.

In the case of the partial baffle and no baffle cells, short circuiting of the cells can occur. These open flow machines were developed to handle larger tonnages in bulk flotation circuits.

16.2.2. Pneumatic Flotation Cells
In these types of cells, pulp and air are injected into the cell through a nozzle to produce intimate contact between air and particles. The air jet is used not only to provide aeration but also to suspend the particles and provide circulation. This usually means that an excessive amount of air must be used, and as a result these types of machines are not as common as mechanical cells in plants. Examples of pneumatic cells is the Davera cell, the Column cell and the Jameson cell.

Column flotation is a pneumatic cell that uses a tall column of pulp rather than a traditional cell. Air is introduced at the bottom of the column and feed is introduced countercurrently near the top of the column. In column flotation air bubble agitation is not sufficient to keep large particles in suspension so that residence times are short in comparison to a bank of mechanical flotation cells. Originally developed in Canada in the 1960's as cleaning cells, this type of cell has become common in the flotation circuit of new plants, both as roughing and cleaning cells with diameters up to 4 or 5 m.

The majority of float cells in use are the mechanical type. The choice of which flotation cell to use is governed by both metallurgical performance and personal liking.
16.2.3. Laboratory Flotation Machines

The two most important requirements of laboratory flotation machines are reproducibility and performance similar to commercial operations. These two criteria are not always satisfied. The basic laboratory machines are scaled down replicas of commercial machines such as Denver, Wemco and Agitair. In the scale down, there are inevitable compromises between simplification of manufacture and attempts to simulate full scale performance. There are scaling errors for example in the number of impeller and stator blades and various geometric ratios. Reproducibility in semi-batch testing requires close control of impeller speed, air flow rate, pulp level and concentrate removal.

16.2.4. Flotation Cell Requirements

Regardless of which type of flotation cell is used to achieve mineral or coal flotation, a machine has two main requirements:

1. Suspension
2. Aeration

In suspension, it is essential that the impeller or air jet of the machine is capable of keeping the solids in the pulp in suspension. If the degree of agitation is inadequate then solids, particularly the largest particles will tend to settle out. Some settling out, for example in the corners of the cell, is not serious but significant sanding of the cell floor will upset pulp flow patterns within the cell and prevent proper contact between suspended particles and air bubbles. Particles not in suspension cannot make effective contact with air bubbles.

Effective aeration requires that the bubbles be finely disseminated, and that the air rate is sufficiently high, not only to provide sufficient bubbles to make contact with the particles but also to provide a stable froth of reasonable depth. Usually the type and amount of frother will be able to influence the froth layer, but the frother and air rate can both be used as variables.

The difficulty facing the flotation designer is that the cell performance is a strong function of the size of the particles to be floated, and that flotation feeds contain a wide range of particle sizes. For any given particle size, the effects of impeller speed and bubble diameter can be summarised as follows [1]:

1. If the impeller speed is too low, the particles are not maintained in suspension, but settle in significant quantities at the base of the cell.
2. If the impeller speed is too high, the turbulence in the cell is sufficient to rupture the bond between the particle and bubble, and so the recovery drops.
3. If the bubble size is too low, the bubble are too small to give sufficient buoyancy to the particles to lift them to the top of the pulp.
4. If the bubble size is too large, the fewer will be the number of bubbles created for a constant air flowrate. Since the overall rate of flotation depends on the number as well as the size of the bubbles, the recovery will drop.

This sets the boundaries for the optimum conditions of impeller speed and bubble size for flotation of any feed. If the feed size range is broad, then the optimum conditions for flotation of the coarse particles may be considerable different to the optimum conditions for the flotation recovery of the fine particles.

The pressure near the centre of the rotating impeller is lower than the ambient pressure at the same point if the rotating impeller were not present. This is due to the centrifugal pressure
gradients induced by the rotation. The pressure near the impeller may be so low as to be less than the hydrostatic pressure in the pulp so that a pipe placed near the impeller and open to the atmosphere may suck air into the impeller region. This is known as induced air and the practice of introducing air into the impeller region is called sub-aeration. Common practice in coal flotation is to use this induced air as the only aeration mechanism. In mineral flotation it is common to supercharge the air to provide a slight excess pressure to give a greater amount of air per unit volume of pulp.

The amount of air specified by cell manufacturers is usually in the range 0.5 – 2 m$^3$ air per minute/m$^3$ pulp, and for use in coal flotation, the figure is towards the bottom end of this range.

The impeller behaves as a pump and circulates the slurry around the cell. The volumetric flowrate, $Q_{VL}$, for simple flat-bladed impellers is given approximately by [1]:

$$Q_{VL} = 0.75 \omega D^3 \text{ m}^3/\text{min}$$

(15.4)

where $\omega$ = the speed of rotation (revolutions/min) and $D$ = the impeller diameter (m).

Flotation impellers would be expected to follow a similar equation, although a slightly different constant may be found. The circulation rates are very high. For example, a 14.2 m$^3$ cell with an impeller of diameter 0.84 m, rotating at 114 rpm, would have an internal circulation of 51 m$^3$ per minute, thus circulating the cell contents between three and four times a minute. The interaction of the liquid circulating in the cell due to the impeller and the air introduced into the impeller generates the size and distribution of bubbles found in the cell.

The effect of air flowrate, $Q_{VA}$, on the formation of bubbles in the cell is described below. The ratio $Q_{VA}/D^3$ is called the air-flow number. In general, for a fixed impeller diameter and speed:

1. At very low rates ($Q_{VA}/\omega D^3 < 0.02$), the air enters the core of the vortices formed behind the tips of the blades, with a strong outwards velocity component due to the pumping action. The bubble size and number is small.
2. At higher rates ($0.02 < Q_{VA}/\omega D^3 < 0.05$), cavities form behind the impeller blades, becoming increasingly large as the air rate increases. Bubbles form from the breakup of the trailing edges of the cavities. The bubble size and number increases.
3. Larger gas rates ($Q_{VA}/\omega D^3 > 0.05$) the impeller FLOODS. The gaps between the blades are blinded by large slugs of air which form very large bubbles.

As the air rate continually increases, the power consumption decreases, because an increasing proportion of the space in the impeller is occupied by air. Increasing the air rate leads to a lower liquid circulation rate, to the extent that the suspended particles may settle out. The general behaviour of the power ratio (the ratio of power consumed in the cell to the power consumed with no air flow) versus the air-flow number is shown in Fig. 16.5.

The onset of flooding coincides with a sudden drop in the power consumption, and is influenced somewhat by impeller design. For best operation a cell should operate well below the flooding gas velocity. Flooding results in very large bubbles, which are of little value for flotation. For example it is found that a reduction in air flow to an induced air flotation cell by closing off part of the air intake can substantially improve the recovery.
16.3. Flotation Circuits

Flotation is carried out as a continuous operation in a series or bank of cells. This increases the floating time, allowing ample opportunity for particle-bubble attachment to occur. The residence time of particles in the bank of cells range from 5 to 15 minutes. The rate at which the particles float will depend on particle composition, pulp density, particle size and degree of turbulence in the cell. The arrangement of a number of cells in series allows the collection of different products from the various cells. For example, liberated particles in general float more rapidly than composite particles so that a high grade concentrate can be collected from the first few cells in a bank and froth from the remaining cells can be collected as a middling concentrate.

Quite often the grade of concentrate recovered from a single stage of flotation is not high enough and requires re-floating in one or more stages of flotation referred to as cleaner or recleaner stages. The series of cells that produce the initial concentrate is called the rougher stage and any subsequent retreatment of the rougher tailings is referred to as scavenging. The scavenger section of the flotation circuit is given higher reagent dosages and long flotation time to float as much valuable mineral as possible and maximise recovery. On the other hand the cleaner stage experiences milder flotation conditions of lower pulp density and lower reagent concentrations to reduce entrainment in the froth and to ensure only the high grade particles will float. To maximise the grade from the cleaner cells, the residence times is generally shorter. For example, rougher and scavenger flotation might take 10 minutes and cleaner flotation, 3 minutes.

The concentrate product from the scavenger cells and/or the tailings from the cleaner or recleaner cells are usually low grade because of locked or composite particles and physically entrained gangue minerals. Also it may contain free valuable mineral particles, present due to physical entrainment, odd shape or size or surface contamination. These low grade products maybe retreated by a re-grind stage and further flotation. The details of the regrinding practice depend largely on the ore characteristics. For example the presence of composites in
any concentrate (such as the rougher concentrate) in any great number would dictate that the concentrate be sent for re-grinding.

An example of some flotation circuits arrangements are shown in Fig. 16.6. A flotation circuit usually contains some provision for handling fluctuations in the flowrate of ore to the plant, either minor or major. Any minor fluctuations can be smoothed out by incorporating an agitator/conditioning tank between the grinding section and the flotation circuit. This is used to maintain a constant rate of feed to the flotation cells and to condition the feed with initial reagent prior to entering the cell. In addition, recovery is often improved by some degree of staged addition of reagents down the bank of cells. In some cases, additional conditioning time is provided by adding reagents to the grinding mill feed and/or discharge.

To accommodate large fluctuations of flow rate, for example if part of the grinding circuit is shut down for maintenance, the flotation circuit can be run in a number of identical modules operating in parallel. If the flow rate drops dramatically then this can be handled by shutting down one or more banks of cells. The more parallel modules built in, the more flexible the plant, but the greater are the control problems involved.

The number and size of cells required in a plant is determined by a number of factors, primarily the tonnage flow rate of material through the circuit. A large number of small cells gives greater flexibility and metallurgical performance whereas a small number of large cells of the same total capacity have a smaller capital cost, less floor area per unit volume and lower power consumption and lower operating costs. With the higher tonnages of lower grade ore now being treated by the minerals industry, the trend is towards large volume flotation cells. The choice of flotation cell size best suited to any specific application must consider the pulp flow rate, the number of parallel modules in the flotation section, the minimum number of cells in each row required to eliminate pulp short-circuiting and the required pulp residence time. Economic considerations favour large cells in rougher-scavenger operations since it is this part of the flotation circuit that handles the largest tonnages. Some of the largest cells available are the Outokumpu TankCells having a volume of 300 m$^3$ and Wemco's 250 m$^3$ Smartcell™.

Flotation feed is generally the ball mill cyclone overflow but in cases where some of the valuable minerals are liberated at coarse sizes and are recirculated in the cyclone underflow, these can be recovered before overgrinding by treating the cyclone underflow by flotation. A flotation cell designed to treat the coarse cyclone underflow is the Skimair® or Flash flotation cell. This is designed to handle a coarse feed at a relatively high % solids as experienced in the cyclone overflow. Since the fine particles have been removed in the cyclone overflow entrainment of non-floating fines is minimised and the flash flotation concentrate generally is suitable for the final concentrate.

16.4. Flotation Kinetics

16.4.1. Batch Flotation
The concentrate obtained from a batch flotation cell changes in character with time as the particles floating change in size, grade and quantity. In the same way, the concentrate from the last few cells in a continuous bank is different from that removed from the earlier cells. Particles of the same mineral float at different rates due to different particle characteristics and cell conditions.

The recovery-time curve of a batch test is generally of the form shown in Fig. 16.7. The recovery of any particular mineral rises to an asymptotic value $R_e$, which is generally less than 100%. The rate of recovery at time $t$ is given by the slope of the tangent to the curve at $t$, and
Fig. 16.6. Flotation cell arrangements.
the rate of recovery at time $t_1$ is clearly greater than the rate at time $t_2$. There is a direct relationship between the rate of flotation and the amount of floatable material remaining in the cell, that is:

$$\frac{\text{slope at } t_1}{(R_m-R_1)^n} = \frac{\text{slope at } t_2}{(R_m-R_2)^n} = k \quad (16.5)$$

or  

Flotation rate = $k \times (\text{concentration in the cell})^n \quad (16.6)$

The flotation rate is equal to the rate of change of concentration of floatable material in the cell, and Eq. (16.6) may be re-written as the differential Eq. (16.7).

$$-\frac{dC}{dt} = kC^n \quad (16.7)$$

This is the basic rate equation, in which the value of $n$ denotes the order of the equation and $k$ is the flotation rate constant.

**16.4.2. First Order Rate Equation**  
Integration of Eq. (16.7) with $n = 1$ (first order), gives:

$$C = C_0e^{-kt} \quad (16.8)$$

where  

$C_0$ = the concentration of valuable material in the cell at zero time, and  

$C$ = the concentration of valuable material remaining in the cell at time $t$.  

---

Fig. 16.7. Time-recovery plot from batch flotation test.
Taking the logs of Eq. (16.8):

\[
\ln\left(\frac{C_0}{C}\right) = k t \tag{16.9}
\]

and if the experimental values of \( \ln (C_0/C) \) are plotted against \( t \), the graph should be a straight line of slope \( k \) if \( n = 1 \).

As the maximum recovery is seldom 100%， if the concentration of valuable material remaining unfloated in the cell after infinite flotation time is \( C_\infty \), then Eq. (16.9) is more accurately expressed as:

\[
\ln\left(\frac{C_0-C_\infty}{C-C_\infty}\right) = k t \tag{16.10}
\]

In terms of recovery, \( R \), by definition is given by:

\[
R = \left(\frac{C_0-C}{C_0}\right) \tag{16.11}
\]

and after prolonged flotation times:

\[
R_\infty = \left(\frac{C_0-C_\infty}{C_0}\right) \tag{16.12}
\]

therefore dividing Eq. (16.11) by Eq. (16.12) gives:

\[
\frac{R}{R_\infty} = \left(\frac{C_0-C}{C_0-C_\infty}\right) \tag{16.13}
\]

or

\[
R = R_\infty \left(\frac{C_0-C}{C_0-C_\infty}\right) \tag{16.14}
\]

From Eq. (16.10):

\[
\left(\frac{C_0-C_\infty}{C-C_\infty}\right) = e^\mu \tag{16.15}
\]

Inverting and subtracting from one gives:

\[
1 - \left(\frac{C-C_\infty}{C_0-C_\infty}\right) = 1 - e^{-\mu} \tag{16.16}
\]
then: \( \frac{C_0 - C_m - C + C_m}{C_0 - C_m} = \frac{C_0 - C}{C_0 - C_m} = 1 - e^{-\left( T_0 + \frac{C_0 - C_m}{C_0 - C_m} \right)} \) \hspace{1cm} (16.17)

and substituting into Eq. (16.14) gives:

\[
R = R_e \left( 1 - e^{-ak} \right)
\]

In the derivation of these equations, it has been assumed that the only independent variable has been the concentration of floatable material, and that everything else has remained constant such as the bubble concentration, size and size distribution, reagent concentrations, cell operation etcetera. If the operator were to alter the air setting, for example, half way through the test, the time-recovery plot would not be a smooth curve, and the same value of \( k \) would not apply before and after the air had been altered.

When other variables are not held constant, the rate equation may be written in general the form:

\[
\frac{dC}{dt} = k \prod C_i^{n_i}
\]

where \( \prod \) = the general product,

\( C_i \) = the concentration of any species which might affect the rate of flotation of the particular species in question, and

\( n_i \) = the order of the equation with respect to \( C_i \).

For example, \( C_i \) might denote the concentration of floatable species, air, gangue and reagents. For any given flotation test, we may assume that the reagent concentrations etc. remain constant, and the equation simplifies to:

\[
\frac{dC}{dt} = k C_S^n C_A^r
\]

where \( C_S \) and \( C_A \) are the concentrations of floatable particles and air bubbles, respectively.

\( k \) is thus a complex function involving reagent concentrations, particle and bubble sizes, induction times, flotation cell design, rate of froth removal, power input and previous treatments and will only be constant as long as these conditions remain constant. In a well-controlled laboratory experiment these may all be maintained reasonably constant, then by varying \( C_S \) and \( C_A \) in turn, the exponents \( n \) and \( r \) can be determined. Since the rate of aeration is usually also kept constant, Eqs. (16.19) and (16.20) simplify to Eq. (16.7).

For any given conditions, \( k \) is a quantitative measure of the probability of the particles of a species being recovered in the concentrate. It can be used to compare different reagent conditions in the same cell, or different flotation cells treating the same pulp. Each particle type in an ore (for example, chalcopyrite, pyrite, pyrrhotite and quartz) will have its own value of \( k \) in any test, and the ratios of these values are a quantitative measure of the selectivity of the operation.

\( k \) is not a measure of the recovery of a mineral in an operation. The recovery is a function of \( k \) and the time of flotation. But the rate constant, \( k \), is the basic concept of the kinetic
approach to the flotation process, and is the factor whose accurate determination is necessary for all of the following developments in flotation.

16.4.3. **Second Order Rate Equation**

The flotation rate equation is given by Eq. (16.7). For second order flotation, \( n = 2 \), and integration over the limits \( t = 0, \ C = C_o \) and \( t = t, \ C = C \):

\[
- \int \frac{dC}{C^2} = k \int dt
\]

\[
\left[ \frac{1}{C} \right]_C^{C_o} = k t
\]

\[
\frac{1}{C} - \frac{1}{C_o} = \frac{C_o - C}{C C_o} = k t
\]

\[
C_o - C = C C_o k t
\]

\[
C_o = C(1 + C_o k t)
\]

or

\[
C = \frac{C_o}{(1 + C_o k t)}
\]

and to plot graphically this can be written as:

\[
\frac{C_o}{C} = 1 + C_o k t
\]

For a maximum recovery of \( R_m \) less than 100%, then:

\[
\frac{dC}{dt} = -k(C - C_m)^n
\]

and on integration:

\[
C = \frac{C_o + C_m(C_o - C_m)k t}{1 + (C_o - C_m)k t}
\]

It is important to always work in terms of floatable material rather than total material. That is, always take into account the value of \( C_o \) or \( R_m \) (see example 16.1). For graphical plotting, Eq. (16.27) becomes:

\[
\frac{C_o - C_m}{C - C_m} = 1 + (C_o - C_m)k t
\]
Therefore a plot of \((C_0 - C_a)/(C - C_w)\) vs. \(t\) will yield a straight line if the flotation rate is second order \((n = 2)\). An analogous equation can be written for recovery (from Eq. (16.5)):

\[
\frac{dR}{dt} = k(R_a - R)^2
\]  

(16.31)

which on integration gives:

\[
R = \frac{R_a^2kt}{1+R_a^2kt}
\]  

(16.32)

from which:

\[
\frac{t}{R} = \frac{1}{R_a^2k} + \frac{t}{R_a}
\]  

(16.33)

or

\[
\frac{1}{t} = \frac{kR_a^2}{R} - kR_a
\]  

(16.34)

is the form for graphical representation. Data for very short periods of flotation are required to be able to distinguish second order kinetics by this graphical method.

---

**Example 16.1**

A sample of coal is floated in a small laboratory cell using a kerosene collector (H1) and a dithiophosphate promoter (DP) in combination with H1. Concentrates were collected at 0.5 minute intervals up to 3 minutes then a final concentrate at 7 minutes. The concentrates were dried, weighed and then assayed by ashing. From the data given below, calculate the flotation rate constant of the coal for the two reagents conditions.

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<th>Mass, g</th>
<th>Ash %</th>
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<td>83.1</td>
</tr>
</tbody>
</table>

Volume of cell = 250 mL.
**Solution**

Step 1: Calculate the concentration of coal in the cell (test HI).
The percentage of coal in the cell is calculated from the ash content by assuming the % coal = 100 - % ash. This is not strictly correct but for this example it will be accurate enough. The initial concentration for the collector HI is then given as:

\[
C_0 = \frac{\text{mass of coal in the feed}}{\text{volume of cell}} = \frac{461.8 \times (100 - 58.5)}{250 \times 100} = 0.767 \text{ g/mL}
\]

For the concentration remaining in the cell after the first half minute:

\[
C = \frac{\text{mass of coal remaining in the cell}}{\text{volume of cell}} = \frac{\text{mass of coal initially} - \text{sum of coal floated up to time } t}{\text{volume of cell}}
\]

\[
C = \frac{[461.8 \times (100 - 58.5)] - [43.6 \times (100 - 13.2)]}{250 \times 100} = 0.615 \text{ g/mL}
\]

similarly for float concentrates at 1.0 - 7.0 minutes:

Repeating the above calculations for the remaining H1 and DP/H1 tests gives:

<table>
<thead>
<tr>
<th>Time mins</th>
<th>(H1) C g/mL</th>
<th>(DP/H1) C g/mL</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.767</td>
<td>0.753</td>
</tr>
<tr>
<td>0.5</td>
<td>0.615</td>
<td>0.575</td>
</tr>
<tr>
<td>1.0</td>
<td>0.554</td>
<td>0.484</td>
</tr>
<tr>
<td>1.5</td>
<td>0.514</td>
<td>0.394</td>
</tr>
<tr>
<td>2.0</td>
<td>0.479</td>
<td>0.325</td>
</tr>
<tr>
<td>2.5</td>
<td>0.461</td>
<td>0.275</td>
</tr>
<tr>
<td>3.0</td>
<td>0.444</td>
<td>0.236</td>
</tr>
<tr>
<td>7.0</td>
<td>0.383</td>
<td>0.178</td>
</tr>
</tbody>
</table>

Step 2: Estimate the infinite concentrations in each test.
Plotting the concentration versus flotation time gives the following graph:
From this plot, the values of \( C_\infty \) are estimated at 0.36 and 0.14 for the H1 and DP/H1 test respectively.

Step 3: Determine the order of the kinetics and determine the rate constants.
The concentration ratios are calculated as follows:

For float test H1:

At time \( t = 0.5 \) minutes,
Decrease in cell concentration during the flotation tests

\[
\frac{C_0 - C_\infty}{C - C_\infty} = \frac{0.767 - 0.36}{0.615 - 0.36} = 1.60 \text{ and}
\]

\[
\ln \left[ \frac{C_0 - C_\infty}{C - C_\infty} \right] = \ln(1.60) = 0.47
\]

Repeating the above calculations for the remaining H1 and DP/H1 tests gives the following table. The last two columns in this table are calculated on the basis of total coal in the cell rather than floatable coal.

<table>
<thead>
<tr>
<th>Time (mins)</th>
<th>(\frac{C_0 - C_\infty}{C - C_\infty})</th>
<th>(\ln(\frac{C_0 - C_\infty}{C - C_\infty}))</th>
<th>(\frac{C_0}{C})</th>
<th>(\ln(\frac{C_0}{C}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0.47</td>
<td>1.25</td>
<td>0.22</td>
</tr>
<tr>
<td>0.5</td>
<td>1.60</td>
<td>0.47</td>
<td>1.25</td>
<td>0.22</td>
</tr>
<tr>
<td>1</td>
<td>2.10</td>
<td>0.74</td>
<td>1.39</td>
<td>0.33</td>
</tr>
<tr>
<td>1.5</td>
<td>2.64</td>
<td>0.97</td>
<td>1.49</td>
<td>0.40</td>
</tr>
<tr>
<td>2</td>
<td>3.42</td>
<td>1.23</td>
<td>1.60</td>
<td>0.47</td>
</tr>
<tr>
<td>2.5</td>
<td>4.05</td>
<td>1.40</td>
<td>1.67</td>
<td>0.51</td>
</tr>
<tr>
<td>3</td>
<td>4.84</td>
<td>1.58</td>
<td>1.73</td>
<td>0.55</td>
</tr>
<tr>
<td>7</td>
<td>17.64</td>
<td>2.87</td>
<td>2.00</td>
<td>0.69</td>
</tr>
</tbody>
</table>

For first order, plot columns 1 and 3 according to Eq. (16.10) which will give a straight line of slope \(k\). For second order, plot columns 1 and 2 according to Eq. (16.30) will give a straight line of slope \((C_0 - C_\infty)k\).
The first order plot shows that when plotting the data according to Eq. (16.9) (uncorrected concentration) the DP/H1 combined reagent has a rate constant of approximately twice that of the HI test. When applying the correction for unfloatable coal (cell contents at infinite time) the two reagents give a similar rate constant. Thus it is important in calculating the flotation rate constant to always consider the floatable material. The difference between the two reagents in this instance is not the effect on the rate of flotation of the coal but rather the effect on the infinite recovery which is 80% and 53% for the DP/H1 and HI reagents respectively. The same effect of corrected and uncorrected concentrations is shown in the second order plot.

Neither the first order or second order plots show a perfect straight line over the whole timed concentrates. The coefficient of determination, $r^2$, is similar for both plots at 0.87-0.94 for the corrected data.
Using the second order plot, a linear regression on the data gives:

\[
\begin{align*}
  k_{HI} &= \text{slope } = 0.4628 \text{ min}^{-1} \text{ for reagent HI and} \\
  k_{DP} &= \text{slope } = 0.4632 \text{ min}^{-1} \text{ for reagent DP/HI}
\end{align*}
\]

\[
\begin{align*}
  k_{HI} &= \frac{\text{slope}}{(C_0 - C_\infty)} = \frac{2.011}{(0.767 - 0.36)} = 4.94 \text{ min}^{-1} \text{ and} \\
  k_{DP} &= \frac{\text{slope}}{(C_0 - C_\infty)} = \frac{1.9237}{(0.753 - 0.14)} = 3.14 \text{ min}^{-1}
\end{align*}
\]

16.4.4. Non-Integral Order

The rate equation can be considered, in general, to be somewhere between first order and second order, approaching first order for flotation of single minerals or in very dilute pulps, and approaching second order for low-grade ores or more concentrated pulps. This is represented by the equation:

\[
k^i t = \frac{R^i}{R^i_\infty} \left[ I - \left( \frac{R^i}{R^i_\infty} \right) - \left( \frac{R^i}{R^i_\infty} \right) \ln \left[ I - \left( \frac{R^i}{R^i_\infty} \right) \right] \right]
\]

(16.35)

where \(R_\infty\) is the total mineral present of all species \(i\).

When only one mineral is involved or if the solid to liquid ratio is very low (\(R^i_\infty = R^i\)) then this equation simplifies to the first-order equation and if \(R^i \ll R\), Eq. (16.35) reduces to a form of the second order equation. Thus Eq. (16.35) is the general equation describing the flotation process, based on the kinetic model of flotation.

16.4.5. Experimental Results

The confirmation of the order of the rate equation is usually obtained graphically as has been done by many investigators for both first and second order kinetics. It has been suggested [2] that a more sensitive test would be to investigate the effect of variation in floatable mineral concentration (that is, pulp density variation) on the time required for a 50% change in recovery. If the half time remains constant with different pulp densities then the rate equation would be first order, if the half time was not constant then higher order kinetics would be involved.

However, in the graphical determination of the order of the rate equation there is sufficient difference between the first and second order curves that provided the experiments are carried out accurately, it is not difficult to distinguish between first and second order or if in fact neither rate law holds.

These rate equations assume that the rate coefficient for each mineral species is constant. However, by considering different functions for the distribution of rate coefficients for the mineral species, the first order rate equation can be applied to most cases of experimental
batch data. Dowling et al [3] evaluated a number of first order batch flotation models available in the literature. Some of the models considered were:

1. The classical first order kinetic model described by Eq. (16.8) which assumes a monodisperse feed with particles of constant floatability. If the plot of $\ln \left( \frac{C_0}{C} \right)$ is not linear then either the rate is not first order or the floatability of particles is not constant. If the rate is first order but the floating particles in the pulp do not have identical properties, i.e., there is a continuous range of rate constants for the mineral being recovered, the integrated form of the rate equation for a semi-batch process becomes:

$$
C = C_0 \int_0^\infty \exp(-kt) f(k,0) dk \quad (16.36)
$$

where $f(k,0)$ represents a continuous distribution of rate constants.

The distribution of rate constants may arise from intergrowths of minerals or a distribution of particle sizes. The problem then is related to the accurate estimation of the distribution of rate constants. Different interpretations of this distribution of rate constants give rise to a number of first order rate models:

2. The Klimpel form of the first order equation is given by [4]:

$$
R = R_a \left[ 1 - \frac{1}{kt} (1 - \exp(-kt)) \right] \quad (16.37)
$$

where $k$ = rate constant representing the largest allowable value of a rectangular distribution.

The difference between the classical model and the Klimpel model is in the representation of the rate constant distribution as uniform or rectangular. A rectangular distribution is one in which the quantity is constant over a fixed interval. For example, the rate constant has a fixed or constant value over a limited property range. For low and high values of the property, the rate constant is zero. Compare this with the classical first order model where the rate constant is assumed to have a constant value for all property values.

3. Kelsall [5] proposed a flotation model incorporating two rate constants, describing a fast floating component and a slow floating component. The use of two rate constants was considered to give a better approximation to the distribution of particle floatabilities than could be obtained with a single rate constant. The mathematical description of the model as modified by Jowett [6], is given by:

$$
R = (R_a - \phi)(1 - \exp(-k_s t)) + \phi(1 - \exp(-k_s t)) \quad (16.38)
$$

where $\phi$ = fraction of flotation components with the slow rate constant, $k_F$ = fast rate constant (min$^{-1}$) and $k_s$ = slow rate constant (min$^{-1}$).
This model attempts to describe non linear rate data (log[C/Co] vs time) by the sum of two straight lines.

4. Another form of the distribution of rate constants is a modified gamma function proposed by Loveday [7] and Inoue and Imaizumi [8]. In terms of recovery, the model becomes:

\[ R = R_e \left( 1 - \left( \frac{k}{k+t} \right)^p \right) \]  

(16.39)

The model has three parameters. The gamma distribution can be simplistically described as being made up of the sum of P exponential distributions.

5. Flotation results may be represented by an expression analogous to an equation describing the time concentration for a series of fully mixed reactors [9]. In recovery terms this may be expressed in a mathematical form, similar to the Gamma model:

\[ R = R_e \left( 1 - \left( \frac{1}{t+\frac{k}{1+\frac{t}{k}}} \right)^p \right) \]  

(16.40)

This model describes the first order time-recovery of a component from a monodisperse feed with an exponential distribution of floatabilities.


16.4.6. Continuous Flotation

If we consider a continuous, single-cell operation under steady state conditions then \( \frac{dC}{dt} = 0 \) and the batch rate equation breaks down. From a feed of constant quantity and quality the cell produces a concentrate and a tailing whose characteristics also do not change with time. For such a cell the rate constant (or specific flotation rate) is defined by Eq. (16.41), where \( M_{s(C)} \) is the mass of solid concentrate, and \( M_{s(T)} \) is the mass of solid tailing.

In terms of the fractional recovery in the single cell, assuming all of the mineral is floatable, (that is \( R_e = 1 \)), then Eq. (16.42) is obtained.

\[ k = \frac{\text{flotation rate (mass / unit time)}}{\text{mass of floatable material remaining in the cell}} = \frac{M_{s(C)}}{M_{s(T)}} \]  

(16.41)

\[ k \lambda = \frac{M_{s(C)}}{M_{s(T)}} \]  

(16.42)

where \( \lambda = \) the nominal residence time of the pulp in the cell (cell volume/volume flow rate of tailings), 
\( M_{s(C)} = \) mass of solid in the concentrate, and 
\( M_{s(T)} = \) mass of solid in the tailing.
then \[1 + k \lambda = \frac{M_{S(C)}}{M_{S(T)}} = \frac{M_{S(T)} + M_{S(C)}}{M_{S(T)}} = \frac{M_{S(F)}}{M_{S(T)}} \tag{16.43}\]

where \(M_{S(F)} = \text{mass of solid feed.}\)

then the fractional recovery in the cell, \(R'\) (the recovery from the feed to that cell), is given by:

\[R' = \frac{M_{S(C)}}{M_{S(F)}} = \frac{M_{S(T)}}{M_{S(T)}} = \frac{k \lambda}{1 + k \lambda} \tag{16.44}\]

For several such cells in series, where the volume of concentrate recovered from each cell is small compared with the total flow, the fractional recovery in the first cell, from the feed to the first cell, will be:

\[R_1 = R_1 = \frac{M_{S(C)}}{M_{S(F)}} = \frac{k \lambda}{1 + k \lambda} \tag{16.45}\]

The fractional recovery in the second cell from the feed to that cell (that is, from the tailings of cell 1) is also given as:

\[R_2 = \frac{k \lambda}{1 + k \lambda} \frac{M_{S(C)}}{M_{S(F)}} \tag{16.46}\]

The recovery from the cell 2 in terms of the original feed to cell 1 will be:

\[R_2 = \frac{M_{S(C)}}{M_{S(F)}} = \frac{M_{S(C)}}{M_{S(T)}} = \frac{k \lambda}{1 + k \lambda} \left(1 - R_1\right) = R_1 \left(1 - R_1\right) \tag{16.47}\]

and the total recovery from \(N\) cells will be:

\[R = R_1 + R_1 \left(1 - R_1\right) + R_1 \left(1 - R_1\right)^2 + \ldots + R_1 \left(1 - R_1\right)^{N-1} = 1 - \left(1 - R_1\right)^N \tag{16.48}\]

Substituting Eq. (16.45) into Eq. (16.48) we obtain:

\[R = 1 - \left(1 + k \lambda\right)^N \tag{16.49}\]

Eq. (16.49) is also obtained by substituting \(t = N t_R\) into Eq. (16.18) where \(t_R\) is the effective residence time, given by:

\[t_R = \frac{\ln(1 + k \lambda)}{k} \tag{16.50}\]
This shows that under semi-batch conditions, the residence time for the same recovery as in a bank of N cells is N \( t_R \). For a single continuous cell where N = 1, Eq. (16.49) can be written as:

\[
\frac{R}{(1-R)} = k\lambda
\]  

(16.51)

so that if the residence time is varied and the recovery measured, a plot of \( R/(1-R) \) vs time will give a straight line of slope k. The use of Eqs. (16.45) and (16.49) is illustrated in example 16.2.

### Example 16.2
Consider the treatment of a very simple ore that contains 5 t/h of pyrite and 95 t/h of mineral quartz, and that the ore is fed to a bank of cells so that the retention time in each cell is two minutes. If we sample the feed, concentrate and tailings from any cell we may calculate the recovery of the two minerals in the cell from the feed to that cell. If the recoveries are calculated as:

- \( R_{\text{pyrite}} = 37.5\% \) and
- \( R_{\text{quartz}} = 3.8\% \)

Substituting in Eq. (16.45) the rate constant is calculated as:

\[
\frac{37.5}{100} = \frac{k\lambda}{1+k\lambda}
\]

\[
0.375(1+k\lambda) = k\lambda
\]

\[
0.375 + 0.375k\lambda = k\lambda
\]

\[
0.375 = k\lambda(1-0.375)
\]

\[
k_{\text{pyrite}} = \frac{0.375}{\lambda(1-0.375)} = \frac{0.375}{2 \times 0.625} = 0.30 \text{ min}^{-1}
\]

and similarly, \( k_{\text{quartz}} = 0.02 \text{ min}^{-1} \)

Using Eq. (16.49) the products of a bank of any number of cells can be calculated. For example, from six cells in series the recoveries from the feed will be:

- \( R_{\text{pyrite}} = 1-(1+0.30 \times 2)^{-6} = 0.9404 \) or 94.0% and
- \( R_{\text{quartz}} = 1-(1+0.02 \times 2)^{-6} = 0.2097 \) or 21.0%
The tonnages of the two minerals in the concentrate will be:

\[ Q_{\text{pyrite}} = Q_{\text{feed(pyrite)}} \times R_{\text{pyrite}} = 5.0 \times 94.0\% = 4.70 \text{ t/h} \]

\[ Q_{\text{quartz}} = Q_{\text{feed(quartz)}} \times R_{\text{quartz}} = 95.0 \times 21.0\% = 19.92 \text{ t/h} \]

and the total concentrate mass = 4.70 + 19.92 = 24.62 t/h

and the grade of the concentrate will be:

\[ \text{grade} = \frac{M_{\text{pyrite}} \times 100}{M_{\text{feed}}} = \frac{4.70}{24.62} \times 100 = 19.1\% \text{pyrite by mass} \]

Similarly, the recovery of each mineral, and hence the total tonnage of concentrate and its grade, can be calculated for any change in feed rate (change in the value of \( \lambda \) in Eq. (16.49)).

Kinetic equations therefore appear to be suitable for at least some useful estimations in the plant. The model developed so far is based upon a rate equation of any order but, for their application, the value of \( n \) must be known. We must also know whether the value of \( n \) remains constant for all conditions of flotation, or whether we have to apply rate equations of different order in different circumstances.

Similarly, not only will different minerals have different values of \( k \), but the same mineral will have different values of \( k \) under different conditions of reagent addition and cell operation. In fact \( k \) is used to determine the optimum conditions of reagent addition and cell operation. If simulations are to be developed for the purposes of prediction and control, it will be necessary to be able to predict the various values of \( k \) under different circumstances.

16.4.7. Laboratory Testing of Kinetic Relationships

**Batch Testing**

The technique involved in batch flotation testing to verify the kinetic model of flotation involves the collection of the concentrate over carefully timed intervals, drying, weighing, assaying each concentrate and the final tailing, calculating the values of \( C_0 \), \( C_\infty \) and several values of \( C \) at different flotation times, and plotting:

\[ \ln \left( \frac{C_0 - C_\infty}{C - C_\infty} \right) \text{ versus } t \text{ for first-order or} \]

\[ \left( \frac{C_0 - C_\infty}{C - C_\infty} \right) \text{ versus } t \text{ for second-order} \]

The batch flotation technique, however has many disadvantages which can be summarised as follows:
1. The concentration of floatable material in the cell at any time \( t \), is obtained only from the difference between the mass in the feed and the mass floated up to time \( t \). This implies that material must be removed immediately it appears in the froth, otherwise it will appear as non-floated. This immediate removal is physically impossible in the early stages of the test when rapid flotation is occurring and when the most accurate data should be obtained. Towards the conclusion of the test, when \( C \) is approaching \( C_\infty \) the term \( \ln (C_0 - C_\infty)/(C - C_\infty) \) is subject to considerable error.

2. Rapid changing of concentrate trays in the early stages of the test is subject to high relative errors in timing.

3. The concentrate contains fluid from the cell, so that the pulp level in the cell is continually falling. Even if the pulp level, and hence the volume of pulp in the cell, can be measured the aeration and agitation characteristics of the cell may change with decreasing pulp volume. Thus the level of the pulp should be maintained constant by the addition of fluid to the cell.

4. The fluid removed with the concentrate contains a higher concentration of any surface active agent, particularly frother, than the pulp. The concentration of any such reagent in the pulp must therefore decrease with time. This can be made up by adding make-up water containing reagents dissolved in the appropriate amounts, but the determination of what these amounts should be is virtually impossible.

5. A further criticism of the batch procedure is the difficulty of denoting zero time. There is a time lag between turning the air on in a batch cell and the formation and removal of concentrate. Any time in a batch test, however, may be denoted as zero time, and it is far better to let the test proceed until froth removal is under control before taking experimental samples.

Better reproducibility of flotation results from batch cells has centred on improvements to the subjective process of froth removal. For serious kinetic studies, laboratory cells have been designed or modified to improve or eliminate the manual scraping of froth from the top of the cell. For example, the Australian Standard 2579.1 [10] for hard coal froth flotation testing specifies a modified Denver laboratory cell comprising a Perspex deflector block for pushing the froth around the impeller shaft to the front of the cell for easier, more convenient and more reproducible froth removal. In a similar manner, the Leeds flotation cell was developed to give accurate control over impeller speed, air-flow rate, pulp level and froth removal [11]. This cell design involves a stationary, slightly inclined froth deflector plate placed above the cell so that it just touches the top of the froth and deflects it forward over the cell lip. Automatic pulp level control is also incorporated into the designs. Froth removal by the use of mechanical paddles are also used in laboratory flotation cells in an attempt to eliminate human bias in concentrate removal.

**Steady-State Testing**

For improved scale-up between laboratory measured rate constants and plant values, the laboratory test should be carried out in a continuous cell and ideally in a minimum of 2 cells in series. If the feed pulp to a continuous flotation cell is maintained constant in all respects, the cell will attain steady-state conditions rapidly (after the passage of 2-3 cell volumes). Thereafter the quantity and quality of the concentrate and tailing will not change with time, and they can be sampled accurately. Two methods of varying the solids concentration in the cell may be used:
1. Changing the feed pulp density or
2. Changing the feed rate.

Changing the feed pulp density immediately poses the problem of how reagent additions should be varied to provide an identical chemical environment from test to test. It seems inherently more reliable to start each test with an identical feed pulp, and to vary the feed rate to the cell.

The success of the steady-state method depends upon obtaining a representative sample of the pulp in the cell. If the cell acts as an ideal mixer, the new feed entering the cell is rapidly distributed, the pulp in the cell is uniformly mixed, and the tailing stream emerging is an accurate sample of the pulp in the cell in all respects. The mixing characteristics of flotation cells can be obtained using tracer techniques. These tests showed that the behaviour of water in a continuous cell indicated ideal mixing in some but not all cases. The mixing behaviour of the solids should be tested separately as water will tend to pass through a cell faster than the solids.

Once a continuous cell has settled down to steady-state conditions the product streams will not vary with time unless and until the feed changes in some respect. In a laboratory test, with limited amounts of feed pulp and agitation volume, a change if feed is inevitable. If all of the requirements of the continuous technique can be met (constant feed, steady state separation, ideal mixing in the cell), it is far superior to the batch technique. The investigation of the steady-state condition can be carried out by analysing samples of the concentrate and tailing over consecutive time intervals. If they remain unchanged, the system may be considered to be in steady-state. The tailing, however, is to be taken as a sample of the pulp in the cell, and this can only hold if the cell is acting as an ideal mixer.

If an impulse of tracer is introduced into an ideal mixer and its concentration at time \( t_0 = y_0 \), it can be shown that its concentration at time \( t \) is given by:

\[
y = y_0 \exp \left( -\frac{t}{\lambda} \right)
\]

If the logarithm of the concentration \( y \) is plotted against \( t \), a straight line will indicate ideal mixing and the slope of the line will be the reciprocal of the nominal residence time (Fig. 16.8).

For more than one cell in series, the residence time of particles in the bank will change. The residence time for a series on \( N \) cells is given by:

\[
P(t) = \left( \frac{N}{t} \right)^N t^{N-1} e^{\frac{-N}{\lambda}} (N-1)!
\]

(16.53)

where \( P(t) \) = the probability that particles will have a residence time \( t \),

\( N \) = number of tanks or cells in series,

\( \lambda \) = the mean residence time.

The residence time distribution is shown in Fig. 16.9 where the mean residence time is 2 minutes. The curve for \( N = 1 \) corresponds to an ideally mixed cell as shown in Fig. 16.8.
For a single cell of volume, $V$, most of the particles exit the cell straight away, though at extended times, beyond 5 minutes, there are still some particles exiting the cell. Increasing the number of cells to 2 and decreasing each cell volume to $V/2$ to maintain the same total volume shows that it takes a bit longer before particles of the tracer start to exit the last cell and more come out at about the same time. If the number of cells is increased to 50, and the cell size decreased to $V/50$, it takes even longer for particles to exit the last cell and the peak of residence times becomes sharper and approaches plug flow behaviour with the mean residence time of 2 minutes. Most particles have the same longer residence time and hence will have a greater chance to float in a large number of small cells compared to a small number of large volume cells.

Fig. 16.10 shows the results of a series of tests where the retention time is varied and recovery is plotted according to Eq. (16.51).
Fig. 16.9. Residence time distribution for a bank of \( n \) cells in series.

One graph shows the results corrected for the unrecoverable portion of the mineral and the other graph uncorrected. Both are good straight lines, indicating the suitability of a first order rate equation to describe the recovery of a mineral species in the flotation process. The slope of the graph is equal to the flotation rate constant and illustrates the importance of correcting for unfloatable material in determination of the true rate constant.

Fig. 16.10. Relationship between recovery and retention time.
The majority of researchers who have used the more accurate, steady-state method of testing have produced evidence supporting a first order equation. If, however, the micro-mechanisms of the flotation process and those factors that affect the flotation rates of particles are considered, the use of a first order equation to describe the flotation of all of the particles of a species is likely to be an oversimplification of the problem.

16.5. Factors Affecting the Rate of Flotation

In the kinetic model we have been considering the flotation process as analogous to a chemical reaction in which instead of colliding and reacting ions or molecules we have interaction between mineral particles and air bubbles. The flotation rate constant can be used to quantify the effect of numerous variables on the flotation process. This can be carried out in laboratory flotation machines with the knowledge that the scale-up of the laboratory batch rate constant may not be accurate though, in general, the trends in rate constant variation with changing flotation variables will be followed.

16.5.1. Impeller Speed

There are several factors of cell operation that are important in the determination of the flotation rates. If it is assumed that the collision of particles with freely moving bubbles is the dominant mechanism, and that bubble precipitation provides a favourable preliminary step to collision and adhesion, then for a given flow rate of air to a cell, an increase in impeller speed should cause more air solution and precipitation, and should break up the undissolved air into finer bubbles. This increases the rate constants of all of the mineral particles up to the point where the agitation is so intense that bubble-particle combinations are disrupted by the severe turbulence, and flotation rates fall again. This is illustrated in Fig. 16.11.

16.5.2. Air Flowrate

For a given impeller speed, an increase in the amount of air to a cell may be expected to increase the value of the rate constant for a species. Fig. 16.12 shows the effect of increasing

![Fig. 16.11. Relationship between flotation rate constant k and impeller speed.](image-url)
air concentration on the rate constant. As the volume of air increases the rate constant at first increases but at a particular air flowrate the slope of the graph decreases. When more air is forced through the impeller, its residence time in the shear zone is decreased, so that, although the concentration of air in the cell increases, this air is in the form of larger bubbles and the value of the rate constant remains almost unaffected.

### 16.5.3. **Particle Size**

The probability of collision and adhesion of a particle with a freely moving bubble must vary with the size of the particle because of:

1. its projected area
2. its inertia which will govern whether or not a particle may cut across flow lines around the bubbles
3. the possibility of its being thrown off the bubble even after adhesion has taken place due to disruptive turbulence
4. the extent to which collision may distort the bubble and alter the time of contact, and
5. the effect of particle size upon the induction time.

These factors all assume that the particles are homogeneously mixed in the cell. In cells treating coarse and dense particles, such as in flash flotation cells, the coarsest may be concentrated in the lower part of the cell, where their chances of collision with air bubbles are greatly reduced. In any given cell, considering the size of the bubbles produced and the disruptive forces present due to the turbulence of the pulp, there is a maximum size beyond which particles cannot be floated in mechanical cells. This maximum is about 420 microns with sulphide ores and is correspondingly coarser for a lighter material like coal. Once any particle becomes attached, its chances of being pulled off the bubble again decreases with decreasing size.
In practice, these physical effects mean that different sized particles of the same mineral, under the same chemical conditions, must be expected to have different rate constants. Fig. 16.13 and 16.14 show the variation of flotation recovery and rate constant with particle size for the flotation of a coal sample.

For a sulphide float, the maximum rate constant relates to an intermediate size of about 35 microns, whereas for the less dense coal, the optimum size for maximum flotation recovery is 100–200 microns.

![Fig. 16.13. Relationship between recovery and particle size for coal for increasing flotation time (data from [12]).](image1)

![Fig. 16.14 Relationship between flotation rate constant k and particle size for coal (data from [12]).](image2)
16.5.4. Reagents

Collectors

The functions of a collector is to transform the hydrophilic surface of the freshly broken mineral particles to the hydrophobic state. The effectiveness of a collector may be measured by either:

1. contact angle between an adhering air bubble and the mineral surface, or
2. the induction time required for adhesion to take place.

Both of these parameters give a good indication of the probability of adhesion taking place upon contact. In any series of tests, the contact angle formed between an air bubble and a particle surface increases with collector concentration up to a maximum value, \( \theta_{\text{MAX}} \), which is characteristic of the collector, and to some extent, independent of the mineral. For example, ethyl xanthate gives a characteristic contact angle of 60° with several different minerals.

Other xanthates give a \( \theta_{\text{MAX}} \) value which increase with the length of the hydrocarbon chain in the xanthate ion, varying from about 50° for methyl xanthate to about 95° for a 16-carbon chain xanthate. The iso-xanthates, those in which the hydrocarbon chain is branched rather than having all the carbon atoms in a straight line, have slightly higher maximum contact angle values for the same number of carbon atoms.

If collectors other than xanthates are used, the value of \( \theta_{\text{MAX}} \) depends again on the length of the hydrocarbon chain, and not on the nature of the polar group in the collector ion. Therefore, other things being equal, \( \theta_{\text{MAX}} \) for the ethyl mercaptan C₂H₅-SH is the same as \( \theta_{\text{MAX}} \) for sodium ethyl xanthate.

In the laboratory it has been shown that the flotation rate constant increased with collector concentration up to a maximum, and this maximum rate constant varies with the type of collector; that is, the length of the hydrocarbon chain. Under laboratory conditions, the maximum contact angle requires a certain collector concentration and takes time for the adsorption of the collector to take place. In some systems, the maximum contact angle may not be achieved for an hour or more. In a continuous plant, this would mean that the longer the pulp remained in the circuit, the larger the contact angle would become and the higher would be the rate constant.

In a freshly ground ore, it has been shown that the surfaces of the particles of any one mineral differ from place to place. The exposed surfaces may be those of different crystallographic planes, or impurities may be present in very small amounts. The outcome is that the collector is not adsorbed evenly over the whole surface, but in patches. If more collector is added, it tends to be adsorbed on the same sites, rather than on clean areas, so that the equilibrium condition is has xanthate adsorbed in these areas, with bare patches in between. This can only result in the contact angle being smaller, or the induction time being greater, than that observed in the laboratory.

Although the actual values of the contact angles and induction times obtained in the laboratory are not generally applicable to plant conditions, the trends demonstrated will almost certainly be followed in the plant and it can be expected that the rate constants of the particles in a plant will vary according to these trends. In practice, the amount of xanthate added is considered to be sufficient to establish an equilibrium contact angle after only a very brief conditioning time and that xanthate addition is unlikely to be rate-controlling. If, however, starvation amounts of reagent are used to obtain selectivity, small changes in this quantity may have profound effects. The addition of xanthate to a copper ore for example
will not only affect the chalcopyrite but also the pyrite and pyrrhotite that may be present. If the amount of xanthate added is sufficient that the chalcopyrite is in the plateau region of response, we may expect that the extra xanthate may have little effect upon the rate constant for the chalcopyrite. The gangue sulphides, however, may not have been in their plateau regions, and the same increase in xanthate may have a great effect on the rate constants of these sulphides.

Any given xanthate addition produces its own values of \( k \) and \( C_\infty \) for any given mineral, and an increase in xanthate may alter both \( k \) and \( C_\infty \), resulting in very different recoveries and grades. Stage addition of reagents may have these same effects.

**Frothers**

The main function of the frother is to permit the formation of a froth that is sufficiently stable to hold the floated minerals so that they may be removed as a concentrate. It has other important effects on the flotation process that may be interpreted in terms of the flotation rate constants of the minerals.

The ideal froth is one in which the entrapped gangue particles drain but the valuable minerals are held for long enough to be removed into the concentrate launder. Frother type and frother concentration both affect the quality of the froth. Too little froth gives an unstable froth which tends to collapse; a little too much frother may cause the froth to be too tight, allowing very little drainage; more frother causes the froth again to become unstable. All these effects will alter the rates of recovery of the various mineral types and hence their effective flotation rate constants. Fig. 16.15 shows the effect of frother concentration on the flotation rate constant.

The concentration of frother also affects the physical conditions in the pulp. Increase in frother concentration causes the air to form finer and more numerous bubbles, increasing the rate constants of all the minerals. Bubbles in the presence of any surface-active agent tend to maintain their spherical shape, and are distorted by collision to a lesser extent. A high concentration of frother, if it is given sufficient time to be adsorbed and oriented at the liquid-gas interface, tends to prevent the adhesion of bubbles to hydrophobic mineral particles.

All of these factors affect the flotation rate constants of the mineral particles in a given flotation cell. Within the general operating limits, an increase in frother concentration will cause an increase in the rate constants of all the particles.

**Modifying agents**

Generally a collector and a frother is not sufficient in a flotation system. Even in an operation producing only one concentrate, the separation is not only of a sulphide from a non-sulphide, for example, but also a sulphide from other sulphides. By the use of xanthate alone, the rate constants for the gangue sulphides will generally be of the same order as that of the chalcopyrite, and high grade concentrates can only be obtained by the addition of depressants, which greatly reduce the values of the rate constants of the gangue sulphides.

### 16.6. Application of Kinetic Equations

#### 16.6.1. Practical Considerations

A simple case of the application of the kinetic rate equation for continuous flotation to the prediction of the products of a bank of cells was given in example 16.1. The use of this equation, however, depended on four simplifying assumptions, namely:
Fig. 16.15. Relationship between flotation rate constant $k$ and frother concentration.

1. the cells in the bank are identical in all physical respects. That is, no change in physical environment occurs which might have an effect on the values of the rate constants,
2. the effective chemical environment of the particles remains constant throughout its flow through the bank. That is, no chemical change takes place which would affect the various values of the rate constant,
3. the volume flow rates of the various concentrates from the cells are negligibly small, and
4. the cells are all ideal mixers and are baffled so that no pulp from any cell may flow into the preceding cell.

**Physical Differences from Cell to Cell**

Individual cell volumes in a bank are usually equal, although in fully baffled cells, the heights of overflow weirs may be varied and may affect the volumes. In normal, steady-state operation, these volumes may be equalised by physical adjustment. Most of the other operating variables will differ to some degree from cell to cell, in particular cell impeller speeds, due to belt slippage, etc., impeller-stator clearances (in some types of cells), due to wear, and air flow rates, due to partial blockage of air passages. In addition, the rate of arrival of air bubbles at the pulp-froth interface can be expected to be a first order function of the concentration of air bubbles in the cell. For an ideally mixed cell, the pulp flowing out of the cell will contain air bubbles at a concentration equal to that in the cell. When flowing from one cell to the next, if the air does not escape from the pulp (in an ordinary overflow weir it probably will; in a cell baffled so that flow from cell to cell takes place below the pulp surface, it should not), the concentration of air bubbles must be expected to rise from cell to cell. The concentration of air in the $N^{th}$ cell of a bank is given by:
\[ C_{AN} = C_{AI} \frac{r^{N-1}}{(1-r)} \]

where
\[ r = \frac{k_A}{k_A + k_{A'}} \]
\[ k_A = \text{rate constant for air removal via froth (dependent on interfacial area/volume ratio)} \]
\[ k_{A'} = \text{rate constant for air removal via tailings (dependent on pulp flow rate)} \]

**Chemical Differences from Cell to Cell**

As the pulp flows down the bank of cells, surface active agents, particularly the frother, will be removed preferentially in the concentrate. This would be expected to reduce the values of the rate constant of all the particle types, although not necessarily by the same amount. The adsorption of xanthate on sulphides is practically irreversible so that a decrease in xanthate concentration would be expected to have little effect after adsorption has taken place. Inorganic reagents show little, or even negative adsorption at gas-liquid interfaces.

It is probable, therefore, that the only important change in chemical environment would be that of frother concentration, except in those cases when chemical reactions are incomplete when the pulp enters the first cell. This may be true of pH adjustment using lime and some depressing and activating reactions.

**Variation in Residence Time**

In most cases of flotation the volume flowrates of concentrate, at least from the first few cells in a bank of roughers, cleaners or recleaners are not negligible and the residence time of the remaining pulp must increase from cell to cell. In this case, if the residence time in the various cells are \( t_1, t_2, \ldots, t_N \), then:

\[ R_1 = \frac{k_{t_1}}{1 + k_{t_1}} \quad (16.55) \]

\[ R_2 = \frac{k_{t_2}}{1 + k_{t_2}}(1 - R_1) \quad (16.56) \]

\[ R_N = \frac{k_{t_N}}{1 + k_{t_N}} \left( 1 - \sum_{i=1}^{N-1} R_i \right) \quad (16.57) \]

The total recovery from the bank of \( N \) cells is \( \sum_{i=1}^{N} R_i \).

There is no algebraic simplification in this case and the expression must be solved numerically.

**Mixing in the Cells**

In a continuous cell, an ideal mixer can be indicated by a straight line. Fig. 16.16 shows the results of tracer tests carried out on fully-baffled rougher cells, using a KBr tracer, and is further evidence of ideal mixing in certain cells. Note, the use of a water soluble ion such as Br\(^-\) will only measure the mixing behaviour and retention time for the water in the cell.
A pulse tracer test on an open flow machine indicated a nominal residence time in the bank of less than half that expected from the ratio of the machine volume to the feed volume flow rate. Since a heavy concentrate was removed, the retention time of the pulp would have been expected to be even greater than this calculated figure. The most likely explanation is that a major portion of the pulp is swept rapidly along the back of the trough (or the front, depending on the direction of impeller rotation), and that the remainder of the available volume is occupied by back-mixed pulp and possibly dead space. This test indicated that the open flow machine does not behave as a series of discrete ideal mixers, and that the absence of effective baffling causes the volume available for pulp treatment to be used inefficiently.

16.6.2 Basic Data for Process Simulation

In this section, we will consider only cells which are fully baffled, and in which ideal mixing takes place and we also assume that a first order rate equation describes the flotation of any particle species, and that the value of the rate constant, k, for any given species remains constant from cell to cell, but not necessarily that retention times remain constant from cell to cell. A value of k, the rate constant may be determined in the laboratory and this value may be expected to vary with changes in some physical and chemical conditions, and the k values obtained are not directly applicable to the plant because the value of k is characteristic of the cell in which the process is carried out. Similarly, we have seen how the retention time in continuous cells may be measured by the use of tracers but we are unable to predict how the retention time varies in a cell with change of feed rate or flotation rate, except in the very simple case where the concentrate volume flow rate is negligible.

There are several methods of determining the values of the rate constants of the various mineral species in a bank of continuous cells, but most of them involve the analysis of a large number of samples (two per cell).
In an *ideal* batch test or in a continuous cell under plug flow conditions, Eq. (16.10) applies if flotation is a first order rate process. This equation does not apply to banks of cells comprising a series of ideal mixers. For such a bank of cells the expression:

\[
\ln \left( \frac{C_0 - C_m}{C - C_m} \right) = N \ln(1 + k \lambda) \tag{16.58}
\]

describes the decay in the concentration of the species in the pulp, where \(N\) is the cell number, and \(\lambda\) is the nominal retention time of the pulp in the cell. This equation is valid only for cases where \(\lambda\) is constant from cell to cell, that is when the volume of concentrate removed is negligible.

In the case of plug-flow, a graph of \(\ln (C_0 - C_m)/(C - C_m)\) versus time will be a straight line of slope \(k\). In the case of a series of ideal mixers, a graph of \(\ln (C_0 - C_m)/(C - C_m)\) versus \(N\) (which is proportional to time \(t\) if \(\lambda\) is constant) will be a straight line of slope \(\ln (1 + k \lambda)\), from which \(k\) may be derived if \(\lambda\) is known.

On the basis of the equation for a bank of ideal mixers, the value of \(k\) may be obtained from the analysis of one sample of ore if accuracy is achieved. Pulp taken from within each cell, providing that concentrate volumes are negligible.

The results of a series of tests based on this method is shown in the Fig. 16.17 and Fig. 16.18.

The method involves the analysis of one sample per cell in the bank, although alternate cells could have been omitted and a single sample of every mineral species could be determined, then every sample would have to be analysed for every species. It is desirable, therefore, that the number of samples to be analysed should be as small as possible. It would also be better if all of the samples on which the analysis is based were taken by automatic samplers, over relatively long periods of time. This is only feasible when sampling the bank feed, concentrate and tailing.

Fig. 16.17. Variation of copper concentrations within a bank of 12 cells.
The retention time may be measured relatively easily by a tracer, such as KBr and using a bromine selective electrode to determine concentration in solution, so that if the retention times are measured in some of the cells and samples of the bank feed, concentrate and tailings are taken, the values of k of the different species may be calculated by an iterative technique as follows:

1. Using an estimated value of the rate constant of any particle type, and the values of the retention times measured experimentally, calculate the recovery that would be obtained at this rate constant, using Eq. (16.45) etc..
2. Compare this calculated recovery with the actual recovery obtained from the plant samples.
3. If the recoveries are not sufficiently close, adjust the estimated value of k and repeat the calculations.
4. Repeat steps 1–3 until the required accuracy is obtained.
5. Repeat steps 1–4 for the other particle types in the feed.

The advantages of this method are:

1. Only three samples are comprehensively analysed.
2. Full allowance is made for variation in retention times.
3. Carrying out the actual retention time experiments in the plant is physically simple and rapid and requires no special equipment.
4. Not all cells in a long bank would have to be subjected to retention time tests. The retention times in consecutive identical cells would be expected to increase according to a reasonably smooth curve relationship.
5. It does not involve the application of any formulae that are algebraic approximations and it may be used on a bank of any number of cells.
16.7. Other Flotation Models
There have been a number of models developed to describe the flotation process, of which the Kinetic Model of flotation is one. A brief description of some of the other mathematical approaches to the flotation phenomenon follows.

16.7.1. Probability Models
The specific flotation rate, rate constant or flotation rate coefficient may be defined as the rate of flotation (mass per unit time) of a pulp constituent divided by the mass of that constituent in the pulp body of the flotation cell. That is:

\[ k = \frac{Q_{MS(c)}}{M_{SP}} \]  

(16.59)

where \( k \) = specific flotation rate,  
\( Q_{MS(c)} \) = mass flow rate of a pulp constituent into the concentrate, and  
\( M_{SP} \) = mass of that constituent in the pulp.

From the hypothesis of Gaudin et al [13], this can be written in terms of the probability of success of a sequence of events such that:

\[ k = P_C P_A F \]  

(16.60)

where \( P_C \) = probability of collision of a given particle with an air bubble, during a given time interval,  
\( P_A \) = probability that the particle will adhere to the bubble after collision, and  
\( F \) = froth stability factor which takes into account occurrences in the froth which may cause the particle to become detached from the bubble and drop back into the pulp.

The froth stability factor is sometimes expressed in terms of further probability terms:

\[ F = P_E P_F \]  

(16.61)

where \( P_E \) = probability of levitation of the bubble-particle aggregate to the froth/pulp interface without detachment and  
\( P_F \) = probability that particles in the froth will survive the drainage of liquid from the froth and be carried into the concentrate.

The probability of collision, \( P_C \), is a function of the radii of particle and bubble, their relative velocity, the density of the particle and the viscosity of the fluid. The probability of adhesion, \( P_A \), corresponds to the likelihood of a particle successfully thinning and rupturing the wetting film and forming an angle of contact that will ensure stable attachment to the bubble surface. In addition, for adherence of the particle to the bubble, the attachment forces must withstand the dislodging forces including gravitational pull, fluid drag and the crowding of adjacent particles at the bubble surface. The probability of attachment will be a function of the equilibrium contact angle, the surface tension between the air and liquid, the particle and bubble radius and the particle density. The probability of emergence, \( P_E \), is a function of
contact angle and particle and bubble size, and the probability of froth drainage, $P_f$, is also a function of the liquid/air surface tension, the contact angle and particle size and density.

In the simple form of the probability model, the mass, $M_{S(T)}$, of a component in the tailing from a single continuous cell at steady state is related to the mass $M_{S(F)}$ of the component in the feed by:

$$M_{S(T)} = M_{S(F)} (1 - P)$$  \hspace{1cm} (16.62)

If the probability of recovery, $P$, is constant and independent of the cell number, $N$, then for a bank of cells:

$$M_{S(T)} = M_{S(F)} (1 - P)^N$$  \hspace{1cm} (16.63)

This simple form of the probability approach is similar to the simplest form of the kinetic model.

16.7.2. Two-Phase Model

This model of the flotation process is based on the flow into and out of a cell and the two-directional transfer of material between two distinct phases, the pulp and the froth as shown in Fig. 16.19.

The mass flow rate balance relationships for Fig. 16.19 are:

Froth phase:

$$\frac{dM_{S(f)}}{dt} = - \frac{M_{S(f)} Q_{V(C)}}{V_f} + a f_r(M_{S(p)}) - b f_r(M_{S(f)})$$  \hspace{1cm} (16.64)

Pulp phase:

$$\frac{dM_{S(p)}}{dt} = Q_{V(f)} C_{S(f)} - \frac{M_{S(p)} Q_{V(T)}}{V_p} + b f_t(M_{S(f)}) - a f_r(M_{S(p)})$$  \hspace{1cm} (16.65)

Fig. 16.19. The two-phase model [14].
where $M_s = $ mass of floatable solid,
$Q_v = $ volumetric flow rate,
$V = $ volume,
$f = $ function relating to the order of kinetics,
a = $ rate coefficient for mass transfer from pulp to froth,
b = $ rate coefficient for mass transfer from froth to pulp.

Subscripts $F$, $C$, $f$, $P$ and $T$ refer to feed, concentrate, froth, pulp and tailing phases respectively.

Assuming mass transfer in both directions is first order kinetics, $f_t(M_s(0)) = M_{s(0)}$, and $f_T(M_s(P)) = M_{s(P)}$, and for constant volume:

$$Q_{V(F)} = Q_{V(T)} + Q_{V(C)}$$  \hspace{1cm} (16.66)

These three equations may be solved for various cases such as the semi-batch or continuous flotation case. The specific flotation rate coefficient, $k$, in the classical flotation model at steady state may be defined as:

$$k = \frac{a}{1 + \frac{bV_f}{Q_{V(C)}}}$$  \hspace{1cm} (16.67)

Although there is considerable evidence that the kinetics of the flotation process interpreted in terms of a one-phase model is first order, it does not follow that when the pulp and froth phases are considered separately each behaves according to first-order kinetics. For example, experiments with sized fractions of iron oxides concluded that the kinetics of forward pulp to froth transfer and the return drainage from froth to pulp were mainly second order. In summary:

1. Multiphase models postulate ideal mixing in each phase and two-way mass transfer between the several phases according to simple kinetics.
2. The two-phase (pulp-froth) model describes the steady-state satisfactorily but it is only marginally successful in describing the transient state. The model does not scale up well.
3. The model does not take account of water flow between the phases, or air flow, although flow in and out of the system is described.
4. Two-phased models can be developed into multiphase models with several layered phases in the froth to account for concentration gradients, and two phases in the pulp to handle residence time effects. Modelling the froth in layers also provides for a degree of plug-flow effect with axial dispersion.
5. Many of the parameters of the two-phase model are difficult to measure and it has not been widely used.

16.7.3. Bubble Surface Area Flux

The Julius Kruttschnitt Mineral Research Centre (JKMRC) investigated the problem of a changing feed floatability as flotation proceeds down a bank of cells and considered the flotation rate of particles according to the following general equation:
Flotation response = particle characteristic x cell characteristic

where the cell characteristics can be considered in terms of the two phases, froth and pulp.

The form of the model is [15]:

\[ k = P S_B R_F \]  \hspace{1cm} (16.69)

where \( P \) = a parameter related to ore floatability, 
\( S_B \) = the bubble surface area flux and 
\( R_F \) = a froth recovery factor.

The bubble surface area flux is defined as the total surface area of bubbles available in the cell per unit cross-sectional area of cell per unit time and hence will depend on the bubble size and velocity.

\[ S_B = \frac{6 J_G}{d_{32}} \]  \hspace{1cm} (16.70)

where \( J_G \) = the superficial gas velocity, m/s and 
\( d_{32} \) = the Sauter mean bubble diameter, m.

The Sauter mean diameter is the diameter of a bubble having the same specific surface (volume per unit surface area) as the whole bubble size distribution. That is:

\[ d_{32} = \frac{\sum n_i d_i^3}{\sum n_i d_i^2} \]  \hspace{1cm} (16.71)

where \( n_i \) = number of bubble of diameter \( d_i \).

The Sauter diameter is measured by a bubble size analyser such as the University of Cape Town Bubble Size Analyser [16] or from digital images such as the McGill Bubble Size Analyser [17]. The superficial gas velocity is measured by capturing a volume of bubbles for a set time in a tube of fixed cross-sectional area.

Factors that affect the bubble size and velocity will determine the bubble surface area flux. These include the cell and impeller design, impeller speed, air flow rate and the frother type and concentration.

The froth recovery factor, \( R_F \), is defined as the efficiency with which the particles arriving at the froth/pulp interface reach the concentrate. This is dependant on the residence time of air in the froth, which is determined by froth depth. For a zero froth depth, \( R_F \) is 100% while an exceptionally deep froth will have a \( R_F \) of zero. \( R_F \) will thus be the ratio of the overall rate constant and the collection zone rate constant and lies between 0 and 100%. Other factors affecting the froth recovery factor are the air flow rate, impeller design, cell design, impeller speed and frother type and concentration.

The ore floatability, \( P \), is affected by the ore mineralogy, particle liberation and particle size, reagent coverage of the particle surface and the pulp chemistry. For a given froth
recovery factor. A plot of rate constant versus bubble surface area flux has been shown to give a straight line, the slope of which, from Eq. (16.69), will be equal to $P [15]$. Substituting Eq. (16.69) into Eq. (16.44) gives:

$$
R = \frac{PS_B R_s \lambda}{1 + PS_B R_s \lambda} = \sum_{i=1}^{n} \frac{PS_B R_s \lambda_i}{1 + PS_B R_s \lambda_i}
$$

(16.72)

16.8. Problems

16.1
An ore consisting of a valuable component A and a gangue, component B, is floated in a rougher/scavenger/cleaner circuit. The rougher/scavenger part of the circuit is shown below:

Rougher bank A consists of 2 x 50 m$^3$ cells
Rougher bank B consists of 10 x 10 m$^3$ cells, and
Scavenger bank consists of 12 x 10 m$^3$ cells

Feed grade
feed rate to rougher bank A = 350 tph
solid density = 2800 kg/m$^3$
water density = 1000 kg/m$^3$
feed density in Rougher A = 30% solids
feed density in Rougher B = 30% solids
feed density in Scavenger = 25% solids

Samples are taken of the concentrates and tailings from each bank of cells at steady state, and analysed for component A and B. The results are as follows:

Rougher bank A
Feed = 1.48% A, 98.52% B
Concentrate = 14.2% A, 85.8% B
Tailing = 0.72% A, 99.28% B

Rougher bank B
Concentrate = 5.7% A, 94.3% B
Tailing = 0.35% A, 99.65% B

Scavenger bank
Concentrate = 2.5% A, 97.5% B
Tailing = 0.19% A, 99.81% B

Assume that the concentrate flow rates are negligible compared to the feed rate.
a. From the assays, calculate the recovery achieved in each bank and from the rougher/scavenger circuit as a whole.
b. calculate the flotation rate constant of components A and B in each bank

16.2
From the values of rate constants obtained in question 16.1, estimate the difference in overall recovery from the rougher/scavenger circuit if the cells were re-arranged to a rougher bank A of 14 x 10 m$^3$ cells, a rougher bank B of 8 x 10 m$^3$ cells and a scavenger bank of 2 x 50 m$^3$ cells. Assuming the rate constants in each bank remain the same in the new configuration.

16.3
A batch flotation test of a copper ore of grade 0.24 % Cu is given below:

<table>
<thead>
<tr>
<th>Time, t (s)</th>
<th>Conc. Mass, g</th>
<th>Assay %</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>24.9</td>
<td>2.47</td>
</tr>
<tr>
<td>40</td>
<td>15.0</td>
<td>1.93</td>
</tr>
<tr>
<td>60</td>
<td>13.2</td>
<td>1.80</td>
</tr>
<tr>
<td>120</td>
<td>35.2</td>
<td>1.34</td>
</tr>
<tr>
<td>300</td>
<td>63.3</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Feed mass = 1036.8 g
Cell volume = 3 litres

Calculate the flotation rate constant, assuming first order kinetics.

16.4
The flotation of magnesite with sodium olate in a batch flotation cell gave the following results:

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Recovery (%)</th>
<th>Time (min)</th>
<th>Recovery (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.0</td>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>20.0</td>
<td>11</td>
<td>52</td>
</tr>
<tr>
<td>3</td>
<td>28.0</td>
<td>13</td>
<td>54</td>
</tr>
<tr>
<td>4</td>
<td>34.5</td>
<td>15</td>
<td>53</td>
</tr>
<tr>
<td>5</td>
<td>40.0</td>
<td>18</td>
<td>55</td>
</tr>
<tr>
<td>7.5</td>
<td>45.0</td>
<td>22</td>
<td>55</td>
</tr>
</tbody>
</table>

Determine the order of the flotation kinetics and hence calculate the rate constant.

16.5
A nickel flotation bank of 12 cells is surveyed and the flotation recovery and residence time in each cell is given below.
1. Calculate the flotation rate constant of the nickel mineral in the individual cells 1, 6 and 12.
2. Calculate the flotation rate constant of the nickel mineral in the first cell, the first 6 cells (cumulative) and the whole 12 cells in the bank.
3. Comment on the results of parts 1 and 2.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16.31</td>
<td>0.39</td>
<td>7</td>
<td>74.81</td>
<td>2.9</td>
</tr>
<tr>
<td>2</td>
<td>33.51</td>
<td>0.8</td>
<td>8</td>
<td>78.31</td>
<td>3.3</td>
</tr>
<tr>
<td>3</td>
<td>46.71</td>
<td>1.2</td>
<td>9</td>
<td>81.01</td>
<td>3.8</td>
</tr>
<tr>
<td>4</td>
<td>56.71</td>
<td>1.6</td>
<td>10</td>
<td>83.11</td>
<td>4.2</td>
</tr>
<tr>
<td>5</td>
<td>64.41</td>
<td>2.0</td>
<td>11</td>
<td>84.81</td>
<td>4.7</td>
</tr>
<tr>
<td>6</td>
<td>70.31</td>
<td>2.5</td>
<td>12</td>
<td>86.11</td>
<td>5.1</td>
</tr>
</tbody>
</table>

16.6
The following laboratory flotation results were achieved on a lead ore. Calculate and compare the flotation rate constants for galena in the rougher and scavenger cells.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Pb rougher</td>
<td>0.5</td>
<td>19.5</td>
<td>Pb scavenger</td>
<td>0.5</td>
<td>1.2</td>
</tr>
<tr>
<td>feed</td>
<td>1.5</td>
<td>38.8</td>
<td>tail</td>
<td>1.5</td>
<td>4.4</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>51.1</td>
<td></td>
<td>4</td>
<td>11.3</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>56.9</td>
<td></td>
<td>8</td>
<td>20.9</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>59.6</td>
<td></td>
<td>11</td>
<td>26.8</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>62.7</td>
<td></td>
<td>20</td>
<td>40.3</td>
</tr>
</tbody>
</table>

16.7
A laboratory flotation test on a copper sulphide ore gave the following results:

<table>
<thead>
<tr>
<th>Time, s</th>
<th>Mass, g</th>
<th>Assay, % Cu</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>30.8</td>
<td>2.98</td>
</tr>
<tr>
<td>40</td>
<td>33.7</td>
<td>1.69</td>
</tr>
<tr>
<td>60</td>
<td>26.0</td>
<td>0.98</td>
</tr>
<tr>
<td>120</td>
<td>50.0</td>
<td>0.83</td>
</tr>
<tr>
<td>200</td>
<td>22.6</td>
<td>0.45</td>
</tr>
<tr>
<td>Tail</td>
<td>768.2</td>
<td>0.08</td>
</tr>
</tbody>
</table>

1. Calculate the concentration, $C_t$, remaining in the 3 litre cell after time $t$.
2. Determine the rate constant
3. If a bank of continuous cells has the same rate constant, how many cells would be required to achieve a recovery of 85% if the cell size is 16 m$^3$ and the feed is 1344 m$^3$/h.

16.8
An ore contains 5% of copper sulphides and 95% of gangue silicates. If the sulphides float at a rate of 0.2 min$^{-1}$ and the gangue floats at a rate of 0.01 min$^{-1}$, what grade of concentrate
would be achieved in a bank of 10 cells treating 100 t/h if the total residence time for the bank is 20 minutes?

What would be the grade after 5 cells?

16.9
A laboratory batch flotation test gave the following results.

Starting mass = 1000 g
Cell volume = 3 L
Feed grade = 3% pyrite

<table>
<thead>
<tr>
<th>Time, s</th>
<th>Mass floated</th>
<th>Assay % pyrite</th>
<th>Time, s</th>
<th>Mass floated</th>
<th>Assay % pyrite</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td>120</td>
<td>205.2</td>
<td>10.81</td>
</tr>
<tr>
<td>20</td>
<td>96.2</td>
<td>9.24</td>
<td>150</td>
<td>208.5</td>
<td>10.97</td>
</tr>
<tr>
<td>40</td>
<td>148.4</td>
<td>9.72</td>
<td>180</td>
<td>209.9</td>
<td>11.06</td>
</tr>
<tr>
<td>60</td>
<td>176.7</td>
<td>10.11</td>
<td>210</td>
<td>210.5</td>
<td>11.11</td>
</tr>
<tr>
<td>90</td>
<td>197.0</td>
<td>10.53</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Determine the rate constant and infinite recovery of sulphide and quartz if the sample only contains these two minerals.

2. Calculate the grade and recovery for a bank of 4 cells (32 m$^3$ each) if the feed rate is 430 m$^3$/h. Assume the same rate constant and infinite recovery as in the batch cell.

16.10
A pyrite/quartz ore (2 components) was subjected to a series of batch float tests. The concentrate from the first test was refloated in a second cell, the concentrate from this cell was refloated in a third cell etc. The tests, conditions and results were as follows:

Test 1:
Feed assay = 4% pyrite/96% quartz
Feed % solids = 30% (mass)
Feed Mass = 1000 g
air flow = 9 L/min
Float Time = 5 minutes

<table>
<thead>
<tr>
<th></th>
<th>Pyrite</th>
<th>Quartz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate constant, k</td>
<td>0.044 s$^{-1}$</td>
<td>0.047 s$^{-1}$</td>
</tr>
<tr>
<td>Infinite recovery, $R_\infty$ (as fraction)</td>
<td>0.705</td>
<td>0.27</td>
</tr>
</tbody>
</table>

Test 2:
Feed % solids = 20 % (mass)
Float Time = 3 minutes

<table>
<thead>
<tr>
<th></th>
<th>Pyrite</th>
<th>Quartz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate constant, k</td>
<td>0.064 s$^{-1}$</td>
<td>0.040 s$^{-1}$</td>
</tr>
<tr>
<td>Infinite recovery, $R_\infty$ (as fraction)</td>
<td>0.591</td>
<td>0.181</td>
</tr>
</tbody>
</table>
**Test 3:**
Feed % solids = 15 % (mass)
Float Time = 2 minutes

<table>
<thead>
<tr>
<th>Pyrite</th>
<th>Quartz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate constant, $k$</td>
<td>0.0815 s$^{-1}$</td>
</tr>
<tr>
<td>Infinite recovery, $R_{\infty}$ (as fraction)</td>
<td>0.52</td>
</tr>
</tbody>
</table>

**Test 4:**
Feed % solids = 12 % (mass)
Float Time = 2 minutes

<table>
<thead>
<tr>
<th>Pyrite</th>
<th>Quartz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate constant, $k$</td>
<td>0.0942 s$^{-1}$</td>
</tr>
<tr>
<td>Infinite recovery, $R_{\infty}$ (as fraction)</td>
<td>0.483</td>
</tr>
</tbody>
</table>

A flotation circuit is available consisting of a rougher bank of 5 cells, cleaner banks of 3 cells, 2 cells and 2 cells (4 banks in total), each cell having a nominal residence time of 1 minute.

1. Assuming the rate data obtained in the laboratory test is appropriate for industrial size continuous cells, calculate the minimum number of cleaning stages required to obtain a concentrate grade of at least 60% pyrite. The flotation feed contains 4% pyrite at a treatment rate of 500 t/h.

2. Calculate the overall pyrite recovery achieved at a concentrate grade of 90% pyrite.

16.11
For a bank of 5 cells having a nominal residence time of 1 minute per cell, the calculated recovery is not the same as a single cell having a nominal residence time of 5 minutes. Why?

16.12
A survey of a copper rougher bank allowed evaluation of the ore floatability, $P$, as $8.3 \times 10^5$ and the superficial gas velocity was 6.5 mm/s and the Sauter mean bubble diameter was 0.78 mm. The copper recovery in the rougher bank was calculated at 60.1% and the mean residence time was 6.1 minutes. If the bubble surface area flux is increased to 88.1 s$^{-1}$ calculate the change in copper recovery.

REFERENCES
Chapter 17. Metallurgical Process Assessment

17. INTRODUCTION

The objective of all mineral processing operations is to concentrate the minerals of interest and reject the unwanted material associated with the ores. The process is often complex as seen in the previous chapters since minerals exist both physically and chemically combined with each other. Ideal clean separation of the mineral from the unwanted fraction, known as gangue or tailings, is therefore almost impossible. In the process of separation therefore a third fraction appears which is a mixture of the gangue constituents and the minerals of interest. This fraction is known as the middlings and is retreated to recover as much of the valuable mineral as possible to increase recovery but at the same time maintain a high product grade.

On repeated treatment, some of the middlings finds their way to the concentrates and some report to the tailings. Thus both the concentrate and the tailings are diluted resulting in a loss of grade of the concentrate. To what extent this loss of grade can be accepted would depend on the economics of the process. The scheme of operation usually followed is represented by Fig. 17.1 where it can be seen that the middlings fraction could be repeatedly treated to recover some mineral of interest that is relatively less contaminated with the gangue minerals.

Each stream has therefore to be assessed to determine the mass and the concentration of the mineral of interest.

This chapter deals with the method of assessing the distribution of minerals in the concentrate, middlings and tailings streams and their recovery.

17.1. Analyses of Constituents

The first step in analyzing the constituents of each stream is to obtain representative samples of the streams. The methods of sampling, the sample size and the errors involved
have been discussed in Chapter 1. Correct representative sampling is a challenge. In industrial large scale continuous operations, where the streams are ever changing, sampling is even more difficult. At best a large number of samples provide data for an average value.

Initial qualitative assessment of the raw material that forms the feed stream to a separating process is done by identifying the minerals present under an optical microscope. To get a better idea, X-Ray diffraction (XRD) and X-Ray Fluorescence (XRF) analysers are employed. These provide quantitative analysis of elements in the rock sample. The nature of interlocking of minerals in rocks and grains is normally assessed by examining polished sections of the rock particles under an optical and/or electron microscopes which can be operated at large magnifications (from 5,000 to more than 10,000 times magnification is common). For observing, the distribution of elements within a grain, polished sections are examined by Scanning Electron Microscope. The Electron Probe Micro-analyser helps to quantify the distributions of different elements within a grain. Recently Scanning Tunnelling Microscopes have been tried to examine individual atoms on the surface while Time of Flight - Secondary Ion Mass Spectrometers (ToF-SIMS) can analyse surface atoms from either mineral surfaces or adsorbed species. These instruments provide a means of understanding the complexities of separation of minerals and also indicate the possibility of liberation or otherwise. These assessments are essentially in the dry state and on small samples.

For estimating quantitatively the presence of elements in continuous operating stream, wet analytical chemical methods are most reliable. However, this is a slow process. To speed up the determinations and to get an equally reliable result Atomic Adsorption Spectrophotometers or Inductively Coupled Plasma atomic emission spectrometers are used where a solution containing the metal ion of interested is aspirated in a flame (AAS) or a plasma (ICP) in which the metal ion is converted into a free atom vapour. A monochromatic light beam is directed through the flame and the amount of radiation of a specific energy level is detected. This electron energy level is specific for an element which helps to identify it. A calibration curve with known amounts of the element and flame intensity helps in quantifying the presence of each element.

In operating plants where analytical results are required with least delay, attempts have been made to analyse the composition in situ while the process is in progress. Both dry and wet methods have been attempted with varying degrees of success. For example, in the iron ore industry, the iron (Fe) content of the ore is examined as it travels on the conveyor belt. The Fe- content is determined by using a neutron-capture, γ-ray transmission and pair production technique. In this method, the ore is irradiated by Ra\(^{226}\) γ-radical placed at the bottom of a conveyor belt. This produces electron-positron pairs. The intensity of the pair production is related to the iron content. Tests by Aylmer, Holmes and Rutherford [1] indicate reliable results are possible with iron ore streams.

In wet streams, as in grinding circuits, continuous quantitative analysis of more than one element in a stream is obtained by the use of On-Stream or In-Stream analysers. Here a radio-active source is used to irradiate elements present in a stream (or slurry). Several elements can be detected and quantified simultaneously. For details, any text book on instrumental analysis can be consulted.

In some operations the feed stream is from a single supply of ore from one mine when sampling and assaying is relatively easy, but in most cases ores from several sources form the feed to a process. In such circumstances either each source is separately sampled and blended at pre-arranged proportion (according to grade and availability) or a composite sample is analysed.

It is a general experience that in industry a sampling error of ± 2% is acceptable.
17.2. Definition of Terms

We need to be clear about the definition of terms used in assessing the different streams in a process.

17.2.1. Mass or Weight

The weight of an object is the effect of gravity acting on the mass of the object and is equal to the mass x gravitational acceleration = M\ g. Therefore the weight of an object will change depending on the value of \( g \) which will change depending on the geological location of the object. In outer space for example, objects are \textit{weightless}. The object in outer space will however still have mass and though weightless, a heavy object will require a larger force to move it than a lighter object. Back on earth, the object on a spring balance will record the weight of the object. A larger gravitational acceleration as experienced at sea level compared to a mountain top for example, will extend the spring more recording a larger weight for the same object mass. The same object on a knife edge or double pan balance will record the mass of the object as the influence of gravity is negated by the equal force acting on the second pan of standard masses. On a single pan electronic balance, the object’s mass is also recorded as the influence of gravity is compensated for electronically. Since mass is the absolute measurement and weight is variable and mass is the usual quantity measured on most balances, unless using a spring balance, this text will refer to mass exclusively.

In a mineral processing operation, the throughputs and capacities of units are expressed as tonnes/h or tonnes/day. These tonnages invariably refer to the dry tonnes passing through a process even if the process is treating a slurry of solid and water. This is principally due to the fact that it is the solid that contains the valuable mineral and hence the mass of this component is of the most concern.

17.2.2. Slurry

A slurry is a suspension of solids in a liquid. It is also referred to as \textit{pulp}. In the metallurgical industry, the liquid is almost always water. The concentration of solids in a slurry, \( C \), is expressed as:

\[
C = \frac{\text{Mass of solids in slurry}}{\text{Mass of solids in slurry} + \text{Mass of liquid}} \times 100 \quad (17.1)
\]

It can also be expressed on a volumetric basis (mass = volume x density) and is often referred to as percent solids usually by mass by also by volume. Thus the % solids of the slurry can be written as:

\[
C = \%S = \frac{M_S}{M_S + M_L} \times 100 \quad (17.2)
\]

Thus a 30\% solids slurry by mass means 30 g (or kg or tonnes) of solids in 70 g (or kg or tonnes) of water.

The concentration may also be expressed as per cent solids in the slurry, that is, if \( \%S \) is the percent solids by mass in the slurry and \( \rho_{SL} \) and \( \rho_S \) the densities of the slurry and solids, it follows that in a slurry:
1. volume fraction of solids = \( \frac{\%S \rho_{SL}}{100 \rho_S} \)  

2. volume fraction of water = \( \frac{(100 \rho_S - \%S \rho_{SL})}{100 \rho_S} \)  

That is,  \[ \frac{\%S \rho_{SL}}{100 \rho_S} + \frac{(100 \rho_S - \%S \rho_{SL})}{100 \rho_S} = 1.0 \]

and

\[ \%S = \frac{100 \rho_S (\rho_{SL} - \rho_L)}{\rho_{SL} (\rho_S - \rho_L)} \]

and  \[ \rho_{SL} = \frac{100 \rho_S \rho_L}{\rho_L \%S + (100 - \%S) \rho_S} = \frac{M_S + M_L}{V_S + V_L} \]  

The mass of 1 litre of slurry = \( \rho_{SL} \times 10^3 \) kg for density in units of kg/m\(^3\).  

The percent solids in a slurry will most often be quoted as mass % rather than volume % since mass is the property of main concern. Percent solids in some plants is also referred to as pulp density, though in reality the two terms mean different things as shown in Eqs. (17.2) and (17.5). The reader should be careful to make the distinction.

The specific gravity of common solids, at different solid-water ratios are given in Appendix A-1 and the pulp properties given in Appendix A-6.

17.2.3. Grade

Grade refers to the concentration of a mineral in a stream and is expressed as a percentage. It is determined by chemical assaying and may be expressed as:

\[ \text{Grade} = \frac{\text{Mass of Mineral constituent in a stream}}{(\text{Mass of Mineral} + \text{Mass of gangue}) \text{in the stream}} \times 100 \]  

For a number of ores, such as base metal ores, grade is quoted in terms of the contained metal rather than the mineral.

17.2.4. Recovery

Recovery describes the amount of mineral or metal of interest that is present in the concentrate in relation to that present in the feed stream. This is usually expressed as a percentage and written as:

\[ \text{Recovery} = \frac{\text{Mass of mineral or metal in the concentrate stream}}{\text{Mass of mineral or metal in the feed stream}} \times 100 \]  

Thus if we take 100 kg of a feed stream containing 2.5% nickel and if 20% by mass of the feed stream forms the concentrate, then the mass of concentrate is 20 kg. If the grade of
nickel in the concentrate equals 10%, then 20 kg of concentrate will contain $0.1 \times 20 = 2$ kg nickel. Hence Ni recovered is $\frac{2}{2.5} \times 100 = 80\%$ by mass.

17.2.5. Distribution

The concept of distribution can be seen from the following illustration. In the above example only 80% of the mineral present has been accounted for in the concentrate, the rest of the mineral must be in the tailings, neglecting the existence of a middling product. Since the mass of the concentrate is 20 kg, then the mass of the tailings must be 80 kg. Let us assume that chemical analysis indicated that the nickel in the tailings was 0.6%. The nickel distribution between the concentrate and tailing product streams is then calculated as illustrated in Table 17.1.

Table 17.1

<table>
<thead>
<tr>
<th>Stream</th>
<th>Mass of stream, kg</th>
<th>Ni Assay, %</th>
<th>Mass of Ni</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feed</td>
<td>100</td>
<td>2.5</td>
<td>2.5</td>
<td>100</td>
</tr>
<tr>
<td>Concentrate</td>
<td>20</td>
<td>10.0</td>
<td>2.0</td>
<td>2.0/2.5 = 80%</td>
</tr>
<tr>
<td>Tailings</td>
<td>80</td>
<td>0.6</td>
<td>0.48</td>
<td>0.48/2.5 = 19.2%</td>
</tr>
</tbody>
</table>

Thus 80% of nickel has been recovered into the concentrate and 19.2% lost in the tailings.

Table 17.1 is almost an ideal case as such clear separation is never possible in mineral processing operations. A fraction invariably appears as middling. The distribution in the middlings fraction is determined in the same manner as described above. That is, the analysis of metal in all the streams is ascertained and the distribution calculated. Provided the assays of the streams are accurate then the distribution or recovery in each of the streams will balance or add up to 100%. In Table 17.1 the total distribution of nickel adds up to 99.2% as a result of sampling or assay errors.

17.3. Material Balance

Material balance estimations is one of the most useful and powerful methods of assessing unit and integrated operations. It is used to calculate recoveries and distributions of valuable components at the end of the monthly accounting period or for estimating quantities in streams (mass or assay) that can’t be easily measured. The results from a mass balance is strongly influenced by the accuracy of stream sampling and assaying. Where difficulties arise in the balancing of a circuit the it is usually one or both of these factors that should be suspect. Streams that are difficult to sample (e.g. cyclone underflow) or difficult to assay (e.g. low value streams such as tailing streams) need special attention. Mass balancing described here in only basic as the complexity in balancing increases with more complex circuits and recycle streams and with less information from sampled streams. The modern spreadsheet programmes are idea for these calculations and specialised mass balancing programs such as JKMetAccount, JKSimMet, Limn and USBC are available for complex calculations.

The basic principle involved is the conservation of mass. For a process at equilibrium or steady state, it can be expressed as:

\[
\text{mass input in all streams to a unit or circuit} = \text{mass output in all streams from a unit or circuit} \quad (17.9)
\]
However, in actual practice, during a continuous operation, a steady state is difficult to maintain with feed variations and process hiccups. Therefore it will be more correct to rewrite Eq. (17.9) as:

\[
\text{mass input in all streams to a unit or circuit} = \text{mass output in all streams from a unit or circuit} \pm \text{holdup within a process} \quad (17.10)
\]

The units are in mass or mass rates, either of the total material or of the constituent of interest, that is:

\[
\text{mass input in all streams of component } i = \text{mass output in all streams of component } i \pm \text{holdup of component } i \text{ within a process} \quad (17.11)
\]

Water can be one of the constituents used in a mass balance for a wet process. In a size separation unit such as a screen or classifier the mass in specific size fractions can be used as components in a mass balance.

For a material balance to provide meaningful data, several samples need to be taken and the variance of errors to be established. Errors originate not only in the procedure of sampling but also in the method of measuring. Napier-Munn et al [2] suggests that:

1. Error = 1% for > 9% (by mass) in a \( \sqrt{2} \) size fraction.
2. Error = \([0.1 + \text{actual } \%] \times 10^{-1}\) for < 9% (by mass) in a \( \sqrt{2} \) size fraction.

Where the mass balance of multi-component system is concerned, the error of each component is more difficult to establish. For an acceptable representative stream, the sum of the squares of errors in each size fraction should be minimum. This would give the best fit of mass split.

Consider the two product process in Fig. 17.2 at steady state. In the metallurgical operation a feed, \( F \), containing a size \( i \) is split into two products with the underflow (classifier) containing the mass fraction \( C \) and (1-C) going to the overflow. \( C \) is the mass split to the underflow (underflow mass/feed mass) and (1-C) the mass split to the overflow (overflow mass/feed mass). Also let the mass fraction of size \( i \) in the feed, underflow and overflow streams be \( m_i (F) \), \( m_i (u) \) and \( m_i (o) \) respectively, then according to Napier-Munn et al. [2] the mass-balance error \( (\Delta_i) \) of each size fraction, (here size \( i \)), will be given by:

![Fig. 17.2. A two product separation process](image-url)
\[ \Delta_i = m_i(F) - C m_i(U) - (1-C) m_i(O) \] (17.12)

The errors in each stream will not be the same but will have to be determined against the variance of each size fraction. The best fit, C, to the underflow is:

\[ C = -\frac{\sum_i \left( m_i(O) - m_i(U) \right) \left( m_i(F) - m_i(O) \right)}{\sum_i \left( m_i(O) - m_i(U) \right)^2} \] (17.13)

where \( \sigma_i \) is the standard deviation for the particle size i over a number of samples from the feed. Eq. (17.13) accounts for errors in each input stream, after which estimates of recovery can be made reasonably accurately.

### 17.3.1 Two Product Formula

In an operation where the interest is to recover a mineral in a single concentrate, the set up is usually called a two product system, producing one concentrate and one tailing stream. To calculate the recovery of the metal or mineral, the material balance method is applied. The resulting formula is known as the Two Product Formula [3].

To derive the formula let us assume that in an operation, samples are taken simultaneously from feed, concentrate and tailing streams and that the weighted average corrected analysis of each stream is:

1. **Feed stream**
   - Feed mass = \( M_F \), and
   - mineral assay in the feed = \( A_F \)

2. **Concentrate stream**
   - Concentrate mass = \( M_C \), and
   - mineral assay in the concentrate = \( A_C \)

3. **Tailings stream**
   - Tailings mass = \( M_T \), and
   - mineral assay in the tails = \( A_T \)

For a two product process, the material balance will be:

Input mass of feed = Output mass of concentrate + Output mass of tailings.

That is: \( M_F = M_C + M_T \) (17.14)

Similarly for the component metal:
From Eqs. (17.14) and (17.15), Mg can be eliminated by multiplying Eq. (17.14) by \( A_T \) and subtracting the result from Eq. (17.15), thus:

\[
M(F)A(F) - M(F)A(T) = M(C)A(C) - M(C)A(T)
\]

and

\[
M(F) (A(F) - A(T)) = M(C) (A(C) - A(T)), \text{ and hence (17.16)}
\]

\[
\frac{M(C)}{M(F)} = \frac{(A(F) - A(T))}{(A(C) - A(T))} \quad (17.17)
\]

Eq. (17.17) can also be written in terms of tailing mass by eliminating the concentrate mass from Eqs. (17.14) and (17.15).

By definition, Eq. (17.8), recovery

\[
R = \frac{M(C) A(C)}{M(F) A(F)} \times 100
\]

and from Eq. (17.17),

\[
R = \frac{(A(F) - A(T)) A(C)}{(A(C) - A(T)) A(F)} \times 100 \quad (17.18)
\]

Eq. (17.18) is applicable in any plant or laboratory situation where two products are involved in an operation. This is illustrated by the Example 17.1

---

**Example 17.1**

A gold ore containing 20 ppm gold was fed to a ball mill in a concentrator at the rate of 200 t/h. The concentrate analyzed 400 ppm and the tailings 0.15 ppm Au. Calculate the recovery and distribution of gold.

**Solution**

Step 1
The given data indicates: \( A(F) = 20 \) ppm, \( A(C) = 400 \) ppm and \( A(T) = 0.15 \) ppm and \( M(F) = 200 \) t/h. The concentrate mass, \( M(C) \) is unknown but can be determined using Eq. (17.17),

\[
\frac{M(C)}{M(F)} = \frac{(A(F) - A(T))}{(A(C) - A(T))} = \frac{20 - 0.15}{400 - 0.15} = 0.0496
\]

\[
M(C) = 0.0496 \times 200 = 9.9 \text{ t/h}
\]
Step 2

Recovery, \( R = \frac{M_{(C)}A_{(C)}}{M_{(F)}A_{(F)}} \times 100 = \frac{9.9 \times 400}{200 \times 20} \times 100 = 99.3\% \)

Step 3

The distribution of gold in the streams is best illustrated in the following table:

<table>
<thead>
<tr>
<th>Stream</th>
<th>Mass, (M) t/h</th>
<th>Gold assay, (A), ppm</th>
<th>(M \times A)</th>
<th>Recovery or Distribution, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feed</td>
<td>200</td>
<td>20</td>
<td>4000</td>
<td>100</td>
</tr>
<tr>
<td>Concentrate</td>
<td>99.94</td>
<td>40</td>
<td>3997.6</td>
<td>99.9</td>
</tr>
<tr>
<td>Tails</td>
<td>100.06</td>
<td>0.02</td>
<td>2.00</td>
<td>0.1</td>
</tr>
</tbody>
</table>

The units of assay can be any mass based concentration unit as long as the same unit is used throughout, e.g. ppm, g/t or mass%.

17.3.2. Three Product Formula

When a process produces three output streams such as an additional middling or second concentrate stream, then the mineral or metal distribution can be calculated using the same principle as that of the two product formula. Such situations frequently occur in metallurgical plants. The recovery formula is generally known as the Three product formula [3].

To illustrate the method, the presence of only two metals, (A and B) is considered here for a metallurgical circuit involving crushing, grinding, classification and flotation. In each unit process, the feed is considered as the concentrate from the previous process. Let us assume that the masses and concentrations of the minerals (metals) in the three streams are as given in Table 17.2

Table 17.2

<table>
<thead>
<tr>
<th>Stream</th>
<th>Mass, (M) t/h</th>
<th>Metal A Assay, %</th>
<th>Metal B Assay, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feed</td>
<td>(M_{(F)})</td>
<td>(A_{(F)})</td>
<td>(A_{(B)})</td>
</tr>
<tr>
<td>Concentrate 1</td>
<td>(M_{(C1)})</td>
<td>(A_{(C1)})</td>
<td>(A_{(B)})</td>
</tr>
<tr>
<td>Concentrate 2</td>
<td>(M_{(C2)})</td>
<td>(A_{(C2)})</td>
<td>(A_{(B)})</td>
</tr>
<tr>
<td>Tails</td>
<td>(M_{(T)})</td>
<td>(A_{(T)})</td>
<td>(A_{(B)})</td>
</tr>
</tbody>
</table>

Considering the overall material and metal balances of the system the following equations apply:

\[ M_{(F)} = M_{(C1)} + M_{(C2)} + M_{(T)} \quad \text{for the overall mass balance} \]  \hspace{1cm} (17.19)

\[ M_{(F)}A_{(F)} = M_{(C1)}A_{(C1)} + M_{(C2)}A_{(C2)} + M_{(T)}A_{(T)} \quad \text{for component A} \]  \hspace{1cm} (17.20)

\[ M_{(F)}A_{(B)} = M_{(C1)}A_{(B)} + M_{(C2)}A_{(B)} + M_{(T)}A_{(B)} \quad \text{for component B} \]  \hspace{1cm} (17.21)
As before eliminating \( T \) from the Eqs. (17.20) and (17.21) the value of \( M_{ci} \) will be given by:

\[
M_{ci} = \frac{\left( A_{A(F)} - A_{A(T)} \right)\left( A_{B(Ci)} - A_{B(T)} \right) - \left( A_{B(F)} - A_{B(T)} \right)\left( A_{A(Ci)} - A_{A(T)} \right)}{\left( A_{A(Ci)} - A_{A(T)} \right)\left( A_{B(Ci)} - A_{B(T)} \right) - \left( A_{B(Ci)} - A_{B(T)} \right)\left( A_{A(Ci)} - A_{A(T)} \right)}
\]  
(17.22)

and similarly:

\[
M_{ci} = \frac{\left( A_{A(F)} - A_{A(T)} \right)\left( A_{B(Ci)} - A_{B(T)} \right) - \left( A_{B(F)} - A_{B(T)} \right)\left( A_{A(Ci)} - A_{A(T)} \right)}{\left( A_{A(Ci)} - A_{A(T)} \right)\left( A_{B(Ci)} - A_{B(T)} \right) - \left( A_{B(Ci)} - A_{B(T)} \right)\left( A_{A(Ci)} - A_{A(T)} \right)}
\]  
(17.23)

The recovery of metal \( A \) in concentrate 1 will be:

\[
\text{Recovery of } A = \frac{A_{A(C1)}M_{(C1)}}{A_{A(F)}M_{(F)}} \times 100
\]  
(17.24)

and the recovery of metal \( B \) in concentrate 2 will be:

\[
\text{Recovery of } B = \frac{A_{B(C2)}M_{(C2)}}{A_{B(F)}M_{(F)}} \times 100
\]  
(17.25)

The application of the method is best understood by Example 17.2

**Example 17.2**

A copper-zinc ore was fed to an integrated mineral processing system at the rate of 250 t/h. The final products were a copper concentrate and a zinc concentrate and tailing. Analysis of each stream were:

<table>
<thead>
<tr>
<th>Stream</th>
<th>Assay</th>
<th>%Cu</th>
<th>%Zn</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feed</td>
<td></td>
<td>25.0</td>
<td>3.1</td>
</tr>
<tr>
<td>Copper concentrate</td>
<td></td>
<td>78.2</td>
<td>6.3</td>
</tr>
<tr>
<td>Zinc concentrate</td>
<td></td>
<td>2.1</td>
<td>55.4</td>
</tr>
<tr>
<td>Tailings</td>
<td></td>
<td>0.7</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Determine the mass flows in the two concentrate streams and the recovery of metal in each product stream.

**Solution**

**Step 1**

To determine the masses of copper and zinc concentrate streams, Eqs. (17.22) and (17.23) can be directly applied. Thus:

\[
\text{Mass of copper concentrate} = 250 \times \frac{(25 - 0.7)(55.4 - 0.8) - (3.1 - 0.8)(2.1 - 0.7)}{(78.2 - 0.7)(55.4 - 0.8) - (6.3 - 0.8)(2.1 - 0.7)}
\]

= 78.4 t/h
Mass of zinc concentrate = \( 250 \times (25.0 - 0.7 \times 6.3 - 0.8) - (3.1 - 0.8 \times 78.2 - 0.7) \)
\( (2.1 - 0.7 \times 6.3 - 0.8) - (55.4 - 0.8 \times 78.2 - 0.7) \) = 2.64 t/h

Step 2
Recovery of copper = \( \frac{78.2 \times 78.4}{25.0 \times 250} \times 100 = 98.0\% \)
and Recovery of zinc = \( \frac{2.64 \times 55.4}{250 \times 3.1} \times 100 = 18.9\% \)

A spreadsheet such as MS Excel can be used to calculate the distributions in each streams and Solver can be used to estimate the concentrate masses without using the three product formulae above. A simple spreadsheet to do this is shown below.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Stream</td>
<td>Mass</td>
<td>Cu Assay</td>
<td>Zn Assay</td>
<td>MiA (Cu)</td>
<td>MiA (Zn)</td>
<td>Recovery</td>
<td>Recovery</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Feed</td>
<td>250.0</td>
<td>78.2</td>
<td>6.3</td>
<td>829.0</td>
<td>775.0</td>
<td>100.0</td>
<td>100.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Cu Conc</td>
<td>78.3</td>
<td>78.2</td>
<td>6.3</td>
<td>613.1</td>
<td>493.5</td>
<td>98.0</td>
<td>63.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Zn Conc</td>
<td>2.5</td>
<td>3.1</td>
<td>6.4</td>
<td>6.6</td>
<td>146.2</td>
<td>0.1</td>
<td>18.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Tail</td>
<td>169.0</td>
<td>0.7</td>
<td>0.6</td>
<td>115.3</td>
<td>136.2</td>
<td>19</td>
<td>17.4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Cells D6, D7 are the unknown concentrate masses
Cell D8 is the feed minus the concentrate masses

Recoveries in cells I6:J8 are calculated in the same manner as in Table 17.1. The sum of the recoveries in the concentrates and tailing should add up to 100% as in cells I5, J5. If the system is not balanced, the sum of the recoveries will be different from 100% and the difference is calculated in cell E14 for copper and cell E15 for zinc. These differences are squared in cells F14:F15 to remove negative numbers and the sum of the squares in cell F16 is minimised using Solver and cells D6 and D7 as variables. The Solver solution is shown in the above spreadsheet and the recoveries or distribution of copper and zinc calculated in cells I6:J8.

**Example 17.3**

A lead-zinc ore was treated at the rate of 100 t/h to an integrated system to produce concentrates of lead and zinc minerals. A closed circuit was chosen so that the middling produced was re-treated. The average lead and zinc concentrations in each stream were:
Determine the mass flowrates of the product stream and the distribution of lead and zinc.

**Solution**

Step 1
Using the two product formula:

\[
\text{Mass of Pb Concentrate} = 100 \times \frac{(1.8 - 0.5)}{(82.0 - 0.5)} = 1.60 \text{ t/h, using the lead assays}
\]

and

\[
= 100 \times \frac{(23.5 - 24.0)}{(2.0 - 24.0)} = 2.27 \text{ t/h, using the zinc assays}
\]

\[
\text{Mass of Zn concentrate} = (100 - 1.6) \times \frac{(24.0 - 1.2)}{(77.1 - 1.2)} = 29.55 \text{ t/h, using the zinc assays}
\]

or

\[
(100 - 1.6) \times \frac{(0.5 - 0.3)}{(1.0 - 0.3)} = 25.56 \text{ t/h, using the lead assays}
\]

A judgement will need to be made as to the reliability of the analytical techniques and/or the sampling procedures before deciding on the accuracy of the calculated values.

Step 2
The distribution of the metals is calculated in a similar manner to Table 17.1 and Example 17.2. The results is shown in the following table. The calculated feed or head assay is a good means of checking the balance if a measured head assay is available.
17.4. Circulating Load

The premise in the methods of assessing the streams so far has been that the circuits were open, that is, maximum product was obtained at reasonable rates in the concentrate. In actual practice it is found that all concentrate do not satisfy the required specifications of size and concentration. Hence the concentrate has to be treated to separate the unwanted fraction. The out of specification fraction is submitted for further separation by passing on to downstream processes or by adding it to the original feed. In the latter case, a continuous circulating load is set up in the system as illustrated in Fig 17.3. The recirculation operation results in improved recovery and grade. Fig. 17.3 illustrates such a situation for a simple grinding circuit where the coarse underflow fraction from the classifier is circulated for retreatment while the overflow fine fraction is the product. The circulation ratio \( C_R \) is defined as the ratio of the flowrates of the circulating stream to the flowrate of the new feed to the mill, which at steady state is equal to the fine product leaving the circuit.

\[
C_R = \text{Circulation Ratio, } C_R = \frac{T}{Q} \times 100 \quad (17.26)
\]

When a classifier returns the coarse fraction of its feed material to the mill, then the total load to the mill is increased and is the sum of the new feed stream plus the coarse returns. Thus in a closed circuit system the circulating load \( C_L \) according to Austin et al [4] is defined as:

\[
C_L = \frac{\text{Rate of new feed} + \text{Amount recycled}}{\text{Rate of new feed}} \times 100
\]

\[
= 100 + \frac{\text{Amount recycled}}{\text{Rate of new feed}} \times 100 \quad (17.27)
\]

Fig. 17.3. Circulating stream in a closed circuit grinding/classification circuit.
This can be written as:

\[ C_L = 100 + C_R \]  \hspace{1cm} (17.28)

The circulating load \( C_L \) and the circulation ratio \( C_R \) are both expressed as a percent. There is some confusion by different authors between the circulation ratio and the circulating load. The circulating load in slurry streams is not easily measured without instrumentation such as density gauges and flowmeters and hence is usually calculated by mass balancing. For a comminution circuit closed with a screen or classifier, a mass balance can be performed on the masses in individual size intervals around the size separator or on the cumulative masses passing or retained at a given size. If individual size intervals are used then a weighted average should be performed on a number of size intervals.

Bond [4] considered the circulating load associated with two general circuit configurations. In the first (Fig. 17.3) the stream returning from the classifier is to the mill feed. In the second (Fig. 17.4) the new feed is classified first and the oversize material sent to the grinding mill.

In both cases the same general formula applies. By a balance of the mass in size interval \( i \), the circulating ratio in the classifier circuit shown in Fig. 17.4 is given by:

\[
\text{Circulating load (C}_L\text{)} = \frac{(\text{Classifier under size - New feed to circuit})}{(\text{Feed to classifier - Classifier over size})} \times 100 \]  \hspace{1cm} (17.29)

Thus if \( m_i(F) \) = mass fraction of size \( i \) in the feed to the circuit,
\( m_i(CF) \) = mass fraction of size \( i \) in the feed to the classifier,
\( m_i(U) \) = mass fraction of size \( i \) in the classifier undersize,
\( m_i(O) \) = mass fraction of size \( i \) in the classifier oversize and
\( m_i(D) \) = mass fraction of size \( i \) in the mill discharge,

then the circulating load for the circuit in Fig. 17.4 can be written as [4]:

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig17_4.png}
\caption{Grinding/classification circuit with recirculating stream.}
\end{figure}
In the case of the circuit in Fig. 17.4, the mass passing through the mill and the mass recirculating in the cyclone underflow are the same, hence the circulation ratio and circulating load will be the same.

For the circuit in Fig. 17.3, the circulating ratio, by a similar mass balance of size fractions will be:

\[
C_R = \frac{\sum |m_{i(U)} - m_{i(O)}|}{\sum |m_{i(D)} - m_{i(O)}|} \times 100 \tag{17.31}
\]

To determine the circulation ratio \(C_R\), it is usual to determine the water to solid ratios of the streams (masses can also be taken), by taking simultaneous samples for the same duration. Considering the water/solid ratio, \(R\), for the different streams as in Fig. 17.4 and:

- \(R_F\) = New feed,
- \(R_D\) = Mill discharge,
- \(R_{CF}\) = Feed to classifier,
- \(R_O\) = Classifier oversize, course stream,
- \(R_U\) = Classifier undersize, fines stream.

Then applying Bond’s concept, the circulation load would be:

\[
C_R = \frac{(R_D - R_F)}{(R_O - R_D)} \times 100 = \frac{(R_U - R_{CF})}{(R_{CF} - R_O)} \times 100 \tag{17.32}
\]

When more than one circulating load is operating in an integrated circuit, the mass balances have to be established for each circuit.

Example 17.4 and 17.5 illustrates the methods of determining circulating load with and without the knowledge of the size distribution of solids in each stream.

**Example 17.4**

An integrated circuit consisted of a crusher, grinding mill and a classifier. The circuit produced 4800 t of ground ore per day. The underflow from the classifier was returned to the mill for re-grinding. The classifier feed contained 45% solids (by mass) and the classifier underflow and overflow streams contained 80% and 20% solids respectively. Calculate the circulation ratio and the circulating load.

**Solution**

Step 1
Determine the liquid/solid ratios in each stream.
Solid-liquid ratios of:

1. Classifier feed stream = \( \frac{100 - 45}{45} = 1.22 \)

2. Classifier underflow (oversize) stream = \( \frac{100 - 80}{80} = 0.25 \)

3. Classifier overflow (undersize) stream = \( \frac{100 - 20}{20} = 4.0 \)

Step 2

Eq. (17.32) may be used to determine circulation ratio \( C_R \):

\[
\text{Circulating ratio} = \frac{(4.0 - 1.22)}{(1.22 - 0.25)} = 2.33 \times 100 = 233\%
\]

Circulating load = 100 + \( C_R \) = 100 + 233 = 333%

**Example 17.5**

A rod mill was integrated with a hydrocyclone and produced a grind at the rate of 250 t/h. Samples taken simultaneously from the discharge of the rod mill and the overflow and underflow streams from the hydrocyclone gave the following results. Calculate the circulating ratio and circulating load.

<table>
<thead>
<tr>
<th>Screen Size</th>
<th>Rod Mill</th>
<th>Hydrocyclone O/F</th>
<th>Hydrocyclone U/F</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4.0 mm</td>
<td>2.0</td>
<td>0.0</td>
<td>36.0</td>
</tr>
<tr>
<td>-4 +2 mm</td>
<td>40.4</td>
<td>0.2</td>
<td>15.2</td>
</tr>
<tr>
<td>-2 +1 mm</td>
<td>63.0</td>
<td>1.8</td>
<td>8.1</td>
</tr>
<tr>
<td>-100+420 µm</td>
<td>5.1</td>
<td>15.6</td>
<td>5.1</td>
</tr>
<tr>
<td>-420+210 µm</td>
<td>16.6</td>
<td>21.2</td>
<td>13.8</td>
</tr>
<tr>
<td>-210+105 µm</td>
<td>19.0</td>
<td>20.4</td>
<td>18.5</td>
</tr>
<tr>
<td>-105+74 µm</td>
<td>5.5</td>
<td>25.0</td>
<td>2.8</td>
</tr>
<tr>
<td>-74 µm</td>
<td>5.1</td>
<td>15.8</td>
<td>0.5</td>
</tr>
</tbody>
</table>

**Solution**

Step 1

Let us choose arbitrarily the following three sizes: -4 +2.0 mm, -1000 +420 µm, -210 +105 µm instead all the eight sizes to illustrate the principle of calculations.

Step 2

Calculate the circulation ratio for each screen interval chosen. Thus

For size -4.0+2.0 mm \( C_R = \frac{42.4 - 0.2}{51.2 - 42.4} = 4.79 \)
A practical but rapid and approximate method to determine the circulating load in a grinding mill and classifying circuit is suggested by Barber-Greene[5]. The procedure is to determine the percent (by mass) of -200 mesh in the classifier feed (mill discharge) and classifier overflow and classifier underflow streams. Then for feed mass flowrate, \( Q_F \), the classifier underflow (Fig. 17.3) will be given by:

\[
Q_{(O)} = Q_F \left[ \frac{m_{(U)} - m_{(D)}}{m_{(D)} - m_{(O)}} \right] \tag{17.33}
\]

where
- \( Q_{(O)} \) = Mass flowrate of the classifier oversize (underflow)
- \( Q_F \) = Mass flowrate of the new feed
- \( m_{(U)} \) = % -200 mesh in classifier undersize (overflow),
- \( m_{(O)} \) = % -200 mesh in classifier oversize (underflow),
- \( m_{(D)} \) = % -200 mesh mill discharge (i.e. classifier feed),

Then the circulation ratio is given by:

\[
C_R = \frac{Q_{(O)}}{Q_F} \times 100
\]

Other screen sizes could be chosen for the calculation as shown in Example 17.5. A similar rapid method can be applied by taking the solid/liquid ratios of the streams as given in Eq. (17.32) and individual size fractions as given in Eqs. (17.30) and (17.31).

### 17.5. Problems

17.1
A calcite sample is ground in a mill in close circuit with a hydrocyclone. Samples were taken from the feed, overflow and underflow streams and their solid to water ratios determined as 2.5, 7 and 0.3 respectively. Calculate the circulating load of the circuit.
17.2
An iron ore company milled ore at the rate of 150 t/h in a ball mill in closed circuit with a classifier. The underflow (course) stream was re-circulated to the mill till a steady value was obtained in the cyclone overflow stream. The specific gravities of each stream were measured as:
1. Mill discharge stream to classifier = SG\(_{\text{MD}}\)
2. Product overflow from classifier = SG\(_{\text{OF}}\)
3. Product underflow from classifier = SG\(_{\text{UF}}\)

Establish a mass balance and derive an expression for the circulating load.

17.3
A copper sulphide ore was crushed and milled in close circuit with a classifier. The overflow from the classifier fed a rougher cleaner flotation circuit at the rate of 500 t/h. The ore assayed 2.75% Cu. The re-circulating load in flotation circuit was 270%. The overall recovery was 76% at a grade of 92%. Calculate:
1. Mass flowrates of the cleaner concentrate and rougher tailings,
2. Grade of copper in the rougher tailings.

17.4
The screen analysis of feed, product and tailings from a classifier were:

<table>
<thead>
<tr>
<th>Size, (microns)</th>
<th>Feed %</th>
<th>Overflow %</th>
<th>Underflow %</th>
</tr>
</thead>
<tbody>
<tr>
<td>850</td>
<td>15.2</td>
<td>0</td>
<td>18.3</td>
</tr>
<tr>
<td>500</td>
<td>8.5</td>
<td>0</td>
<td>10.2</td>
</tr>
<tr>
<td>250</td>
<td>33.6</td>
<td>2.9</td>
<td>39.9</td>
</tr>
<tr>
<td>125</td>
<td>34.2</td>
<td>65.4</td>
<td>27.8</td>
</tr>
<tr>
<td>75</td>
<td>8.5</td>
<td>31.8</td>
<td>3.7</td>
</tr>
<tr>
<td></td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
</tbody>
</table>

The classifier was fed with ground silica at the rate of 50t/h. Estimate:
1. Mass (tonnes) of dry ore per day in classifier underflow,
2. Mass (tonnes) of dry ore per day from classifier overflow.

REFERENCES