Chapter 11. Screening

11. INTRODUCTION

Minerals of interest exist in nature in the dispersed state, as a separate entity, for example native gold particles in silica rock, or in the combined form, like nickel sulphide or chalcopyrite in an host rock. Often due to relative differences in the hardness, friability and crushability between the mineral and host rocks, minerals may be "liberated" by repeated crushing and other comminution processes. The particles produced, having different sizes and shapes, can be separated over screens that allow particles that are less than the aperture of the screen to pass through while retaining the others. Such separations of mineral constituents can be an efficient and cheap method to concentrate a mineral and to reject the gangue constituents in some mineral ores.

Separations of dry materials by screens and sieves are generally attempted down to about 75 microns. Finer materials have a tendency to blind the sieve openings. In such cases, screening in the presence of water helps. Separations of even finer sizes are difficult on a sieve. For such fine material other processes have to be adopted like classification.

In the metallurgical industry a distinction is made between screening and sieving. The mechanism of size separation by both is the same, but screening generally applies to industrial scale size separations while sieving refers to laboratory scale operations.

In this chapter the design of different types of screens and their operation are described.

11.1. Basic Design features in Screens

The three most important design features of screens are:

1. Surface and aperture,
2. Types of screens,
3. Screen movement.

11.1.1. Surface and Aperture

Coarse Screen Surface – Grizzly

For the metallurgical industry coarse scalping screens are generally fabricated by welding steel rails, rods or bars forming grids of a desired pattern. These are usually known as grizzly screens. The selection of rails varies in size from about 7.4 kg/m to about 225 kg/m. The rails usually run parallel to each other for the entire length of the screening surface. The spacings in between are of the order of 5 – 200 mm. For smooth flow of materials the openings are tapered, the top being wider than the bottom. Heavy-duty grizzly bars are cast from manganese steel having double tapers [1]. These are designed to receive lump ore from railroad wagons, tipper cars and other bulk material handling systems that discharge from considerable heights. They are therefore very robustly built.
The rail grizzlies can be installed to operate in a horizontal flat plane, but they are often inclined to aid transport of ore across the screen. The inclination is of the order of 30 - 40°. For sticky ores the inclination could be up to 45°. For very sticky ores, vibrators are employed to facilitate continuous operation.

When rods are used to fabricate grizzlies, they are usually free rotating, or mechanically driven. The rods rest on bearings and rotate in the direction of material flow acting like a conveyor. The space between the rolls is the aperture of the grid.

While designing a grizzly for a specific purpose, the openings between the grizzly bars should be commensurate with the size of the receiving hopper where the product has to be discharged. Usually the maximum distance between the grizzly rails is 0.9 times the maximum hopper opening feeding say, a crusher.

Grizzlies can be designed with more than one deck (usually not more than two). The top deck has a scalping action while the lower deck aims to yield the final size. The two decks produce a coarse, middle and finer fraction. The coarse and middlings have to be recrushed and re-screened to an acceptable size.

**Medium Screens and Screen Surfaces**

These are used for screening medium size particles that are less than 100 mm but greater than about 2 mm. The screens are fabricated from:

1. Plates mainly by drilling or punching to produce a perforated pattern,
2. Woven wire surfaces to various designs.

**Perforated or Punched Plates**

Plates made of plain carbon or alloy steels, including stainless steel are used to make perforated screens. Hard plastics such as polyurethane and rubber are also used with reasonable success. Holes are punched, drilled or cast directly during the manufacturing process of the sheets. Shape of the apertures are usually circular, square, or rectangular. The circular holes are equally spaced at the corners of an equilateral triangle or at the four corners of a square or elongated rectangular pattern. Simple patterns of apertures are illustrated in Fig. 11.1. Hole spacings at 60° are common. Several variations of patterns are industrially available, like staggered squares, holes or slots or combinations of squares and rectangles. In general the square pattern is most accurate but the throughput could be less than the rectangular patterns which have much more open areas.

![Screen perforation patterns on plates: a – circular apertures on a 60° pattern; b – circular apertures on a square pattern; c – rectangular apertures](image-url)
The percent of open area of plates with circular holes, drilled one-half diameter apart, is about 5% more than those drilled on the corners of a square. When the holes are one diameter apart the difference is less. The amount of open area for diagonal and square spacings can be estimated by simple geometry. Thus if $d$ is the diameter of the hole and $s$ the minimum spacing between them, then the percent of holed area for square and diagonal spacings would be (Fig. 11.2):

For diagonal spacing

$$A = \frac{\left(\frac{\pi}{8}d^2\right)}{\sin 60\left(\frac{1}{2}(s+d)^2\right)} = \frac{0.907d^2}{(s+d)^2}$$ (11.1)

and for square spacing

$$A = \frac{\left(\frac{\pi}{4}d^2\right)}{(s+d)^2} = \frac{0.785d^2}{(s+d)^2}$$ (11.2)

The perforated plates are often rubber clad. The rubber sheets have apertures slightly larger than the base plate. The holes in the rubber conform to the product size. The rubber cladding helps to absorb the force of impact of feed material falling onto the screen. They also retard abrasion of the steel and promote a longer screen life. The elasticity of the rubber helps to reduce blinding of the screens. An added advantage of rubber-clad screens is a considerable reduction of noise level. The rubber sheets are about 7mm – 25 mm thick and held down by a steel frame with bars and bolts.

Fig. 11.2. Geometry of open area for diagonal and square placement of circular openings.
Woven Wire Screens

For woven screens, wires of uniform cross section are usually taken for both warp and weft strands. Occasionally the diameter of the warp is greater than the weft.

The wire material used depends on the environmental circumstances. Thus plain carbon steel wires are used for general purposes but for corrosive atmospheres stainless steel wires are used. Other types of metal wires commonly used are brass, bronze, monel metal (Ni-Cu alloys) and different types of aluminium alloys. Wires or threads made of plastics material, especially polyurethane are increasingly being used for areas where strong acidic, caustic or wet environments prevail.

When screens are woven with straight profile wires with circular cross-section, the wires have a tendency to move during the screening operation. Crimped wires help to lock the wires in place. Weaves with double crimped wires are now common. For smoother operation the weave is designed to provide a flat top.

The patterns of weaves are usually square, but rectangular weaves with length to width ratio of 2 or more are also common in the mineral industry. Matthews [2] suggests that for a crimped wire mesh, a rectangular aperture is stable with a slot ratio of 12:1 with large wire and 4:1 with small wire.

Wire screens are mounted on frames and held down tightly by strips of metal (or plastics) and held down firmly using bolts. For large screens appropriate supports are spaced. These support strips occupy space and therefore reduce the effective screening area. Several alternative methods of holding the screens have been devised, like side hooks.

Since the advent of different types of plastics in the form of wires and threads, industrial screens with fabricated plastic are common. The usual plastic wire thickness ranges from 5 – 25 mm. Plastic screen cloths are woven to produce square or rectangular slots that are in line or staggered. The slots are set either parallel to the direction of the flow or across. The open areas of different weaves and patterns depend on the dimensions of the wires. The common types of apertures, their dimensions and the corresponding open areas are given in Table 11.1.

Table 11.1
Screen data for rubber screen fabric [2].

<table>
<thead>
<tr>
<th>Type of Aperture</th>
<th>Aperture dimension, mm</th>
<th>Open Area, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square hole, in line</td>
<td>35 190</td>
<td>49 – 63</td>
</tr>
<tr>
<td>Square hole, staggered</td>
<td>8-30</td>
<td>33 – 44</td>
</tr>
<tr>
<td>Round hole, staggered</td>
<td>12-190</td>
<td>30 – 48</td>
</tr>
<tr>
<td>Slotted hole, staggered (flow parallel to slot)</td>
<td>2 x 25</td>
<td>28 – 41</td>
</tr>
<tr>
<td>Slotted hole, staggered (flow across slot)</td>
<td>0.30 x 40</td>
<td>23 – 42</td>
</tr>
<tr>
<td>Slotted hole, staggered (flow across slot)</td>
<td>1.5 x 25</td>
<td>23 – 42</td>
</tr>
<tr>
<td>Slotted hole, staggered (flow across slot)</td>
<td>0.14 x 25</td>
<td>23 – 42</td>
</tr>
</tbody>
</table>

The available aperture per unit area of screen is the most important criteria of screens. The apertures may be determined if the diameters of the weft and warp wires are known. Fig. 11.3 shows the warp and weft wires of a woven screen cloth with square openings and the rectangular aperture of a typical profile bar screen. It can be seen that the available screening area is the space between the materials forming the aperture. This space is expressed as a percent of the area of the screen. If we assume that the screen wires have round sections of
diameter \( d_1 \) and \( d_2 \), forming a square aperture, and if \( A_1 \) and \( A_2 \) were the clear areas, then for the square screen, open area \( A_1 \) must be equal to area \( A_2 \) or equal to any such area \( A_N \). If we also assume that the distances between them were as shown, and \( d_w = d_{w1} = d_{w2} \), then from Fig. 11.3A, the percent clear open area of the screen, \( A_O \), will be:

\[
A_O = \left( \frac{L_A}{L_A + d_w} \right)^2 \times 100
\]  

where

- \( A_O \) = open area expressed as percent,
- \( L_A \) = aperture, and
- \( d_w \) = diameter of wire (or horizontal width of bar or plates, if used).

For a rectangular opening, the open area will be given by:

\[
A_O = \frac{L_{A1} L_{A2}}{(L_{A1} + d_{w1})(L_{A2} + d_{w2})} \times 100
\]  

where \( L_{A1} \) and \( L_{A2} \) are the aperture dimensions and \( d_{w1} \) and \( d_{w2} \) the wire diameters.

When the screens are set at an angle \( \theta \) to the horizontal then the effective aperture will be diminished and will be equal to the projection of the actual screen aperture. The available area will then be modified as \( \text{Area} \cdot \cos \theta \).

For parallel bar screen surface (Fig. 11.3B) the open area is:

\[
A_O = \frac{L_A}{L_A + d_w} \times 100
\]  

Fig. 11.3. A = square or rectangular opening between wires, bars or strips; B = parallel openings between wedge wires.
The mesh of a screen is defined by the relation \( M = (L_A + d_w)^{-1} \) for measurements in inches or \( M = 25.4 (L_A + d_w)^{-1} \) for measurements in millimetres. When \( M \) is substituted in Eqs. (11.3) and (11.4), the mesh size may be calculated. For example the mesh number of a square opening of screen will be:

\[
M = \sqrt[25.4^2 A_o}{100 L_A^2}
\]

(11.6)

The use of these expressions for designing screens is illustrated in Example 11.1.

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Example 11.1

A stainless steel woven wire screen with a square aperture had an aperture 3.18 mm square. The diameter of the wire was 1.2 mm. Determine:

1. The percent open area when the screen was operated in an horizontal position,
2. The percent open area when the screen was operated at a slope of 20°,
3. The mesh size of the screen.

Solution

Step 1

As the entire screen is fabricated with wire of diameter of 1.2 mm, Eq. (11.3) is applicable for the horizontal screen.

Thus for an horizontal screen:

\[
A_o = \left( \frac{L_A}{L_A + d_w} \right)^2 .100
\]

\[
= \left( \frac{3.18}{3.18+1.2} \right)^2 .100
\]

= 52.7%

Step 2

For an inclined screen:

Effective percent open area \( A_{oe} = A_o \cos 20° = 49.5% \)

Step 3

Square opening in mesh, \( M = \sqrt[25.4^2 A_o]{100 L_A^2} \)
Choosing the screen aperture to pass a specific size of particle depends on the angle of inclination of the screen, the amplitude and frequency of the vibration in a way that is not easily predicted. Fig. 11.4 however shows a first estimate of the size of screen aperture required for a given maximum particle size reported by various sources. The screen aperture guidelines provided by Metso [3] for inclined screens are the particle size + 5-10% for a wire mesh, + 25-30% for rubber screen surfaces and + 15-20% for polyurethane screens. The data from Deks Tyer [4] are for inclined polyurethane screens sizing natural grain material. The data from Taggart [5] are for square mesh screens mounted horizontally and with a steeply sloping surface. Taggart also provides empirical data for round apertures that are 20-30% larger than the equivalent square aperture to pass the same size particle.

\[
\text{Screen opening size} = \sqrt{\frac{25.4^2 \times 52.7}{100 \times 3.18^2}} = 6 \text{ mesh (approximately)}
\]

Fig. 11.4. Estimate of the screen aperture required to pass a given particle size [3,4,5].

11.1.2. Types of Screens
The usual industrial screen is either a stationary or dynamic type. They may be described as:

1. Stationary and straight,
2. Stationary and curved,
3. Vibrating straight,
4. Vibrating and curved, or
5. Cylindrical and revolving.
In mineral processing plants, the use of cylindrical screens (*trommels*) is limited to washing of ores for removing clayey material, desliming, oversize (scat) separation at a mill discharge and in dewatering operations. As their use is limited, these screens have not been considered here.

**Stationary and Straight Screens Surfaces**

Stationary screens are operated either in horizontal or inclined planes. The inclination is to assist material transport and is consistent with the angle of repose of the material. A relatively steep installation is preferred for higher throughputs but the quality of separation is likely to be affected as the effective aperture and open area are decreased. An aperture above the separation size can be selected to overcome this problem.

During the process of screening, particles on the screen deck encounter the apertures where they either fall through or are held back. Obviously particles larger than the aperture opening cannot pass through. A fraction of particles, although smaller than the aperture also do not pass through the first time they encounter an aperture as they fall across the apertures and are held back. In subsequent encounters, the probability of passing through is increased. Particles that are flaky are more likely to have similar problems. Particles that are elongated, but with cross section less than the aperture, will pass through provided they approach the aperture at an appropriate angle. Fig. 11.5 shows the effect of shape and size of particles during screening. Both A and C particles are prevented from passing through, A being larger in size than the aperture while C is elongated with one dimension greater than the size of the opening. Particle C will however pass through in any subsequent encounter if it approaches the screen at a suitable angle as shown in aperture D. Particle size B will always pass through. Thus both shape and size are of importance in a screening operation. Particle sizes that are near to the aperture size are the most difficult to screen. It is a general observation that particles having a size 0.75 to 1.5 times the aperture are the most difficult to screen.

When a screen is overloaded such that the top layer does not come in contact with the screen surface, the top layer will be discharged as oversize while containing fine particles. In

![Fig. 11.5. Behaviour of particle size and shape at screen surface. Particle A is too big to pass through in any orientation; particle B will pass in any orientation; elongated particles can pass through only in orientation D but not if it lies flat on the screen in orientation C.](image-url)
such cases the movement of the bottom layer of particles on the screen, aided by the movement of the screen, will promote the possibility of particles at the top surface approaching the screen surface. Increasing the length of the screen and the screening time will likewise improve the probability of particles in the stratified top layer approaching the screen surface. Thus both time of screening and the movement of particles on the screen surface are important criteria in the designing and operation of screens.

A less common straight screen is the probability screen where the aperture is considered on the basis of the probability of a certain size of material passing the aperture.

**Stationary Curved Screens**

The commonly used stationary and curved screen is known as the DSM screen or sieve bend. It is named after the Dutch State Mines who first developed and introduced it to the mineral industry. It is used for wet screening and for dewatering slurries.

These screens have screening surfaces made of stainless steel wedge-bars fixed parallel to each other across a frame shown schematically in Fig. 11.6. The stainless steel wires are tapered from about 2mm down to about 1mm. They are bent forming a 40° to 60° concave with a radius of curvature between 900 mm and 2000 mm depending on the length of the screen. The bars spacings are from 0.35 mm to 3.5mm. A uniform flow of slurry is discharged over a weir on to the curved surface. Alternately, multiple nozzles are spread across the width to disperse the slurry uniformly over the screening surface. The commercial sizes range from about 750 mm to about 2500 mm in length and about 50 mm to 2400 mm in width.

![Fig. 11.6. Schematic diagram of a sieve bend](image-url)
The feed arrangements induce enough potential for gravity forces to act and for the slurry to gravitate down the screen. The curvature of the screen helps the slurry to cling to the surface by centrifugal force. The surface tension of the fluid also contributes to the flow of slurry against the screen surface. The pressure against the screen depends on the stream thickness at any point of the screen, the density of feed and the angle that the centre of the screen makes with the horizontal, θ (see Fig. 11.6). The pressure differential across the screen slit as a result of gravity, at any point x, is given by the relation derived by Fontein [6] as:

$$\Delta P = D \rho_{SL} g \sin \theta$$

(11.7)

where $D$ = thickness of slurry at any point $x$
$\rho_{SL}$ = density of slurry
$\theta$ = the angle that the centre of the curvature makes with the horizontal

and the liquid pressure, $\Delta P_C$, against the wedge wire screens (bars) due to centrifugal forces is:

$$\Delta P_C = \frac{D \rho_{SL} V_{SL}^2}{R}$$

(11.8)

where $V_{SL}$ is the slurry feed velocity, and $R$ the radius of curvature of the screen.

In deriving the total pressure Fontein considered the pull due to surface tension of the liquid, thus deriving the total pressure drop across slot per unit slot width as:

$$\Delta P_T = \Delta P_G + \Delta P_C + \Delta P_Y$$

(11.9)

where $\Delta P_Y = \frac{T \mu}{W}$ is the pull of the liquid in a radial direction due to surface tension $\gamma$ across the width of slot, $W$ and $\mu$ a coefficient that can be determined experimentally.

According to Stavenger [7], in order to maximise the water split to the screen undersize, the velocity of slurry should be high (12–18 m/s) when the slit width is small (50–150 microns). For larger spacings (350–3000 microns) the velocity may be as low as 3 m/s.

During industrial operations if the particle size in the slurry is less than 200 microns the sieve bends tend to blind rapidly. Feeding at a higher velocity or incorporating a rapped or vibrated screen assembly tends to clear the material between the wedges. According to Fontein [6] however, the blockage can be prevented when the Reynolds number ($Dv \rho/\mu$) is 1000 or greater and that blockage is most likely when the Reynolds number is in the region of 300. ($D$ represents the slit width, $\nu$ the velocity, $\rho$ and $\mu$ the density and viscosity of the fluid respectively).

Size separations take place at each encounter of the slurry with the screen bar where the slurry is cut and sliced off, taking with it a fraction of the fines present in the slurry. Each bar therefore encounters classified slurry. The course fraction in the slurry with a size greater than the spacings between the wedge bars continue to travel over the screen surface and collect at the end of the screen. The amount of slurry sliced off at each aperture depends on the distance between the slots and the radius of curvature of the screen. Fontein [6] quantitatively determined the amount sliced off each time, $I_{SL}$ as:
\[ I_{SL} = \frac{L_A^2}{2R} \]  

(11.10)

where \( L_A \) = slot aperture and 
\( R \) = the radius of curvature of the screen.

The size of separation is directly related to the wedge bar spacing. A log-log plot of slot spacing and separation size is linear as indicated in Fig. 11.7 [7].

This figure indicates that the separation size of a sieve bend varies from 50–95% of the slot aperture. For separation sizes less than 200 microns the incidence of screen blinding is high. In such cases the velocity of particles over the screen is increased by application of hydraulic pressure on the slurry. Alternatively arrangements are made to tap or vibrate the screens and agitate the surface by a combination of tapping and vibrations. As a rule of thumb the thickness of the slurry layer passing through the aperture should be less than half the slot opening to avoid clogging of the screen.

In the metallurgical industry, size separations by sieve bends are usually confined to the range of 200–3000 μm though feed sizes can be up to 12 mm. The sieve bends commonly used are gravity fed at a slurry velocity of about 180 m/min and solids in the slurry as high as 50%. Typical bar spacings range between 0.35 to 3.5 mm with angles 45° to 50°. Where pressure is employed, the angles and the length of sieves are about 270° and length about 2300 mm.

During operation, the top edge of the screens wear out thereby affecting the thickness of the slurry layer passing through the screen and hence the size of separation. Industrial curved screens are therefore designed to flip the surface around when the top edge becomes worn in  

Fig. 11.7. Relation between bar spacing and diameter of separation of particles [7].
order to expose the lower end of the wedge bar to the descending stream of slurry thus increasing the operating life of the screens.

11.1.3. Vibrations and Movement of Straight and Curved Screens
Blinding of screens during operation is one of the most contentious and difficult factors that a screen designer has to face. A partial and probable solution is to use a design that is clad with hard rubber or plastics. In practice however, no screen is really free from blinding. The most effective way to reduce blinding is to impart vibratory or circular motion to the screen.

To impart the motion, the screen surfaces are rigidly fixed on to a frame. The frame in turn is fixed to moving devices that are either mechanically or electrically driven. Several ingenious methods of movement and vibration of screen surface have been devised over the years. These have been classified according to the manner of movement. Some authors like, Colman and Tyler [1], have preferred to classify screens according to the number of bearings which are mechanically responsible for different movements. In Table 11.2 an attempt is made to classify screens according to the manner of motion and also incorporating Colman’s concept of bearings. As the movement of the shaft also controls the screen motion this is also included in Table 11.2.

The vibrating devices are mounted either at the feed end, centre of the screen frame or near the discharge end. The vibrations are controlled by large steel springs attached to the bottom of the frames or by suspended hangers and cables. Air cushions are also used. Some novel devices include use of bouncing balls that strikes the screen under surface and help to keep the apertures from blinding. Stretching plain carbon wires, 1.0–1.22 mm in diameter, in grooves about 254 mm apart along the entire length of the screen induces similar action. The wires are taught and vibrate against the screen loosening any accumulation in the apertures.

Camshafts sitting on eccentric bearings, connecting rods or cranks impart reciprocating movements. Slow reciprocation is of the order of 150 rpm with stroke length varying between 75–100 mm. A fast reciprocation is 200–300 rpm at 25–75 mm stroke.

Table 11.2
Design and movement of screens [8].

<table>
<thead>
<tr>
<th>Screen Motion</th>
<th>Shaft No</th>
<th>Shaft Type</th>
<th>No. of Bearings</th>
<th>Throw</th>
<th>Stroke length, mm</th>
<th>Frequency, rpm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oscillating, Linear</td>
<td>1</td>
<td>Eccentric</td>
<td>2</td>
<td>Circular</td>
<td>&lt;25 ***</td>
<td>500-2500</td>
</tr>
<tr>
<td>Vibratory Forward</td>
<td>1</td>
<td>Eccentric</td>
<td>2</td>
<td>Circular</td>
<td>15-30</td>
<td>25-500</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Double Eccentric</td>
<td>4**</td>
<td>Circular Forward, positive</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Reciprocating</td>
<td>2</td>
<td>Reciprocating and Eccentric</td>
<td>4</td>
<td>-</td>
<td>25-75</td>
<td>200-300</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>75-100</td>
<td>150</td>
</tr>
<tr>
<td>Sifter, Circular</td>
<td>4</td>
<td>Eccentric</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Sifter, Gyrotrary*</td>
<td>4</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>500-600</td>
<td>-</td>
</tr>
<tr>
<td>Sifter, Circular</td>
<td>4</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

* Movement circular at feed end and reciprocating at discharge end.
** Two for bearings and two for shafts
*** Stroke length usually less than 10 mm
The primary objective for imparting vibrations to screens is to aid segregation of fines through the bed to the screen surface, dislodge accumulations at the apertures and to keep the screen active at all times. However, the shape of particles, moisture content and the number of times a particle is able to approach the screening surface complicate the process. The probability, \( p \), of a particle passing through a screen has been shown by Gaudin [9] to be:

\[
p = \left( \frac{L_A - d_p}{L_A + d_w} \right)^2
\]

where \( d_p = \) the particle size of which 50% passed through the screen.

This probability is affected by the amplitude, frequency and direction of vibration. Miwa [10] has estimated the number of presentations (\( N \)) of particles on to the screen surface by considering the effective aperture \( (L_{AE}) \), the diameter of the wire \( (d_w) \) of the screen, the length of screen \( (L) \) and taking a particle size, \( d_{50} \), at which 50% of the material of this size passes or is retained on the screen. The expression is given by:

\[
N = \frac{1}{L} \left[ \frac{0.833 \left( L_{AE} + d_w \right)}{L_{AE} - d_{50}} \right]^2
\]

The value of \( N \) includes vibration (amplitude and frequency) and other variables related to screening. Therefore \( N \) can be regarded only as an index of vibration and therefore of screening.

It should be noted that:

1. Increasing the amplitude of vibration initially increases the percentage passing through a screen. After reaching a peak, a further increase of vibration decreases the amount passing through.
2. Change in frequency has little effect on the amount passing through the screen.

### 11.2. Operation of Straight Screens

#### 11.2.1. Basic Considerations

Two criteria are used to assess screen performance, **Capacity** and **Efficiency**. Capacity is simply the quantity of material fed to the screen per unit time per unit area of screen surface. In reality capacity should be quoted along with efficiency. Capacity and efficiency are generally conflicting quantities. Any screen can have its capacity increased, but this is likely to be achieved at the expense of efficiency.

The basic purpose of screening is to separate particles larger or smaller than the aperture of a screen. An ideal screening condition would be to have a monolayer of a mixture of sizes of particles on the screen surface so that the probability of each and every particle passing or not passing can be determined. As illustrated in Fig. 11.5 the passage of each particle will depend on its size, shape and the angle at which it reaches an aperture. To attain the required angle, a particle may require several presentations. If the screen was sufficiently long it could eventually approach the aperture at the appropriate angle and pass through. If the length was
insufficient then in spite of the particle size being smaller than the aperture it may report as oversize. Thus the length of the screen is important. The length of screen provides the screening efficiency and the width indicates the throughput rate. Ultimately the probability of passing will depend on the initial mass of particles in the feed stream that would pass after N approaches to the aperture and the mass fraction remaining that had not passed. This can be written as:

\[
\frac{M}{M_i} = (1-p)^{NL}
\]  

(11.13)

where \( M_i \) = the initial mass of undersize in the feed stream, 
\( M \) = the mass of undersize remaining on the screen after N attempts, 
\( NL \) = the number of presentations per unit length of screen and 
\( p \) = the probability of a particle passing through the screen.

Eq. (11.13) implicitly assumes that the probabilities of all particles passing in every attempt are equal. Substituting the value of \( p \) from Eq. (11.11) in Eq. (11.13) gives:

\[
\frac{M}{M_i} = \left[ 1 - \left( \frac{L_A - d_p}{L_A + d_w} \right) \right]^{NL/L}
\]  

(11.14)

On simplifying Eq. (11.14) by neglecting the higher powers and replacing \( d_p \) by \( d_{50} \) as the particle size that is equally split between the overflow and underflow, we get Eq. (11.15) as:

\[
d_{50} = L_A - \frac{0.833 (L_A + d_w)}{L^{0.5} N^{0.5}}
\]  

(11.15)

Eq. (11.15) indicates that if the \( d_{50} \) was experimentally determined along the length of the screen of known wire diameter (\( d_w \)), then the aperture and number of attempts could be determined.

The screening process however is complicated by the fact that in practice a layer of particles are charged on to a screen and the probability of particles at the bottom of the layer and against the screen surface passing through the screen will be greater than those at the top of the layer. Also the particle size distribution on the screen surface along the length will be different as illustrated in Fig. 11.8 where it can be seen that screening along the length of screen is not uniform. Two factors are in operation. First, due to uneven stratification of the particle layers on the screen surface, undersize particles may not reach the screen surface due to excessive bed depth. Secondly, the probability of passing when the undersize particles do reach the surface. Thus Eq. (11.15) is not exactly applicable to a real situation. It can however be used as an indicator.

From the above discussions it is seen that absolute separation of different sized particles using a screen is difficult as it involves probabilities of movement of particles at different stages that may be difficult to determine.

A screen is said to behave perfectly if, in a mixture of different sizes of materials, all material of a particular size less that the screen aperture is separated from the mix. In general, a screening operation does not produce a perfect separation therefore it is necessary to express
Assuming that in a continuous screening process of a material, the mass flow rate of solid feed is given by $Q_{MS(F)}$ and the distribution of the overflow and underflow rates is:

Mass flow rate of solid in the overflow $Q_{MS(O)}$
Mass flow rate of solid in the underflow $Q_{MS(U)}$
Mass fraction of undersize in the feed $m_u^{(F)}$
Mass fraction of undersize in the oversize $m_u^{(O)}$
Mass fraction of undersize in the undersize $m_u^{(L)}$

Then the screen efficiency, $E_o$, based on the oversize will be:

$$E_o = \frac{Q_{MS(O)} (1-m_u^{(O)})}{Q_{MS(F)} (1-m_u^{(F)})}$$  \hspace{1cm} (11.16)$$

and the screen efficiency, $E_u$, based on undersize:

$$E_u = \frac{Q_{MS(U)} m_u^{(U)}}{Q_{MS(F)} m_u^{(F)}}$$  \hspace{1cm} (11.17)$$

The overall efficiency, $E = E_o \times E_u$
Substituting values from Eqs. (11.16) and (11.17), the overall efficiency of screens would be:

\[ E = \frac{Q_{MS(O)} (1 - m_{U(O)})}{Q_{MS(F)} (1 - m_{U(F)})} \times \frac{Q_{MS(U)} m_{U(U)}}{Q_{MS(F)} m_{U(F)}} \]  
(11.18)

The values of \[ \frac{Q_{MS(O)}}{Q_{MS(F)}} \] and \[ \frac{Q_{MS(U)}}{Q_{MS(F)}} \] can easily be determined from a material balance of the system.

11.2.2. Material Balance of a Screen in Operation

In any screening operation the size analysis of the feed, oversize and undersize indicates the partition of a particular size. Fig. 11.9 shows the cumulative distribution curves where size 1 is the coarsest and size 12 the finest screen size used.

In a dynamic system at steady state, the mass flow rate of feed material charged for screening must be equal to the sum of the mass flow rate of material discharged in the overflow and underflow. Mathematically therefore:

\[ Q_{MS(F)} = Q_{MS(O)} + Q_{MS(U)} \]  
(11.19)

As the screen partitions the total feed material into overflow and underflow streams, we can write, using the above symbols:

\[ Q_{MS(F)} m_{U(F)} = Q_{MS(O)} m_{U(O)} + Q_{MS(U)} m_{U(U)} \]  
(11.20)

Fig. 11.9. Distribution of particles over a screen.
Substituting the value of $Q_{MS(U)}$ from Eq. (11.19) in Eq. (11.20) and rearranging we have:

$$
\left[ \frac{Q_{MS(O)}}{Q_{MS(F)}} \right] = \left( \frac{m_{U(F)} - m_{U(U)}}{m_{U(O)} - m_{U(U)}} \right) 
$$

(11.21)

Similarly substituting the value of $Q_{MS(O)}$ from Eq. (11.19) in Eq. (11.20) we get:

$$
\left[ \frac{Q_{MS(U)}}{Q_{MS(F)}} \right] = \left( \frac{m_{U(O)} - m_{U(I)}}{m_{U(O)} - m_{U(U)}} \right) 
$$

(11.22)

Eqs. (11.21) and (11.22) can now be substituted in Eq. (11.18) to give the efficiency of the screen as:

$$
E = \left[ \frac{m_{U(F)} - m_{U(U)}}{m_{U(O)} - m_{U(U)}} \right] \left[ \frac{m_{U(O)} - m_{U(I)}}{m_{U(O)} - m_{U(U)}} \right] \left[ 1 - m_{U(U)} \right] \left[ \frac{m_{U(U)}}{m_{U(F)}} \right] 
$$

(11.23)

The use of the Eq. (11.23) is illustrated in example 11.2.

Because the efficiency Eqs. (11.16) and (11.17) use laboratory measured data using square mesh sieves, these equations are meant for square mesh industrial screens and are not strictly applicable to rectangular mesh. Using these formulae and square mesh laboratory screening data of rectangular industrial mesh screen products, calculated efficiencies in excess of 100% are possible. To overcome this problem, Leonard [11] defined the efficiency of screens in terms of the amount of total misplaced material (fines in oversize and coarse in undersize). Thus:

$$
E = \frac{\text{rate of feed} - \text{rate of undersize in oversize fraction} - \text{rate of oversize in undersize fraction}}{\text{rate of feed}} 
$$

For a material balance of the undersize product, using Eqs. (11.19) and (11.20), the distribution of the undersize can be expressed as given in Eq. (11.22). Substituting these terms into the Leonard’s efficiency equation and simplifying we get:

$$
E = \left( 1 - m_{U(O)} \right) - \left[ \frac{m_{U(F)} - m_{U(O)}}{m_{U(U)} - m_{U(O)}} \right] \left( 1 - m_{U(O)} - m_{U(U)} \right) 
$$

(11.24)

Leonard’s method is illustrated by example 11.2.

Osborne [8] considered the efficiency of a square aperture screen as the ratio of the amount that actually passes through the screen to the amount that should pass through the screen. The screen efficiency then is:
Example 11.2
A gold ore is screened through a 30 mm screen. The average size distribution of the feed, oversize and undersize were determined and graphed below. Determine the efficiency of the screen.

Solution (Efficiency 1)
From the graph we can see that for a 30 mm separating size, \( m_{U(F)} = 46\% \), \( m_{U(O)} \) in oversize = 7.5% and \( m_{U(U)} \) in undersize = 90%.

Using Eq. (11.23) and directly substituting the values:

\[
E = \frac{100 Q_{MS(U)}}{Q_{MS(F)}} = \frac{100}{m_{U(F)}} \left( \frac{m_{U(F)} - m_{U(O)}}{m_{U(U)} - m_{U(O)}} \right)
\]

(11.25)

The efficiency is 83.4%.

Note: Usual screen efficiencies encountered in industry are of the order of 60% to 85%.

Solution (Efficiency 2)
Substituting values into Leonard’s Eq. (11.24):

\[
E = \left[ \frac{(0.46 - 0.90)}{(0.075 - 0.90)} \right] \left[ \frac{(0.075 - 0.46)}{(0.075 - 0.90)} \right] \left[ \frac{1 - 0.075}{1 - 0.46} \right] \left[ \frac{0.90}{0.46} \right] = 0.834
\]
Example 11.3
From a crushed quartz sample the fraction less than 2 mm had to be removed by screening. The feed sample contained 35\% of minus 2 mm material. After screening the oversize fraction contained 10\% of minus 2 mm size and the undersize contained 82\% of minus 2 mm size.

Determine the efficiency of the screen.

Solution (Efficiency 1)
Substituting the values into Eq. (11.23):

\[
E = \left[ \frac{0.35 - 0.82}{0.10 - 0.82} \right] \left[ \frac{0.10 - 0.35}{1 - 0.35} \right] \left[ \frac{0.82}{1 - 0.35} \right] = 73.5\%
\]

Solution (Efficiency 2)
Using Eq. (11.24) expressed as percentage, i.e.,

\[
E = (100 - 10) - \left[ \frac{35 - 10}{82 - 10} \right] (100 - 10 - 82)
\]

\[
E = 87.2\%
\]

Solution (Efficiency 3)
Substituting values into Eq. (11.25):

\[
E = \frac{100}{0.35 \left( \frac{0.35 - 0.10}{1 - 0.35} \right)} = 99.2\%
\]
11.2.3. Screen Efficiency and the Tromp Curve
Since a feed may contain a whole range of particles of different properties, such as grade or size, then the separation efficiency may be different for different particles. That is, we need to take into account the amount of misplaced material that can occur or the difficulty of separation of some of the particles.

In 1937, Tromp [12] introduced a graphical method of assessing separation efficiency which is universally used and is alternatively referred to as; Tromp Curve, Partition Curve or Performance Curve.

We can refer to any characteristic in the feed or any other stream, in general terms as characteristic \( i \), where \( i \) can refer to a size interval for size separators. The amount of misplaced material to an output stream is referred to as the partition coefficient (also called the distribution factor or probability factor). The partition coefficient is then defined as:

\[
\text{partition coefficient} = \frac{\text{mass of material of characteristic } "i" \text{ in a stream}}{\text{mass of material of characteristic } "i" \text{ in the feed}} = \frac{M_{i\text{os}}}{M_{i\text{fs}}} (11.26)
\]

It may be expressed as a fraction or a percentage. The partition coefficient is essentially the recovery of a given characteristic (size in this case) to a stream, usually the positive response stream, but not always. To some extent it incorporates a measure of the grade as well since it indicates how much of each particle characteristic is present in the output stream. For example, the partition coefficient tells us how much undersize to oversize particles are there in the stream and if the fine sizes are enriched in the valuable mineral then an indication of the grade follows. However grade is not really a factor in this measure of efficiency or performance, it depends on the particle characteristic the separator is using to generate the output streams. For example, a process separating on particle size can work efficiently if there is no change in grade between the feed and the output streams, e.g. if the feed was all the same mineral.

Having obtained the partition coefficient, this is plotted against the mean separating characteristic of the fraction to generate the performance curve, as shown below. The mean values plotted may either be the arithmetic mean or the geometric mean.

A Perfect Separation
Let us consider a screen as our separating unit and the screen aperture is 2 mm. In a perfect separation, any particle that is less than 2 mm should go through the screen and hence the amount remaining on the screen at the completion of the process (the oversize) should be zero. Any particle that is greater than 2 mm should remain on the screen and hence the amount of this material in the oversize product should be 100%. That is, the partition coefficient for -2 mm material in the oversize product will be zero and the partition coefficient for the +2 mm material in the oversize product will be 1.0 or 100%. The performance curve will then have the shape of the solid line in the Fig. 11.10. That is, there will be a sharp jump from 0 to 1.0 (or 100%) at the separation point. This separation point is referred to as the \( d_{50} \).

If the partition coefficient is calculated with respect to the negative response stream instead of the usual positive response stream, the performance curve will have the same shape but will be a mirror image about the \( d_{50} \). The \( d_{50} \) point will be the same, either way.

The performance of any separator depends on three factors:

1. the characteristic composition of the feed (e.g., the size distribution, the density composition etc.),
2. the value of the size where separation occurs, and
3. the sharpness with which the unit separates the feed.

Often the performance criterion required is the ability of the separator to make a sharp separation and to compare different separators this must be free of the influence of feed composition and the size of separation. Only then can the performance of units treating different feeds and separating at different sizes be compared directly.

The performance curve is a convenient way of showing the sharpness of separation, however, as a means of comparison between different separating units, a numerical figure is better for describing the deviation from ideal behaviour. These numerical figures are based on the error between the actual curve and the line of perfect separation, and are termed the *probable error, error area or ecart probability* (see Fig. 11.11).

One way of quantifying the deviation from the perfect separation is to determining the area between the performance curve and the ideal curve provided the partition coefficient values range from 100 to 0 (that is, there is no by-pass or short circuiting of material). This area is termed the error area. If several performance curves are plotted on the same axes then this area provides a means of comparing the sharpness of separation.

Another method of characterising the performance curve is to determine 50% of the difference between the separating size at a partition coefficient of 0.75 (or 75%) and 0.25 (or 25%). This figure may be referred to as the Ecart probability ($E_p$), the probable error or the probable deviation.

$$E_p = \frac{d_{75} - d_{25}}{2} \tag{11.27}$$

If the performance curve is a straight line between the $d_{75}$ and $d_{25}$ points then the probable error is a measure of the slope of this curve, through the $d_{50}$ or 50% point.
Fig. 11.11. Quantifying the deviation from ideal performance.

That is, slope = \( \frac{\Delta Y}{\Delta X} = \frac{0.75 - 0.25}{d_{75} - d_{25}} \)  

and from the definition of \( E_p \) above,

\[
E_p = \frac{1}{4 \times \text{slope}} 
\]

or, the probable error is proportional to the reciprocal of the slope.

So as the slope of the performance curve approaches the vertical (infinity), the probable error approaches zero or the smaller the probable error, the greater the sharpness of separation (the closer to a perfect separation the performance of the separator becomes).

The degree of misplacement of material is not symmetrical about the 50% horizontal. For example the misplacement in the fine fraction may be greater than the misplacement in the coarse fraction since the fines have to segregate to the screen surface before it leaves the unit whereas it is more difficult for coarse particles to enter the undersize unless there are worn or broken wires on the screen surface.

In this case of asymmetrical performance curves, we could define;

a 75% partition error = \( d_{75}/d_{50} \), and

a 25% partition error = \( d_{50}/d_{25} \)  

(11.30)
to give a more precise description of the deviation from ideal behaviour, but usually the
probable error, $E_p$, is satisfactory.

When performance curves were first developed, the performance curves, which were
drawn for gravity separations, were believed to be independent of the characteristic (density)
of separation. With time however it became apparent that the curve tended to steepen as the
density of separation decreased. That is, at low densities, the separation tended to be sharper
than those at higher separation densities. The French research organization, Cerchar (Centre
D’Etudes et de Recherches de Charbonnage de France) was the first to recognise the
relationship between sharpness of separation and separation density. They consequently
coined the term Imperfection, designated $I$, which was defined as:

$$I = \frac{E_p}{d_{50} - 1}$$  \hspace{1cm} (11.31)

They believed that the $d_{50}$ increases in proportion to $(d_{50} - 1)$. Since then, it has been
suggested by other workers that the formula for Imperfection should be:

$$I = \frac{E_p}{d_{50}} = \frac{d_{50} - d_{25}}{2 d_{50}}$$  \hspace{1cm} (11.32)

The usefulness of the Imperfection as a sharpness of separation criteria, independent of the
$d_{50}$, has been questioned and the reader is refer to the work by Peng et al. [13] for a detailed
discussion of these criteria.

The method usually employed to draw the Tromp curve and its interpretation is illustrated
in Example 11.4.

---

**Example 11.4**
The size fractions of a screen feed, oversize and undersize stream sample are given in the
table below. The oversize represented 62.5% of the feed mass flow rate. Draw the Tromp
curve for the separation and determine:

1. The separating size,
2. The probable error,
3. The imperfection.

**Solution**
The solution is best understood by following the calculations shown in Table 11.3.

In the table:
Columns A and C are the analyses of the oversize and undersize streams
Column B = Column A x yield in oversize (0.625 in this example)
Column D = Column C x yield in undersize (0.375 in this example)
Column E = Sum of columns B and D giving the reconstituted feed
Column F = Partition Coefficient = $B/(B + D)$. 
Table 11.3
Sizing data for a screen oversize and undersize fraction

<table>
<thead>
<tr>
<th>Size, microns</th>
<th>Mean Size, microns</th>
<th>Oversize stream</th>
<th>Undersize stream</th>
<th>Calculated Feed</th>
<th>Partition Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mass %</td>
<td>Mass in sample</td>
<td>Mass %</td>
<td>Mass in sample</td>
<td>E = B + D</td>
</tr>
<tr>
<td>16000</td>
<td>37.5</td>
<td>23.44</td>
<td>0.5</td>
<td>0.19</td>
<td>23.63</td>
</tr>
<tr>
<td>8000</td>
<td>32.0</td>
<td>20.00</td>
<td>1.0</td>
<td>0.38</td>
<td>20.38</td>
</tr>
<tr>
<td>4000</td>
<td>13.0</td>
<td>8.13</td>
<td>10.6</td>
<td>3.98</td>
<td>12.10</td>
</tr>
<tr>
<td>2000</td>
<td>7.4</td>
<td>4.63</td>
<td>12.1</td>
<td>4.54</td>
<td>9.163</td>
</tr>
<tr>
<td>1000</td>
<td>3.6</td>
<td>2.25</td>
<td>15.0</td>
<td>5.63</td>
<td>7.875</td>
</tr>
<tr>
<td>500</td>
<td>2.5</td>
<td>1.563</td>
<td>18.0</td>
<td>6.75</td>
<td>8.313</td>
</tr>
<tr>
<td>250</td>
<td>2.0</td>
<td>1.25</td>
<td>20.0</td>
<td>7.50</td>
<td>8.750</td>
</tr>
<tr>
<td>125</td>
<td>1.5</td>
<td>0.94</td>
<td>19.8</td>
<td>7.42</td>
<td>8.363</td>
</tr>
<tr>
<td>-125</td>
<td>0.5</td>
<td>0.31</td>
<td>3.0</td>
<td>1.13</td>
<td>1.438</td>
</tr>
<tr>
<td></td>
<td>100.0</td>
<td>62.5</td>
<td>100.0</td>
<td>37.5</td>
<td></td>
</tr>
</tbody>
</table>

The partition coefficient may now be plotted on a semi-log paper as shown in Fig. 11.12.

Fig. 11.12. Tromp curve of screen data.

Reading off Fig. 11.12 it can be seen that:

1. The separation size, \( d_{50} = 2800 \mu m \) and the \( d_{25} \) and \( d_{75} = 1200 \) and \( 6600 \mu m \) respectively,

2. The efficiency \( E_p = \frac{d_{75} - d_{25}}{2} = \frac{6600 - 1200}{2} = 2700 \)

3. Imperfection \( I = \frac{2700}{2800} = 0.96 \)
11.2.4. Bed Depth

The bed depth of material on the screen affects the efficiency and the performance of a screen. Fig. 11.8 shows that the profile of a bed of material on the surface of a screen is far from uniform. The feed end of the screen surface is overloaded while the rest of the screen surface is thinly spread with the material. The fraction of particles in the feed stream that is smaller than the sieve openings and occupying upper layers of the feed stream need time and agitation to work their way down to the screen surface. Agitation of the screen surface imparts fluid properties to the bed of particles. The material on the screen expands and the larger particles tend to travel up. The smaller particles tend to gravitate down the voids created by the expansion of the bed. The stratification of the bed has the added advantage that it helps to minimise the agitation of the smaller particles by holding them down on the surface of the screen. Thus the depth of the bed, the rate of feed and the inclination of the screen are of major importance to the screen operation. Too thick a bed will tend to delay stratification, while too thin a bed reduces the efficiency as it allows unconstrained movement of particles on the screen. Hence the bed thickness at the discharge end is more important than the feed end. The bed thickness at the discharge end will in turn depend on the length of the screen. According to Matthews [2] for screens of length from 1.8 m, the bed depth at the discharge end should be a minimum of about 1.5 – 2.0 times the average particle size, and for screen lengths of about 7.2 m the thickness of the discharge end should be about 2.5 to 3.0 times the average particle size. Thus if the screen length was 2 m and the average particle size 850 microns, then the minimum bed height at the screen discharge end should be about 1.3 mm.

Manufacturers have charts available relating the capacity (which includes the width of the screen) and the depth of bed. The general relationship relating bed depth, feed rate and width of screen, according to Osborne [8] is:

\[
D = \frac{50 Q_o}{3 W_E \nu_F \rho_B} \quad (11.33)
\]

where 
- \(D\) = bed depth, mm,
- \(Q_o\) = tonnage of oversize material, t/h,
- \(\nu_F\) = transport rate across the screen, m/min,
- \(W_E\) = effective width of the screen, m,
- \(\rho_B\) = bulk density, t/m\(^3\).

The effective area of the screen (the total area minus the area of clamps and fittings) is given approximately by the equation:

\[
A_E = (W - 0.15) L \quad (11.34)
\]

where \(W\) and \(L\) are the width and length of the screen in meters respectively.

Some authors [14,15] have related the bed thickness with the bulk density of the material to be screened. The general conditions are:

1. For material of bulk density 1.6 t/m\(^3\), the bed depth at the feed end should not exceed 4 times the size of the aperture,
2. For material of bulk density 0.8 t/m\(^3\), the bed depth should not be greater than 2.5 – 3 times the size of aperture.
The bed depth is also related to the slope of the screen. While a quantitative relation between these parameters has not been established, the following observations can be made [2]:

1. For screen widths of 0.6 – 2.5 m the inclination should not be less than 16° and a maximum of about 26° for capacities 15 – 270 t/h.
2. When the slope is greater than 20° the capacity is markedly reduced as the effective aperture area is reduced by 0.93 times.
3. For longer screens, eg. 4.8 meters, screen manufacturers recommend a further addition of 2° and for screens about 6 meters, 4° should be added.

11.3. Capacity and Screen Selection of Straight Screens

The above discussion indicated that the capacity of a screen is related to the screen characteristics and the material characteristics. The screen characteristics include:

1. available area,
2. aperture (size and type),
3. slope,
4. method of vibration, and
5. number of decks.

The material characteristic include:

1. size and shape of material ,
2. moisture content,
3. rate of throughput, including depth of material layer,
4. dry or wet screening.

The capacity of a screen is referred to either in terms of the oversize or in terms of the undersize product streams. For a square mesh sieve at a slope of 18°, where maximum efficiency is expected, Taggart [5] suggests that the capacity of a screen should be based on that size fraction in the feed that is most difficult to separate. This fraction is described as the percent of critical size and the basic function is described here as \( F_B \).

\[
F_B = \frac{73.14 L_A \rho_B}{C} \tag{11.35}
\]

where
- \( F_B \) = basic feed rate, t/h/m of screen width,
- \( L_A \) = aperture in mm (square mesh),
- \( \rho_B \) = mass of material/m³, and
- \( C \) = percent of critical size taken as the percent of feed between the critical size of 0.75 to 1.5 of the screen aperture.

The actual feed rate, \( F \), in tonnes per hour per meter of screen width, is related to the basic feed rate, \( F_B \), by the relation:

\[
F = F_B R \tag{11.36}
\]
The factor \( R \) is a function of the screen efficiency and the vibration intensity. The relation between \( R \) and the efficiency factor was determined experimentally for screens of different lengths. Typical curves for 1.8 m and 2.4 m length screens are reproduced in Fig. 11.13.

The efficiency factor is the fraction of true undersize in the screen oversize based on the amount of near size particles (percent of critical size, \( C \)). Thus if the feed analysis is known then \( C \) can be established and \( R \) determined from efficiency and vibration intensity values using Fig. 11.13. From the \( R \) values, the actual screen feed rate can be determined using Eq. (11.36).

For example, if the feed size distribution has 32% greater than the screen aperture and 25% of the feed is between 0.75 and 1.5 times the screen aperture and after screening, the oversize contains 10% of undersize material then;

\[
\text{Efficiency factor} = \frac{\text{mass of undersize in the oversize}}{\text{mass of critical size}}
\]

\[
= \left( \frac{\text{fraction of undersize in the oversize}}{C} \right) \times \left( \frac{\text{mass of oversize in the feed}}{\text{mass of undersize in the oversize}} \right)
\]

\[
= \frac{m_o \cdot M_o}{C (1-m_o)}
\]

\[
= \frac{0.10 \cdot 32}{25(1-0.10)} = 0.14
\]

Fig. 11.13. Screen capacity factors [5].
The above considerations are for screens with normal vibration intensity. For vibration intensities above normal, the relation between R and the efficiency factor has to be established. Screen manufacturers normally supply these.

Having estimated the actual feed rate the dimensions (L x W) of the screen can be estimated using the expression [5]:

\[
W = \frac{t/h \text{ of total feed}}{\text{Actual feed rate per meter}} = \frac{Q}{F} \quad (11.38)
\]

A much simpler method to determine capacity and screen dimensions is to consult empirical screen performance data produced by screen manufacturers like Hewitt-Robins and Nordberg (Metso). The method followed is similar to that advocated by Taggart [5] and modified by Colman and Tyler [1] and Kelly and Spottiswood [14]. Their procedure is summarised as follows:

The basis of the calculations is Eq. (11.36) which is re-written as:

\[
Q = AF_B C_R \quad (11.39)
\]

where

- \( Q \) = mass rate of flow, (t/h), also taken as the t/h of undersize in the feed, or the total feed to the screen deck depending on the data source,
- \( C_R \) = Combined correction factor,
- \( A \) = Area of open surface.

The basic production rate is a function of screen aperture. The relation between aperture and basic capacity has been determined empirically and for some screen types is given in Fig. 11.14. Though most metallic ores have similar screening characteristics, other materials may have different screening characteristics so the appropriate specific capacity curve should be used. The difference between the data sets reflects the difference in the definition of base capacity and the different standard screening conditions used.

The correction factor \( C_R \) takes into account all the variables listed below. These corrections factors have been determined empirically and are based on the conditions assumed for deriving the basic equation. In all, eleven factors are identified. These are designated by various symbols by different screen manufacturers and are numbered \( C_1-C_{11} \) in this book. The descriptions of these variables are as follows. The values of the corresponding factors are given in Figs. 11.16-11.23 and Tables 11.3-11.5.

- \( C_1 \) = mass factor
- \( C_2 \) = open area factor
- \( C_3 \) = \% oversize material
- \( C_4 \) = \% undersize (fines) in the feed
- \( C_5 \) = screen efficiency factor
Fig. 11.14. Relation between aperture and Base Unit Capacity expressed as tonnes per hour per square meter for different screen types.

Gluck [16] – using a bulk density of 1.6 t/m³; Nordberg (Metso) [18]– 50% oversize in feed, 25% half size, slope 20°, 92-95% efficiency; Osborne [8] – 60% open area.

\[ C_R = C_6 C_7 C_8 C_9 C_{10} C_{11} \]  

The significance of these factors and the method of determination are:

**C1: Mass Factor**

The correction factors were derived at normal vibrating speeds of screens using a material of bulk S.G. 1.602 t/m³ which was considered as standard. Factors for higher vibrations are available from the manufacturers. Where the bulk density of a specific material is different.
from the standard, it has to be corrected by taking the ratio of the specific gravities [8]. That is:

\[
\frac{F}{F_B} = \frac{\rho}{1.602}
\]  \hspace{1cm} (11.41)

where \( F \) = capacity at the actual bulk density and 
\( F_B \) = capacity at the standard bulk density (1.602 t/m\(^3\))

Hence the correction factor \( C_1 = \rho/1.602 \)

**C2: Open Area Factor**

While deriving the basic equation, the capacity for different open areas of screens were determined using standard woven wire screens having square apertures. Commercial screens differ from this standard. Correction factors were therefore determined by simple formulae such as:

\[
C_2 = \frac{\% \text{Open Area of screen}}{A_{OB}}
\]  \hspace{1cm} (11.42)

where \( A_{OB} \) = the base open area used.
eg. 50% [16,17], 60% [8], 100% [14], variable [18].

The open area used by Nordberg (Metso) [18] in Eq. (11.42) changes depending on the screen aperture. The relationship between the base open area and screen aperture is shown in Fig. 11.15.

**C3: Correction Factor for Oversize**

The standard oversize in the feed is mainly taken as 25%. When the oversize percent in the feed is greater than 25%, then stratification of bed layer is incomplete, which leads to a screening error. This error has to be allowed for and a correction is made for different percentages of oversize. The correction factor, \( C_3 \), from several sources is reproduced in Fig. 11.16.

**C4: Correction Factor for Undersize (Fines)**

The fines are defined as the percent less than half the screen aperture. It is a convenient measure of the ease of screening. By convention, 40% fines in the feed is taken as the standard case. This is used to establish the basic unit capacity of screens. The difficulty or otherwise of screening is therefore related to 40% fines content in the feed. This is assigned a factor equal to 1.00. Factors for different fines content have been derived over a range of undersizes. Factors for the percent half size or the percent of feed passing half the aperture size were determined and are plotted as correction factor \( C_4 \) in Fig. 11.17.

In a multi-deck screen, the percentage of half size in the feed to the screen is expressed as the percentage of the feed to the deck under consideration. For example, in a double deck screen, if the total screen feed contains 35% passing half the lower deck screen aperture size and 70% passing the upper deck aperture size then the percentage half size for the bottom screen is \( 35/70 = 50\% \) (see Fig. 11.18).
Fig. 11.15. Base open area versus screen aperture [18].

Fig. 11.16. Correction factor $C_3$ for percent oversize in the feed.

Nordberg (Metso) [18] – 50% oversize in feed.
Fig. 11.17. Relation between percent half size and Correction Factor $C_4$
Gluck [16] – 40% half size, Osborne [8] – 40% half size,

Fig. 11.18. Half size percentage for multi-deck screens
**C5: Screen Efficiency Factor**

In industrial screening, efficiencies of 100% is not achievable. Hence 90 – 95% efficiency is considered as the maximum for normal wire screens. During scalping operations Colman [15] suggests that the efficiency should be taken as 80 – 85%. For normal wire screens, the correction factors for different efficiencies have been determined and are reproduced in Fig. 11.19.

**C6: Deck Factor**

Fig. 11.8 shows that the effective screening does not take place immediately at the charging end of the screen as the material has to travel some distance for stratification to take place. When a bottom deck is set up the effective screening takes place further down the screen. Thus the effective screening area is reduced. The reduction of area at the top deck is not significant. Fig. 11.20 indicates the possible manner of the loss in effective screening area in deck 2.

The correction factor for the top deck is therefore considered as unity. For the lower decks, the correction factors are given in Table 11.4 as recommended by Colman [1,15] and Gluck [16].

Table 11.4
Correction for the number of decks (Deck Factor, C₆) [15-18].

<table>
<thead>
<tr>
<th>Deck position</th>
<th>Correction factor, C₆</th>
</tr>
</thead>
<tbody>
<tr>
<td>top deck, (No.1)</td>
<td>1.0</td>
</tr>
<tr>
<td>second deck (No. 2)</td>
<td>0.9</td>
</tr>
<tr>
<td>third deck (No. 3)</td>
<td>0.8</td>
</tr>
<tr>
<td>fourth deck (No.4)</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Fig. 11.19. Desired Efficiencies for varying loads, factor C₅.

Fig. 11.20. Inactive area on screen due to deck position.

Fig. 11.21. Correction Factor $C_7$ for the slope of the screen.
Nordberg (Metso) [18] – 20° slope standard, Gluck [16]– 15° slope standard.
C7: Correction due to the Screen Slope
It is usual to set the screen between 18 and 25° in a normal close circuit crushing operation. Increasing slope results in increased speed of movement of material but in so doing could result in a reduced effectiveness of the screen. The correction factor for different screen inclinations are indicate in Fig. 11.21.

C8: Correction for Aperture Slot Shape (Slot Factor)
Fig. 11.1 illustrates some commonly used aperture shapes. The basic flow rate calculations are based on a square aperture. For non square apertures, a correction factor applies. Except for the round apertures all others regular apertures may be described by the aperture length to width ratio (L/W). Some slight differences between the correction factor values have been published and some data are included in Table 11.5.

<table>
<thead>
<tr>
<th>Aperture shape</th>
<th>Gluck</th>
<th>Colman</th>
<th>Nordberg (Metso)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L/W</td>
<td>C₈</td>
<td>L/W</td>
</tr>
<tr>
<td>square</td>
<td>1</td>
<td>1</td>
<td>&lt;2</td>
</tr>
<tr>
<td>rectangular</td>
<td>&gt; 6</td>
<td>1.6</td>
<td>&gt; 25</td>
</tr>
<tr>
<td>rectangular</td>
<td>3 - 6</td>
<td>1.4</td>
<td>4 - 25</td>
</tr>
<tr>
<td>circular</td>
<td>2 - 3</td>
<td>1.1</td>
<td>2 - 4</td>
</tr>
</tbody>
</table>

C9: Correction for Particle Shape
Fig. 11.5 illustrates the effect of particle shape on screening. Shapes of irregular particles are difficult to describe. The divergence from sphericity or cube can be described in terms of the length/width ratio. An elongated particle is defined as a particle having a length to width ratio greater than 3 and a size between 0.5 and 1.5 times the aperture size. Correction factors have been determined as a function of the percentage of elongated particles in the feed and the various data values are plotted in Fig. 11.22.

C10: Correction Factor for Wet Screening
Water is added during industrial screening for purposes, including:

1. as an aid to screening
2. removal of accumulations in the apertures which tend to block screens
3. reducing dust

Too much water however is inadvisable as it could unnecessarily lead to agglomeration. Colman recommends 15 L/min to 25 L/min per cubic meter of feed for efficient wet screening (1 - 2.5 vol. %, Gluck [16]). But this would depend severely on the composition of the gangue content. For example the bentonite and kaolinite content could lead to sticky material. This can be obviated by a different size of screen openings. The assistance given by water in screening is dependent on the screen aperture. It is generally observed that when the feed size is 25 mm or greater, the error due to water is minimal (wet screening is less effective). As the aperture decreases the correction factor varies as indicated in Fig. 11.23.
Fig. 11.22. Correction factor for particle shape, $C_9$; Gluck [16].

Fig. 11.23. Correction Factor $C_{10}$ for wet screening at different apertures. (Gluck [16], Colman [15], Nordberg (Metso) [18]).
C11: Correction FACTor for Moisture Content

Most ores have inherent and surface moisture. On mining and storage, part of the surface water tends to evaporate. The inherent moisture content is difficult to remove and is only slightly reduced on exposure to air. Most of the inherent water is therefore retained. When the total moisture content is 5% or less, the ore is considered more or less dry and generally the screening operation is satisfactory. This condition is considered to have a correction factor of 1. When the moisture is retained the factor is taken as 1.25. When the ore does not contain hygroscopic material, the factor is 0.85 and for sticky, hygroscopic material the factor is taken as 0.75 (Table 11.6)

Table 11.6
Correction factor C₁₁ for feed condition [16,17].

<table>
<thead>
<tr>
<th>Condition</th>
<th>C₁₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moist or dirty stone, muddy or sticky</td>
<td>0.75</td>
</tr>
<tr>
<td>Moist ore from underground, &gt; 14% (vol) moisture</td>
<td>0.85</td>
</tr>
<tr>
<td>Dry quarried rock &lt; 4-10% (vol) moisture</td>
<td>1.0</td>
</tr>
<tr>
<td>Dry uncrushed material, dried or hot material</td>
<td>1.25</td>
</tr>
<tr>
<td>Wet screening with sprays</td>
<td>1.75</td>
</tr>
</tbody>
</table>

The application of Eq. (11.39) to determine the area of screen for a given flow rate and solid characteristics is illustrated by Example 11.5.

Example 11.5
The size distribution of a dry crushed ore as determined by a standard sieve analysis is given below. The ore was stockpiled and then withdrawn at the rate of 40 t/h for screening on an 850 micron square mesh screen. A screening efficiency of 79% was desired. The open area of the screen was 70%. The bulk density and moisture content of the mineral was 2.7 t/m³ and 18% respectively. The screen is inclined at 15 degrees to the horizontal and particle shape is estimated at 10% elongation. Determine the surface area and the screen size to be used.

<table>
<thead>
<tr>
<th>Size (microns)</th>
<th>% Passing</th>
<th>Size (microns)</th>
<th>% Passing</th>
</tr>
</thead>
<tbody>
<tr>
<td>3350</td>
<td>100</td>
<td>210</td>
<td>24.2</td>
</tr>
<tr>
<td>1680</td>
<td>75.3</td>
<td>105</td>
<td>18.4</td>
</tr>
<tr>
<td>850</td>
<td>49.1</td>
<td>75</td>
<td>10.6</td>
</tr>
<tr>
<td>420</td>
<td>33.9</td>
<td>-75</td>
<td>6.0</td>
</tr>
</tbody>
</table>

Solution Using the Gluck data

Step 1: Base capacity
From Fig. 11.14, the basic capacity of a screen at 850 microns = 6.2 t/h/m²
Step 2: Correction factors

1. The mass correction factor \( C_1 = \frac{2.7}{1.60^2} \) = 1.685
2. The open area factor, \( C_2 = \frac{70}{50} = 1.4 \)
3. From the size distribution of the ore, the percentage of the feed greater than the screen aperture (850 microns) = 100 – 33.9 = 66.1%. From Fig. 11.16 the correction due to over-size material in the screen feed, \( C_3 = 0.95 \)
4. The percentage of the feed less than half the aperture (420 microns) is 24.2%. From Fig. 11.17, \( C_4 = 0.7 \)
5. For a screen efficiency of 79% and Fig. 11.19, the correction factor for screen efficiency, \( C_5 = 1.3 \)
6. For a single deck screen the factor \( C_6 = 1 \)
7. For a screen inclined at 15 degrees, the slope factor from Fig. 11.21, \( C_7 = 1.03 \)
8. For a square aperture, from Table 11.4, \( C_8 = 1 \)
9. At an elongation figure of 10%, \( C_9 = 0.95 \) (Fig. 11.22)
10. For dry screening, the wet screening factor, \( C_{10} = 1 \) (Fig. 11.23)
11. For a moisture of 18%, the correction for feed condition, \( C_{11} = 0.85 \) (Table 11.5)

The overall correction factor is given by:

\[
C_R = 1.685 \times 1.4 \times 0.95 \times 0.7 \times 1.3 \times 1.03 \times 1 \times 0.95 \times 1 \times 0.85 \times 0.74 \times 1.696
\]

Step 3. Screen Area

From Eq. (11.39); \( A = \frac{F}{F_R \times C_R} = \frac{40}{6.2 \times 1.696} = 3.8 \text{ m}^2 \)

Available screen sizes having areas close to the calculated value are (Table 11.7):
3052 x 1219 mm (area 3.72 m²) and 3659 x 1219 mm (area 4.46 m²)

These calculations are an indication only. Screen manufacturers should be consulted for data pertaining to specific screening equipment.

Table 11.7

<table>
<thead>
<tr>
<th>Width mm</th>
<th>Area m²</th>
<th>Width mm</th>
<th>Area m²</th>
<th>Width mm</th>
<th>Area m²</th>
<th>Width mm</th>
<th>Area m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>254</td>
<td>0.31</td>
<td>308</td>
<td>1.24</td>
<td>1524</td>
<td>5.57</td>
<td>2134</td>
<td>10.41</td>
</tr>
<tr>
<td>305</td>
<td>0.37</td>
<td>610</td>
<td>1.49</td>
<td>1219</td>
<td>5.95</td>
<td>1829</td>
<td>11.15</td>
</tr>
<tr>
<td>356</td>
<td>0.43</td>
<td>914</td>
<td>1.67</td>
<td>1524</td>
<td>6.50</td>
<td>2134</td>
<td>11.71</td>
</tr>
<tr>
<td>356</td>
<td>0.54</td>
<td>914</td>
<td>2.23</td>
<td>1829</td>
<td>6.69</td>
<td>2438</td>
<td>11.89</td>
</tr>
<tr>
<td>610</td>
<td>0.56</td>
<td>1219</td>
<td>2.23</td>
<td>1524</td>
<td>7.43</td>
<td>2134</td>
<td>13.01</td>
</tr>
<tr>
<td>406</td>
<td>0.62</td>
<td>914</td>
<td>2.79</td>
<td>1829</td>
<td>7.80</td>
<td>2438</td>
<td>13.38</td>
</tr>
<tr>
<td>610</td>
<td>0.74</td>
<td>1219</td>
<td>2.97</td>
<td>1524</td>
<td>8.36</td>
<td>2438</td>
<td>14.86</td>
</tr>
<tr>
<td>406</td>
<td>0.74</td>
<td>1219</td>
<td>3.72</td>
<td>1829</td>
<td>8.92</td>
<td>2438</td>
<td>17.84</td>
</tr>
<tr>
<td>508</td>
<td>0.93</td>
<td>1219</td>
<td>4.46</td>
<td>2134</td>
<td>9.10</td>
<td>2438</td>
<td></td>
</tr>
<tr>
<td>508</td>
<td>1.08</td>
<td>1524</td>
<td>4.65</td>
<td>1524</td>
<td>9.29</td>
<td>2438</td>
<td></td>
</tr>
<tr>
<td>610</td>
<td>1.11</td>
<td>1219</td>
<td>5.20</td>
<td>1829</td>
<td>10.03</td>
<td>2438</td>
<td></td>
</tr>
</tbody>
</table>
11.4. Operation of Curved Screens

11.4.1. Capacity of Curved Screens
The capacity of a curved screen surface (sieve bend), like straight screens, is a function of screen open area. In addition, the greater the curvature of the screen surface the greater is the centrifugal force and therefore a greater capacity is expected. The capacity and separation of the oversize and the undersize of curved screens depend on:

1. the feed layer thickness which is related to the feed rate,
2. the radius of curvature. This is significant when less than 760 mm and velocities greater than 3 m/s,
3. the angle $\theta$ subtended against the horizontal, see Fig. 11.6,
4. the Reynolds number $\left(\frac{L \cdot v \cdot \rho}{\mu}\right)$ where $L$ is the slot width, $v$ the velocity of slurry through the slot, $\rho$ the density of the slurry and $\mu$ the viscosity of the slurry,
5. the kinematic viscosity $\left(\mu/\rho\right)$,
6. the slot width,
7. the surface tension,
8. the shape of the wedge bars, i.e. triangular or rectangular,
9. the mode of vibration (where employed),
10. the number of slots and the slot spacing.

The influence of the above variables on the overflow and underflow streams has not been quantitatively established satisfactorily. However it is generally observed that relatively smaller radii of curvature result in comparatively higher centrifugal force and therefore more capacity. If we consider the ratio of underflow to feed rate as the measure of capacity then the conditions contributing to maximum capacity will be:

1. the greatest ratio of length of screen opening to thickness of the feed layer,
2. the maximum slot width on the screen,
3. the Reynolds number (Re) is a maximum, that is when the viscosity is a minimum and the product $(L \cdot \rho \cdot v)$ is a maximum,
4. the maximum thickness of bed that does not promote stratification.

The capacity would be adversely affected by:

1. a small angle $\theta$, and
2. a low feed velocity

At low Reynolds numbers, up to approximately 300, the ratio of the undersize capacity to the feed capacity increases but above 400 the Reynolds number does not have any further significant effect on capacity (Fig. 11.24).

The sieve bends commonly used for metallurgical operations are gravity fed having angles between 45° and 50° and with typical bar spacings between 0.15 to 3.0 mm. The capacity is up to 4.3 m$^3$/min/m width. They are used for classification of feed sizes ranging from 100 $\mu$m to 12,000 $\mu$m with feed solid content as high as 45% by volume.
11.4.2. Rapid Method to Determine Sieve Bend Size
Stavenger [7] has recommended a rapid method for estimating the sizes of sieve bends. Like Taggart [5] he has considered the productivity at size ranges less than and more than 300 microns and a minimum feed flow velocity of 3 m/s. The calculations are based on two statistical relations:

1. Bar spacing and diameter of separation (Fig. 11.7) and
2. Bar spacing and capacity (Fig. 11.25)

For example, from Fig. 11.7, considering a separation size which is approximately 50% of the slot width, a separation size of 1000 \( \mu \)m will require a 2.2 mm slot width. Then from Fig. 11.25, for an 800 mm screen length and a slot width of 2.2 mm then screen capacity would be 4.5 m\(^3\)/min/m of screen width.

11.5. Modelling of the Screening Process
The screening process involves material transport along the screen, the probability of undersize particles passing through the apertures of the screen and stratification of fine particles in the particle bed to the screen surface. Modelling of these processes can be quite complex with the result that predicting screening performance has been based on empirical data of the basic capacity per square meter of screen surface combined with correction factors for deviation from the standard screening conditions. These predictions are only an estimate and give no prediction of particle size distribution or screen efficiency.

Under certain circumstances screening and sieving can be represented as a rate process though in practical situations a number of overlapping processes may occur.
11.5.1. Two Process Treatment
Ferrara and Preti [20] proposed that during screening, particles are subjected to two distinctly different types of condition, crowded or separated screening depending on their position on the screen deck and the feed and vibration conditions. This leads to two different rate processes.

**Crowded Screening**
Crowded screening occurs when the flow rate is above a critical value \( F_c \) such that the material bed is so thick that only particles in the layer immediately in contact with the screen are capable of passing through the screen. The eventual passage of a particle under crowded conditions depends on two distinct statistical phenomena:

1. the probability of a particle reaching the screening surface through stratification and  
2. the probability that the particle will pass through the screen.

As long as the upper layers are capable of replenishing the contact layer, that is, probability 1 is higher than probability 2 and particles are hindered by neighbouring particles from passing through the screen, the rate of passage will remain constant and will be given by:

\[
- \frac{dF_L}{dL} = k_c \tag{11.43}
\]

where \( F_L \) = mass flow rate on the screen per unit width, at a distance \( L \) from the feed point, and  
\( k_c \) = rate constant for the crowded condition.
In practice, a range of particle sizes will be present and a separate equation will apply for each particle size $d_i$, then:

$$-\frac{d(F_{L,m_{iL}})}{dL} = k_{Ci} m_{iL} \quad (11.44)$$

where $m_{iL}$ = mass fraction of particles in size interval $i$ in the bed at distance $L$ from the feed point.

Because $F_L$ and $m_{iL}$ are functions of $L$, this becomes on integration:

$$-\ln(1-E_{iU}) = k_{Ci} \int_0^L \frac{dL}{F_L} \quad (11.45)$$

where $E_{iU} =$ mass of size interval $i$ reporting to the undersize stream as a fraction of the mass of size $i$ in the screen feed (the partition coefficient of the undersize) and

$$(1 - E_{iU}) = E_{iO}, \text{ the partition coefficient of the oversize.}$$

A plot of $\ln(1-E_{iU})$ versus $\int_0^L \frac{dL}{F_L}$ should then give a straight line of slope equal to $k_{Ci}$.

Eq. (11.45) can be used to calculate the screen performance curve provided the rate constant and the function $\int_0^L \frac{dL}{F_L}$ is known.

Ferrara et al. [21] introduced a new variable, $\chi_j$, defined as:

$$\chi_j = \frac{\ln E_{iO}}{\ln E_{iO}} = \frac{k_{Ci}}{k_{Ci}} \quad (11.46)$$

where $j =$ size fraction of particles on the screen that affects the kinetics of particles of size $d_i$ (mean size of particles in interval $i$) and

and $\begin{cases} k_{Ci} \neq 0, & \chi_j \neq 0 \\ k_{Ci} = 0, & \chi_j = 0 \end{cases}$ for $d_i \geq L_A$

where $d_j =$ mean size of particles in interval $j$

The crowded screening equation then becomes:

$$F \left[ \int_0^{d_j} m_{jF} \frac{1}{\chi_j} (E_{iO}-1) \, \text{d} d_i + \int_{L_A}^d m_{jF} \ln E_{iO} \, \text{d} d_j \right] = -k_{Ci} L \quad \text{for } 0 < d_i < L_A \quad (11.47)$$
for \( d_i \geq L_A \) \( k_{Ci} = 0 \) and \( E_{i0} = 1 \)

Incorporating the Gaudin model of screening probability [9], then \( \chi_j \) can be expressed as:

\[
\chi_j = \left[ \frac{L_A - d_j}{L_A - d_i} \right]^{\gamma}
\]

(11.48)

where \( \gamma = 2 \) for square mesh and 1 for wedge wire screens.

If \( d_i/L_A = 0.5 \) then:

\[
k_{Ci} = k_{CS0} 2^{\gamma} \left( 1 - \frac{d_i}{L_A} \right)^{\gamma}
\]

(11.49)

where \( k_{CS0} \) = the kinetic constant in the crowded condition for particles of size equal to half the aperture size (ie. \( d_i/L_A=0.5 \)).

Substituting Eq. (11.49) into Eq. (11.47), screening in the crowded condition is then described by the equation:

\[
L_f = -k_{CS0} 2^{\gamma} \left( 1 - \frac{d_i}{L_A} \right)^{\gamma}
\]

(11.50)

for \( d_i < L_A \)

**Separated Screening**

For mass flows across the screen less than \( F_C \), the particles behave as isolated particles and do not interfere with each other. For these conditions, the quantity of particles that pass through the screen, \( df_i \), in the small incremental length \( dL \) is proportional to \( dL \) and the rate \( F \) at which particles enter \( dL \). Therefore assuming a first order relationship:

\[
-\frac{df_i}{dL} = k_s F_L
\]

(11.51)

For a feed of size interval \( i \):

\[
-\frac{d(F_i m_{si})}{dL} = k_{si} F_i m_{si}
\]

(11.52)

Integration of Eq. (11.52) and including the screen oversize partition coefficient gives:

\[
E_{i0} = \exp(-k_{si} L) \quad \text{for} \quad 0 < d_i < L_A
\]

(11.53)
and for \( d_i > L_A \), \( k_{si} = 0 \) and \( E_{jO} = 1 \).

Substituting the similar relationship to Eq. (11.49):

\[
k_{si} = k_{550} 2^\gamma \left(1 - \frac{d_i}{L_A}\right)^\gamma
\]

(11.54)

into Eq. (11.53) gives the approximate separated screening equation:

\[
E_{jO} = \exp \left[ -k_{550} 2^\gamma \left(1 - \frac{d_i}{L_A}\right)^\gamma L \right]
\]

(11.55)

**Combined Screening**

For screening conditions where both crowded and separated screening occur, the overall oversize efficiency is given by:

\[
E_{iOL} = E_{iOL(C)} E_{iOL(L-C)}
\]

(11.56)

where \( E_{iOL}, E_{iOL(C)}, E_{iOL(L-C)} \) = oversize efficiency for screen lengths \( L, L_C \) and \( L-L_C \),

\( L_C = \) the distance from the feed end to the point of transition from crowded to separated screening condition.

Substituting the expressions for the crowded and separated screening efficiencies (Eqs. (11.45) and (11.55) gives:

\[
E_{iOL} = \exp \left[ -k_{C50} n_d 2^\gamma \left(1 - \frac{d_i}{L_A}\right)^\gamma \right]
\]

(11.57)

where \( n_d = \int_0^{L_C} \frac{dL}{F_L} + \frac{L-L_C}{C} \), and

\[
C = \frac{k_{C50}}{k_{550}}
\]

The variable \( n_d \) is not easily determined but is constant under set operating conditions. Ferrara et al. [21] estimate \( C \) as \( F_C \), the mass flow rate on the screen per meter width at \( L_C \).

The combined parameter \( k_{C50}, n_d \) and \( \gamma \) can be estimated by fitting screening data to Eq. (11.57). This will allow screen efficiencies and product sizes to be modelled. For design work, the separate parameter \( k_{C50} \) needs to be evaluated as well as \( \gamma \).

To determine these parameters, Eq. (11.50) is written in the form:

\[
F \left[ \sum_{j=1}^r m_j \frac{1}{k_j} \left( \frac{E_j}{E_{jO}} - 1 \right) + \ln E_{jO} \sum_{j=r+1}^m m_j \right] = -k_{C50} 2^\gamma \left(1 - \frac{d_i}{L_A}\right)^\gamma L
\]

(11.58)
for $1 \leq i \leq n < L_A$ and $E_{io} = 1$ for $L_A < n+1 \leq i \leq m$

where $m$ = the number of size intervals and
$n$ = the number of size intervals less than the screen aperture.

To evaluate the parameters, the screening process is simulated using Eq. (11.58) for interval $i$ using guessed values of the parameters and minimising the sum of the squares of the residuals:

$$\Phi = \sum (E_{io} - E_{io}^*)^2 z_i$$

where $E_{io}, E_{io}^*$ = experimental and calculated screen oversize efficiency and $z_i$ = a weighting factor.

Ferrara et al. [21] likened the significance of the crowded rate constant, $k_{C50}$, to the basic capacity, $F_B$, in the empirical approach to screen sizing in that they both depend on the screen aperture, the open area of the screen, the aperture shape, vibration characteristics and screen slope. The parameter $\gamma$ affects the ratio of probabilities for different particles passing through the screen.

In screen design, it would be necessary to know how $k_{C50}$ and $\gamma$ varies with screen aperture and the other screening parameters in much the same way as the data exists for the base screen capacity.

---

**Example 11.6**

A set of screening data was used to obtain the screening parameters as given below;

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{C50}$</td>
<td>25 t/h/m$^2$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.8</td>
</tr>
<tr>
<td>Screen length, $L$</td>
<td>3.5 m</td>
</tr>
<tr>
<td>Screen feed/width</td>
<td>60 t/h/m</td>
</tr>
<tr>
<td>Screen aperture, $L_A$</td>
<td>0.004 m</td>
</tr>
</tbody>
</table>

Calculate the screen performance and the oversize and undersize distributions given the following screen feed: Note, in this case, interval 1 is the smallest size interval.

<table>
<thead>
<tr>
<th>Interval, $i$</th>
<th>Screen size, $m$</th>
<th>Mean size, $m$</th>
<th>Mass fraction, $m_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0.0060</td>
<td>-</td>
<td>0.2</td>
</tr>
<tr>
<td>6</td>
<td>0.0040</td>
<td>0.0049</td>
<td>0.3</td>
</tr>
<tr>
<td>5</td>
<td>0.0035</td>
<td>0.0037</td>
<td>0.2</td>
</tr>
<tr>
<td>4</td>
<td>0.0025</td>
<td>0.0030</td>
<td>0.1</td>
</tr>
<tr>
<td>3</td>
<td>0.0015</td>
<td>0.0019</td>
<td>0.08</td>
</tr>
<tr>
<td>2</td>
<td>0.0005</td>
<td>0.0009</td>
<td>0.05</td>
</tr>
<tr>
<td>1</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.07</td>
</tr>
</tbody>
</table>
**Solution**

Substitute the screen parameters into Eq. (11.58) and solve for $E_{10}$. This is easily performed using a computer. A spreadsheet solution is shown below.

For the feed size shown, the number of intervals less than the screen aperture, $n = 5$ and the total number of intervals, $m = 7$. A set of initial starting values for $E_{10}$ are estimated as follows:

<table>
<thead>
<tr>
<th>size</th>
<th>$E_{10}$ (guessed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>0.800</td>
</tr>
<tr>
<td>4</td>
<td>0.300</td>
</tr>
<tr>
<td>3</td>
<td>0.050</td>
</tr>
<tr>
<td>2</td>
<td>0.010</td>
</tr>
<tr>
<td>1</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Then, starting with $i = 1; j = 1 \rightarrow 5$

<table>
<thead>
<tr>
<th>$j$</th>
<th>$\chi_i$ (equ (11.48))</th>
<th>$m_{0F}(E_{10}^{eq} - 1)\chi_i$</th>
<th>$j$</th>
<th>$m_{0F}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>-0.012</td>
<td>6</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>0.669</td>
<td>-0.160</td>
<td>7</td>
<td>0.2</td>
</tr>
<tr>
<td>3</td>
<td>0.323</td>
<td>-0.502</td>
<td>$\Sigma$</td>
<td>0.5</td>
</tr>
<tr>
<td>4</td>
<td>0.093</td>
<td>-0.472</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.008</td>
<td>-0.562</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\Sigma = -1.709$

For $i = 1$, the left hand side of Eq. (11.58) is then:

$LHS = 60(-1.709 + (\ln 0.002 \times 0.5) = -289.0$

and the right hand side of Eq. (11.58) is:

$RHS = -25 \times 2^{1.8} (1 - (0.00025/0.004))^{1.8} \times 3.5 = -271.3$

Using Solver in MS-Excel® to zero the square of the difference between the LHS and RHS values using $E_{10}$ as the variable gives a fitted value for $E_{10}$ of 0.0031. Repeating the procedure for other values of $i$ from 2 to 5 gives the following results:

<table>
<thead>
<tr>
<th>size</th>
<th>$E_{10}$ (fitted)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>0.957</td>
</tr>
<tr>
<td>4</td>
<td>0.586</td>
</tr>
<tr>
<td>3</td>
<td>0.155</td>
</tr>
<tr>
<td>2</td>
<td>0.021</td>
</tr>
<tr>
<td>1</td>
<td>0.0031</td>
</tr>
</tbody>
</table>
From the partition coefficients, the oversize and undersize distributions can be estimated from the feed size distribution. The following graphs show the fitted performance curve and the predicted size distribution of the oversize and undersize.

**Segregation Treatment**

Subasinghe et al. [22] considered screening to be described by two simultaneous first order rate processes; segregation and passage through the screen. Segregation of undersize material through the bed to reach the screen surface depends on the size of the undersize relative to the
surrounding particles, the size distribution in the bed and the screen vibration. Particle
passage through the screen was reported first order under conditions giving rise to a constant
probability of passage. Combining these two processes, Subasinghe et al. obtained the
following equation for the fraction of size i retained on the screen after length \( L \), \( m_i \), as:

\[
E_{i\alpha} = \frac{k_{G} \exp(-k_{G} L) - k_{P} \exp(-k_{G} L)}{k_{G} - k_{P}} \text{ for } 0 \leq i \leq L_A
\] (11.59)

where \( k_{G} \) = rate constant for size i segregating to the screen surface and \( k_{P} \) = rate constant for size i passing through the screen.

Analysis on a set of screening data using Eq. (11.59) to estimate \( k_{G} \) and \( k_{P} \) showed that as
the particle size became small relative to the screen aperture, the segregation rate decreases
and the passage rate constant increases while if the particle size approaches the aperture size
(near size particles), the segregation rate constant increases and the passage rate constant
approaches zero. For intermediate values of \( d_i/L_A \), the value of \( k_{G} \) approaches the value of \( k_{P} \)
and a dynamic equilibrium exists between the two processes.

The variation of \( k_{G} \) and \( k_{P} \) with particle size for this data set were described by the
empirical correlations:

\[
\ln k_{G} = -4.311 + 21.810 \left( \frac{d_i}{L_A} \right) - 54.876 \left( \frac{d_i}{L_A} \right)^2 + 40.544 \left( \frac{d_i}{L_A} \right)^3
\] (11.60)

\[
\ln k_{P} = 0.8779 - 16.744 \left( \frac{d_i}{L_A} \right) + 40.120 \left( \frac{d_i}{L_A} \right)^2 - 37.310 \left( \frac{d_i}{L_A} \right)^3
\] (11.61)

Thus from known values of \( k_{G} \) and \( k_{P} \), the size distribution of the screen undersize can be
estimated. However, evaluation of more data sets are required to determine how \( k_{G} \) and \( k_{P} \)
 vary with equipment and particle characteristics.

Subasinghe et al. [22] observed that plots of the expression in Eq. (11.59) had a similar
shape to a 2 parameter survival function of the Weibull distribution function and that in the
form of a Rosin-Rammler function was adequate to describe the screen products as:

\[
E_{i\alpha} = \exp \left[ \frac{L}{B} \right]^{A}
\] (11.62)

The constants A and B were fitted to third order polynomials and for the same set of
screening data used above:

\[
A = 1.196 - 2.803 \left( \frac{d_i}{L_A} \right) + 15.74 \left( \frac{d_i}{L_A} \right)^2 - 14.13 \left( \frac{d_i}{L_A} \right)^3
\] (11.63)

\[
\log B = 1.000 + 0.147 \left( \frac{d_i}{L_A} \right) - 1.013 \left( \frac{d_i}{L_A} \right)^2 + 2.570 \left( \frac{d_i}{L_A} \right)^3
\] (11.64)

The shape of a performance curves for a vibrating screen is shown in Fig. 11.26. The
upturned end of the curve at fine sizes is attributed to the low proportion of fines in the feed
and the rate of segregation of fines is low at this size, possibly as a result of the fines adhering
to coarser particles.

The JKMRC modelled the screen on the basis of the efficiency curve, described by the
equation:
Fig. 11.26. Efficiency curve for a vibrating screen (after Subasinghe et al. [22])

\[ E_{\text{io}} = \exp \left( -n \frac{A_0}{100} \left( 1 - \frac{d_i}{L_A} \right)^\gamma \right) \]  
(11.65)

where 
- \( n \) = an efficiency parameter which is related to the number of attempts the particle has to pass the screen,
- \( A_0 \) = the percent open area,
- \( d_i \) = particle size, and
- \( \gamma \) = approximately 2

This equation applies for the regular shaped central portion of the curve in Fig. 11.26. The variation of the efficiency factor with respect to the operating conditions is obtained from a set of regression equations of the form [23]:

\[ \ln(n) = K_1 + K_2 F + K_3 P_1 + K_4 P_2 \]  
(11.66)

\[ \ln(n) = K_5 + K_6 F + K_7 P_1 + K_8 P_2 \]  
(11.67)

\[ \ln(n) = K_9 + K_{10} F_2 + K_{11} P_1 + K_{12} P_2 \]  
(11.68)

where \( K_1, K_2, K_3, K_4, K_5 \) and \( K_6 \) are regression constants. \( K_5 \) and \( K_6 \) are usually set at zero.

- \( P_1 \) = percent of the feed in size interval \( i \)
- \( P_2 \) = percent of the feed less than \( d_c \), a critical size close to \( L_A \)
The upturned end of the curve is described by a function $SF$ which is determined from experimental data and is related to the percent fines and the fines feed rate:

$$SF = K_7 + K_8 \cdot 100m_{kF} + K_9 F_k$$  \hspace{1cm} (11.69)

where $K_7$, $K_8$, $K_9$ are regression constants and

$m_{kF} = \text{fraction of the feed that is less than size } d_k$

$F_k = \text{feed rate of material that is less than size } d_k$

$d_k = \text{smallest screen.}$

This fines factor accounts for the fine particles that adhere to larger particles and hence are retained in the oversize fraction. The quantity of this misplaced material is dependant on the surface area of the particles. This is expressed as:

$$A_s \approx \sum_{i=1}^{a} \left[ \frac{V_i}{\left( \frac{d_i + d_{\text{top}}}{2} \right)} \right]$$  \hspace{1cm} (11.70)

where $A_s = \text{total surface area}$,

$V_i = \text{particle volume in interval } i$, and

$d_i = \text{the top size of the interval } i$.

The tonnage of fines (particles less than the finest screen eg. 6.3 mm in example 11.6) that are retained within the oversize fraction, is given by;

$$F_{FO} = SF \times A_s$$  \hspace{1cm} (11.71)

The screen length is scaled from the efficiency factor, $n$.

---

**Example 11.7**

Product from a jaw crusher is screened at 63 mm using a single deck vibrating screen. The parameters of the Whiten and White model have been determined from survey data and are given below.

For a feed of 285 tph and a feed size distribution given below, calculate the oversize and undersize size distributions.
From the size distribution of the feed the cumulative % passing data is calculated:

<table>
<thead>
<tr>
<th>Size, mm</th>
<th>% Retained</th>
<th>Size, mm</th>
<th>% Retained</th>
</tr>
</thead>
<tbody>
<tr>
<td>152</td>
<td>2</td>
<td>63</td>
<td>9</td>
</tr>
<tr>
<td>125</td>
<td>2</td>
<td>45</td>
<td>17</td>
</tr>
<tr>
<td>106</td>
<td>7</td>
<td>31.5</td>
<td>6</td>
</tr>
<tr>
<td>100</td>
<td>4</td>
<td>19</td>
<td>10</td>
</tr>
<tr>
<td>90</td>
<td>12</td>
<td>6.3</td>
<td>10</td>
</tr>
<tr>
<td>75</td>
<td>11</td>
<td>0</td>
<td>10</td>
</tr>
</tbody>
</table>

From this distribution,

\[ F = 285/3.048 = 93.5 \text{ t/h/m} \]

\[ d_k = 6.3 \text{ mm} \]

\[ m_{kr} = 0.10 \]
\( F_k = 93.5 \times 0.10 = 9.35 \text{ t/h/m} \)

Since \( F < F_1 \), using Eq. (11.66);

\[
\ln(n) = 3.5 + (-0.004 \times 93.5) + (0 \times P_1) + (0 \times P_2) = 3.126
\]

and \( n = 22.8 \)

Now from Eq. (11.65), the partition coefficient of the oversize, \( E_{10} \), can be calculated;

<table>
<thead>
<tr>
<th>Interval</th>
<th>Size, ( d_i ) (mm)</th>
<th>( E_{10} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>152</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>125</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>106</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>90</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>75</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>63</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>45</td>
<td>0.242</td>
</tr>
<tr>
<td>9</td>
<td>31.5</td>
<td>0.018</td>
</tr>
<tr>
<td>10</td>
<td>19</td>
<td>0.001</td>
</tr>
<tr>
<td>11</td>
<td>6.3</td>
<td>0.00</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>0.00</td>
</tr>
</tbody>
</table>

For example, considering size interval 8;

\[
E_{10} = e^{\exp \left[ -22.8 \times \frac{64}{100} \left( 1 - \frac{45}{63} \right)^{0.86} \right]} = 0.242
\]

Note, for \( d_i > L_A \), \( E_{10} = 1 \).

The size distribution of the screen oversize and undersize can then be calculated from the screen feed. The table below shows the results and example calculations for one size interval is given below:

for size interval 8,

\[
E_{10} = \frac{\text{mass of size interval 8 in the O/S}}{\text{mass of size interval 8 in the feed}}
\]

\[
\text{mass of size 8 in the O/S} = 0.242 \times 48.45 = 11.73 \text{ tph}
\]

and \( \text{mass of size 8 in the U/S} = 48.45 - 11.73 = 36.72 \text{ tph} \)
The surface area for each size interval is given by Eq. (11.70);

\[ A_{ss} = \left( \frac{48.45}{3.8} \right) \left( \frac{63 + 45}{2} \right) = 0.236 \]

and the total surface area is given by \( A_S = \sum A_{ss} = 4.00 \)

The fines factor is given by Eq. (11.69);

\[ SF = 0.035 + (0.0025 \times 100 \times 0.1) + (-0.0015 \times 9.35) = 0.046 \]

and the tonnes of fines in the oversize;

\[ F_{FO} = 0.046 \times 4.00 = 0.184 \text{ tph} \]

This fines factor should be added to the fines fraction of the screen oversize and subtracted from the screen undersize to give a corrected partition coefficient in the table below.

The efficiency curve and predicted size distributions are shown in the graphs below.
The predicted partition curve for the screen in Example 11.7

Predicted screen oversize and undersize using Eq. (11.65).
11.5.2. Modelling Sieve Bends

In the sieve bend, separation is considered the result of thin layers of slurry sliced off the slurry stream passing over the screen surface and being diverted to the screen underflow. Fontein [6], considered the main parameters in the sieve bend separation are:

1. the ratio $d_{50}/L_{A}$ and
2. the fraction of the feed stream reporting to the undersize stream, $F_u$.

The separation size of the screen should be small compared to the screen aperture ($d_{50}/L_{A}$ small) to minimise blinding, while $F_u$ is a function of the separating size with high values of $F_u$ yielding large separating sizes. Analysing the factors that contribute to the thickness of the diverted layer Fontein derived the equations:

$$
\frac{d_{50}}{L_{A}} = \frac{K}{R} + \frac{K_2}{f(Re_s)} \left( \frac{L_e + L_f g \sin \theta}{v^2} + \frac{K_3 \gamma_s}{\rho_p v^2 L_{A}} \right)^{0.5}
$$

and

$$
F_u = \frac{K_4 L_{A} \gamma_s N}{L_e + L_f g \sin \theta + \frac{K_3 \gamma_s}{\rho_p v^2 L_{A}}} 0.5
$$

(11.72)

where $K$ = constants,
$N$ = number of slots,
$R$ = radius of curvature of the screen surfaces,
$Re_s$ = Reynolds number of the slot, $L_{A} \nu \rho_p / \mu$ ,
$\nu$ = feed velocity,
$L_f$ = thickness of feed layer,
$\theta$ = angle of arc of the screen surface, and
$\gamma_s$ = surface tension.
Fontein showed that at high Reynolds Numbers, the separation size of the DSM screen was around half of the aperture size. This relationship is likely to change however as the screen aperture increases or decreases (Fig. 11.7).

The DSM screen may be modelled on a reduced performance curve. Lynch [24] produced a linear relationship between the corrected d50 and the screen design parameters:

$$\log(d_{50c}) = K_1 \log L + K_2 Q_w w_U + K_3 M_F + K_4$$  \hspace{1cm} (11.73)

where

- $d_{50c}$ = corrected separation size
- $Q_w$ = volumetric flowrate of the feed
- $w_U$ = fraction of feed water split to the underflow
- $M_F$ = mass % solids of the feed
- K = constant

Based on laboratory and plant data, Lynch [24] obtained values of the constants K as follows:

- $K_1 = 1.1718$
- $K_2 = 0.001372$
- $K_3 = 0.0029$
- $K_4 = 2.45$

11.6. Screening and Crushing Circuits

When grizzlies are used to receive ROM ores they are primarily used as a scalping screen and more often as a single deck operation in open circuit. The capacity of scalping screens is given by the screen dimensions, the depth of bed, the bulk density of material and the speed of travel of material on screen surface. The capacity may be written as:

$$Q_s = 6 \times 10^{-3} (D W v \rho_B), \hspace{1cm} \text{t/h}$$ \hspace{1cm} (11.74)

where

- $D$ = Bed depth at the feed end, mm,
- $W$ = Width of screen, m,
- $v$ = velocity of travel, m/min,
- $\rho_B$ = bulk density, kg/m$^3$

Most commercial screening is performed in closed circuit, particularly in crushing and grinding operations. Since these are continuous processes the oversize from the screen is returned continuously for re-crushing. In so doing the original character of the feed changes and results in an altered feed size distribution and change in bulk density. Therefore, the screen size to be used has to be reassessed under the new conditions. The methodology of screen selection however remains the same.

11.7. Problems

11.1

A 5 mm square aperture single deck screen woven with 1.0 mm uniform diameter stainless steel wire was used to classify a crushed and dried mineral having the following sieve analysis:
The bulk density of the mineral was 1.5 t/m³ and the feed rate required was 100 t/h. Estimate:

1. The area of screen
2. The size of screen for effective screening
3. If the screen had two decks estimate the area of each.

11.2
Iron ore of bulk density 2080 kg/m³ containing 5% moisture by volume had the following screen analysis:

<table>
<thead>
<tr>
<th>Size, mm</th>
<th>Cum mass % Passing</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>25</td>
<td>95</td>
</tr>
<tr>
<td>12.5</td>
<td>90</td>
</tr>
<tr>
<td>6.3</td>
<td>75</td>
</tr>
<tr>
<td>3.0</td>
<td>35</td>
</tr>
</tbody>
</table>

The ore had to be screened at the rate of 180 t/h through a 12.5 mm screen. The clamps and strips holding down the screen occupied 12 % of the screen surface. Determine:

1. The effective area of the screen,
2. The bed height to be maintained,
3. The flow rate at 20° inclination of screen.

11.3
A gold ore was crushed in a secondary crusher and screened dry on an 1180 micron square aperture screen. The screen was constructed with 0.12 mm diameter uniform stainless steel wire. The size analysis of the feed, oversize and undersize streams are given in the following table. The gold content in the feed, undersize and oversize streams were; 5 ppm, 1.5 ppm and 7 ppm respectively. Calculate:

1. The mass ratios of the oversize and undersize to the feed,
2. Overall efficiency of the screen,
3. Distribution of gold in the oversize and undersize streams.
Iron ore with a moisture content of 6% was fed to a screen at the rate of 200 t/h. The screen had a square opening of 12.5 mm made of uniform stainless steel wire. The size analysis of the feed was:

<table>
<thead>
<tr>
<th>Size mm</th>
<th>Cum. Mass % Passing</th>
</tr>
</thead>
<tbody>
<tr>
<td>38</td>
<td>100</td>
</tr>
<tr>
<td>25</td>
<td>96</td>
</tr>
<tr>
<td>12.5</td>
<td>84</td>
</tr>
<tr>
<td>6.3</td>
<td>39</td>
</tr>
<tr>
<td>3</td>
<td>16.6</td>
</tr>
</tbody>
</table>

Assume the bulk density of the ore is 1600 kg/m³ and the screen length equals the width. Determine:

1. the size of the screen,
2. the screening efficiency if the feed rate was increased to 250 t/h,
3. the efficiency of screening when the depth of bed on the screen was increased by 10%.

A cassiterite ore (SG 7.0) was crushed in a jaw and cone crusher yielding a product whose average size was 25% greater than 16 mesh. The crushed ore was screened on a 16 mesh screen having a clear opening of 1 mm (wire diameter 0.59 mm) inclined at 20° to the horizontal. Calculate:

1. the screen area required for a feed rate of 60 t/h,
2. the change in feed rate if the slope was reduced to a horizontal position, but maintaining the same efficiency,
3. the percent of fine material in the undersize product when the efficiency was 80%.

Assume a bed porosity of 40%.
11.6
The effective length and width of a vibrating screen was 1.5 m and 10 m respectively. The screen was made of wire 10 mm in diameter with an open area of 70%. The feed size of a mineral to be screened was 48% oversize and 30% less than half the aperture of the screen. The speed of travel of the material over the screen was 15 m/min and the feed rate 50 t/h. The bulk density of the material was 1.8 t/m$^3$. Estimate:

1. The depth of the material on the screen,
2. Comment on the suitability of the screen if the feed rate was increased to 120 t/h.

11.7
A vertical shaft furnace was designed to operate on a coke size of 60 x 30 mm. Coke from a coke oven, after preliminary crushing in a hammer mill, gave the following size analysis:

<table>
<thead>
<tr>
<th>Size, mm</th>
<th>Mass % retained</th>
</tr>
</thead>
<tbody>
<tr>
<td>-100 + 85</td>
<td>14</td>
</tr>
<tr>
<td>-85 + 42.5</td>
<td>31</td>
</tr>
<tr>
<td>-42.5+25</td>
<td>22</td>
</tr>
<tr>
<td>-25 + 18</td>
<td>8</td>
</tr>
<tr>
<td>-18 + 15</td>
<td>3</td>
</tr>
<tr>
<td>-15 + 7.5</td>
<td>8</td>
</tr>
<tr>
<td>-7.5</td>
<td>14</td>
</tr>
</tbody>
</table>

The coke was screened over a single-decked screen with circular holes, inclined at an angle of 25 degrees. The moisture content of the coke was 4%. The feed rate to the shaft furnace was 759 t/day. The maximum permissible bed depth on the screen was 100 mm. Assume that the screen length equals 1.2 times the width, screen open area is 40%, the density of coke is 600 kg/m$^3$, the bed porosity was 40% and the screen efficiency was 40%. Determine:

1. the capacity of the screen,
2. the effective screen area,
3. the travel rate of material over the screen.

11.8
The oversize from a 12.5 mm aperture screen was fed to a crusher. The efficiency of the screen was 80%. The product size from the crusher was 80% minus 12.5 mm at a close set of 12.5 mm and was returned to the screen for sizing. The initial feed to the screen was 120 t/h and the screen undersize was also 120 t/h at steady state. Estimate the recirculating load on the screen.
11.9
The feed to a 3 deck screen gave the following analysis:

<table>
<thead>
<tr>
<th>Feed size, mm</th>
<th>Cum. mass percent passing</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>100</td>
</tr>
<tr>
<td>50</td>
<td>90</td>
</tr>
<tr>
<td>36</td>
<td>70</td>
</tr>
<tr>
<td>24</td>
<td>33</td>
</tr>
<tr>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
</tr>
</tbody>
</table>

The screen was fed at the rate of 60 t/h. The screen opening and the product rate from each deck is given above.

Determine the minimum area of each screen and a suitable final screen size.

11.10
A sieve analysis of a silicious gravel containing 5% moisture is given below.

The ore was to be screened at 6 mm using a single deck square opening screen having a 42% open area. The bulk density of the ore was 3.2 t/m³. Assuming the length/width ratio is 1.5, estimate the area of the screen.

<table>
<thead>
<tr>
<th>Feed size, mm</th>
<th>Mass % retained</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>12.5</td>
<td>30</td>
</tr>
<tr>
<td>6</td>
<td>32</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>
REFERENCES

Chapter 12. Classification

12. INTRODUCTION

After initial liberation of a mineral constituent from its ore by crushing, grinding and screening, separation of minerals by size are normally attempted by a classifying process. In mineral processing operations, classification and separation of mixtures of fine and coarse particles and also of lighter and heavier particles may be performed in a wet or dry state. The majority of separations are carried out in a liquid environment because of an increased efficiency. The basic technique employed is to allow particles to settle under gravity in a liquid medium (usually water). The higher terminal velocity of irregular shaped, coarser, heavier particles allows these particles to reach the bottom of the vessel at a faster rate compared to particles that are smaller and lighter. Removing the settled particles while the others are still settling offers a simple means of a separation. For very small particles, like clay or silt, whose size approaches colloidal dimensions, long times are required to settle and the small difference in settling rates of these fine particles leads to low separation efficiency.

To accelerate the settling rate of these fine particles, centrifugal forces are employed such as in cyclones or hydrocyclones.

In this chapter we shall confine ourselves to the design and operation of the common types of classifiers, namely those that depend on gravity forces alone and those that employ a combination of gravity and centrifugal forces.

12.1. Design Features of Mechanical Classifiers

The design of mechanical classifiers includes a settling tank and a mechanism to remove the settled solids from the bottom of the tank. The settled solids are usually conveyed away by some discharge system while the overflow is collected in launders and pumped away. The classifier designs differ mainly in the mode of removing the underflow and the overflow slurries. Immersed spiral or rakes are generally used for underflow slurries and an open launder carries the overflow. Fig. 12.1 is a sketch of a spiral classifier where the spiral conveyor is installed within the bowl. The spiral operates along the sloping sides of the tank and dredges the thick sludge out of the tank. Fig. 12.2 shows the spiral conveyor replaced by a rake, which drags the sand up the incline for discharge.

Fig. 12.3 shows a submerged rotating rake inside a conical bowl, which collects the settled sand in a well from which it is conveyed or pumped away. Classifiers are either rectangular or circular in shape with the bottom inclined at an angle. The circular tanks are more common.

12.1.1. Spiral classifiers

The shape of the spiral classifier tanks is usually rectangular (Fig. 12.1). The feed is introduced at a position about halfway along the length of the settling tank. The tank slopes range from 14° to 18°. The slope is adjusted such that the top end is higher than the height of the overflow weir. The spirals impede the downward slurry movement resulting in some build up. The sides are therefore raised. Classifiers with raised sides are generally called high or H-type classifiers. In contrast, classifiers with low sides and shallow tanks are known as
Fig. 12.1. Sketch of a spiral classifier.

Fig. 12.2. Sketch of a rake classifier.

Fig. 12.3. Sketch of a Bowl Classifier with spiral conveyor for collecting sand from the tank and discharging to the launder at the top end of the vessel.
S-type classifiers. The S type classifiers have almost gone out of use. The maximum lengths of H type classifiers are about 14 m with widths of 0.5 to 7 m and spirals up to 2400 mm in diameter. The speed of rotation of the spirals varies inversely with size. Thus classifiers with a 300 mm spiral diameter revolve at about 8-20 rpm while the 2000 mm diameter spirals rotate at about 2-5 rpm to give a sand conveying speed of 2-3.5 m/s. The raking capacity of the large classifiers is approximately 200 t/h while smaller classifiers have raking capacities as low as 1.5 t/h. To some extent the capacities depend on the number and design of the helix in the spiral. The helix could be single, double or even triple pitch. The pitch is related to the diameter of the spirals. It is generally of the order of 0.5 to 0.75 times the diameter of the shaft. The number of helix may be single (simplex) or two side by side (duplex) depending on the dimensions of the tank.

Some spiral classifiers have flared sides. This increases the capacity. For example, in a simplex type H classifier, the capacity is increased 1.3 times and for a duplex type H classifier, the capacity may be increased 2-3 times.

The feed size of particles to spiral classifiers is in the region of 150 microns and coarser. The overflow particle size distribution depends both on the height of the weir and a baffle placed before the weir. The baffle is placed within the tank and located at a distance of approximately 38 mm (maximum about 380 mm) from the weir. The flow rate of the overflow stream ranges from 1 t/h to around 40-45 t/h. Increasing the feed flowrate increases the overflow rate, decreases the residence time and increases the fraction of coarse particle sizes in the overflow stream. A slow feed rate, well spread out along the width, is preferred for finer feeds to eliminate or reduce the presence of coarser sizes in the overflow stream.

12.1.2 Rake classifiers
When rakes are used in place of spirals, the classifiers are called Rake classifiers. These are less common than spiral classifiers. The rakes consist of one or more parallel lines of steel plates that hang from a central shaft or shafts. The plates are hinged on to these shafts and have a reciprocating movement. As in spirals, the plates agitate the settling solids and drag the settled particles up the inclined base of the tank. At the end of the stroke the plates rise sharply and then are lowered back into the tank after an eccentric movement to its original position. On repeating the operation the settled matter is conveyed up the inclined slope and finally discharged into the sands launder. The overflow stream passes over a weir at the bottom end of the tank and pumped to the next processing stage.

Typical sizes and stream characteristics of rake classifiers are summarised in Table 12.1 and Table 12.2.

Table 12.1
Rake Classifier summary [1].

<table>
<thead>
<tr>
<th>Description</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>Min. 1.2 m</td>
</tr>
<tr>
<td></td>
<td>Max. 4.8 m</td>
</tr>
<tr>
<td>Tank slope</td>
<td>9.4°</td>
</tr>
<tr>
<td></td>
<td>11.7°</td>
</tr>
<tr>
<td>Rake speed</td>
<td>5 strokes/min</td>
</tr>
<tr>
<td></td>
<td>30 strokes/min</td>
</tr>
<tr>
<td>Capacity</td>
<td>20 t/day/m-width-stroke</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Power</td>
<td>7.6 kWh</td>
</tr>
<tr>
<td></td>
<td>15.2 kWh</td>
</tr>
</tbody>
</table>
Table 12.2
Stream Characteristics of Classifiers [1].

<table>
<thead>
<tr>
<th>Streams</th>
<th>% Solids, mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feed</td>
<td>65 (max.)</td>
</tr>
<tr>
<td>Overflow</td>
<td>1-35</td>
</tr>
<tr>
<td>Underflow</td>
<td>75-83</td>
</tr>
</tbody>
</table>

However, the larger industrial sized rake classifier tanks are around 3.7 m to 12 m in length and 4.5 m to 5 m in width.

12.1.3. Cone classifiers

The cone classifier is the simplest of all of the classifiers, however its use in industry is relatively limited. The classifier vessel is conical in shape. The feed enters the vessel (Fig. 12.3) through a centrally located inlet pipe. Initially the bottom spigot is closed. When the slurry reaches a certain height, the spigot is opened. The settled particles then discharge through the spigot. The finer particles travel with the water to the periphery and overflow into a launder.

The mechanism of settling in cone classifiers was described by Kojovic and Whiten [2] as the settling of coarse particles against an upward flowing overflow stream. The mechanics of settling depended on:

1. Particle size, \(d\)
2. Velocity of slurry in the cone section, \(v\)
3. Overflow volume fraction of solids,
4. Underflow and overflow pulp densities, and
5. Viscosity of the slurry, \(\mu_{SL}\)

In the ideal case, where the particles are considered as perfect spheres and the medium through which they fall as infinite with no wall effect, Stokes Law describes the terminal velocity of the particles as:

\[
\nu_T = \frac{d^2 g (\rho_s - \rho_f)}{18 \mu}
\]  

(12.1)

where
- \(\nu_T\) = terminal velocity
- \(\rho_s, \rho_f\) = density of solid and fluid (liquid or gas)
- \(g\) = acceleration due to gravity
- \(d\) = particle diameter (sphere)
- \(\mu\) = viscosity of the fluid (liquid or gas)

The free fall of the particles depend on the Reynolds number, \(Re\), and the Froude number, \(Fr\). Using these dimensionless numbers, a quantitative estimation of the separation of irregularly shaped particles of different sizes can be obtained. According to Kojovic and Whiten [2] for a cone of vertical height \(H\) and apex diameter \(D_u\) (Fig. 12.4), the dimensionless groups, \(Re\) and \(Fr\), in the cone section and apex sections are:
Reynolds number, Cone section, \( \text{Re}_c = \frac{2 \rho_{SL} v H}{\mu} \) \hspace{1cm} (12.2) \\

Froude number, Cone section, \( \text{Fr}_c = \left[ \frac{\rho_{SL}}{\rho_s - \rho_{SL}} \right] \frac{v^2}{g H} \) \hspace{1cm} (12.3) \\

Reynolds number, apex section, \( \text{Re}_A = 2D_U \sqrt{g H \rho_{SL}} \) \hspace{1cm} (12.4)

Using these dimensionless numbers Kojovic and Whiten derived the underflow solids concentration, \( C_{S(U)} \), and the 50\% size split (\( d_{50C} \)) for cone classifiers as:

\[
C_{S(U)} = \frac{8.56 \exp(2.38 V_{SF}) \text{Fr}_c^{0.07} \text{Re}_A^{0.11} \left[ \frac{d_{50}}{H} \right]^{-0.10}}{\text{Re}_c^{0.07} \left[ \frac{A_U}{A_C} \right]^{0.24}}
\hspace{1cm} \text{and}
\hspace{1cm} (12.5)
\]

\[
d_{50C} = \frac{\exp(7.02 V_{SF}) \text{Fr}_c^{0.38} \left[ \frac{d_{50}}{H} \right]^{0.53}}{\exp(7.05(V_{SF} - V_{SO}) \text{Re}_c^{0.33} \left[ \frac{A_U}{A_C} \right]^{0.48}} \cdot 2H
\hspace{1cm} (12.6)
\]

Fig. 12.4. Cone classifier.
where $V_{SF}$ = volume fraction of solids in the feed  
$V_{SO}$ = volume fraction of solids in the overflow  
$Fr_C$ = Froude number, cone section  
$Re_A$ = Reynolds number, apex section  
$Re_C$ = Reynolds number, cone section  
$A_U$ = cross sectional area of the apex (underflow)  
$A_C$ = cross sectional area of the cone  
$d_{80}$ = particle 80% passing size of the feed

Kojovic and Whiten suggest that both Eqs. (12.5) and (12.6) are applicable for industrial cone classifiers having diameters between 0.073 m and 3 m and feed rates of 1.2 to 5000 L/min.

12.1.4. Bowl classifiers

The bowl classifiers are similar to cone classifiers except that a bowl with relatively shallow sides replaces the deep cone. The feed, in the form of slurry, enters the bowl through a centrally located pipe. The slurry in the bowl is gently agitated by rotating immersed rakes. The relatively heavy particles settle to the bottom of the bowl, which slopes towards the centre of the tank. The settled particles are collected by the submerged rakes and guided to the discharge end by a conveyor for dispatching as the underflow fraction.

The maximum diameter of industrial size bowl classifiers is around 7.8 m. and minimum around 1.2 m. In some bowl classifiers vibrating plates operate just under the surface of the slurry to help break up agglomerated particles. The present tendency is to replace the rakes with vibrating plates.

12.2. Designing the Pool Area of Mechanical Classifiers

In practice the effective area of the bowl appears smaller than the actual bowl size. This is also true for spiral and rake classifiers. The ratio of the effective area to the actual area is known as the areal efficiency. Fitch and Roberts [3] have determined the areal efficiencies factors of different classifiers as shown in Table 12.3.

Table 12.3
Areal efficiency of pool classifiers [3]

<table>
<thead>
<tr>
<th>Classifier</th>
<th>Areal efficiency factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Minimum</td>
</tr>
<tr>
<td>Rake</td>
<td>0.2</td>
</tr>
<tr>
<td>Spiral</td>
<td>0.2</td>
</tr>
<tr>
<td>Bowl</td>
<td>0.4</td>
</tr>
</tbody>
</table>

The percent areal efficiency is affected by the speed of the rake. For submerged rakes, Hitzrot and Meisel [1] determined the relation between the stroke rate and areal efficiency. Their relation is reproduced in Fig. 12.5 where it can be seen that the areal efficiency decreases with increasing stroke rate and therefore with agitation.

For designing the pool area of a classifier, the concept of areal efficiency is necessary. Also it is necessary to estimate the settling forces, the size of the overflow particles, the
volume flow rate of the overflow or underflow stream and the settling rate of the heavier particles. The settling rate in turn depends on the shape of the particles and any disturbance in the pool. Roberts and Fitch [4] and Fitch and Roberts [3] considered these factors and stated that the product of these factors determined the settling rate. In the case of spherical particles, the settling rate is given by:

\[
\text{Settling rate} = v_s H P_s A_{\text{EFF}}
\]

(12.7)

where

- \( v_s \) = the settling rate of spherical particles at infinite dilution (no hinderance)
- \( H \) = the hindrance factor
- \( P_s \) = the shape factor and
- \( A_{\text{EFF}} \) = the areal efficiency factor included to account for a decrease in settling rate resulting from turbulence or contact with other particles in the pool.

To determine the pool area \( A \), it is assumed that the settling rate was related to the volume of water passing over the weir. The quantity of overflow liquid (water) passing over the weir in unit time will be:

\[
Q_{VL(0)} = v_s A H P_s A_{\text{EFF}}
\]

or

\[
A = \frac{Q_{VL(0)}}{v_s H P_s A_{\text{EFF}}}
\]

(12.8)

To apply Eq. (12.8) to non-spherical particles Fitch and Roberts [3] considered \( v_s \) as the settling rate of spheres under ideal conditions, (that is an infinite, undisturbed volume of
water) and the shape factor, $P_s$, as the deviation of the particle shape from a sphere. The values of each parameter were determined in the following manner:

1. **Estimation of $v_s$:**
   Under ideal conditions of settling, the terminal velocity, $v_T$, is given by:

   $$v_T = \left( g \left( \frac{\rho_s - \rho_L}{\rho_L} \right) \left( \frac{\mu}{\rho_L} \right) \right)^{1/3} \text{ m/s}$$
   (12.9)

   where $\rho_s = \text{density of solids, kg/m}^3$, $\rho_L = \text{density of liquid, kg/m}^3$, $\mu = \text{viscosity of liquid, Pa.s}$, $g = \text{acceleration due to gravity, 9.81m/s}^2$.

   While the ideal settling velocity is related to the dimensionless Reynolds number, for non-ideal system, Roberts and Fitch considered a reduced Reynolds number, $Re_R$, defining it as:

   $$Re_R = \left( \frac{d_{so} \sqrt{g \rho_s}}{\mu} \right)$$
   (12.10)

   where $d_{so} = \text{the size of separation}$

   For different values of reduced Reynolds number the values of the dimensionless term $v_s/v$ can be determined. Such a plot is reproduced in Fig. 12.6 for Reynolds numbers varying between 1 and 1000. In practice the value of $Re_R$ is estimated and the value of $v_s/v$ determined from Fig. 12.6. Then from a known value of $v$ the value of $v_s$ is determined.

2. **Estimation of the hindrance factor, $H$:**
   The estimation of the hindrance factor $H$ for separation size $d_{so}$, also involves considerations of the ideal state of settling. Further it is assumed that:

   1. all coarse particles have been separate in the pool,
   2. the concentration of the finer particles ($<d_{so}$) remain unchanged, and
   3. a void fraction, $\varepsilon$, exists between the particles in the settling zone.

   The void fraction $\varepsilon$ was expressed as:

   $$\varepsilon = \frac{1}{1 + (v'_{T}/v_T)}$$
   (12.11)

   where $V_F = \text{volume dilution in the feed, } (V_{L(F)}/V_{S(F)})$, $V_d = \text{volume fraction of solids finer than the } d_{so} \text{ in the feed, } (V_{450}/V_{S(F)})$, $V_{450} = \text{volume of solids finer than the } d_{so} \text{ in the feed}$, $V_{S(F)}, V_{L(F)} = \text{volume of solids and liquid respectively in the feed.}$
The hindrance factor is defined by some power function of the void fraction, which in turn is related to Reynolds number and particle shape. Mathematically this is written as:

$$H = e^{f(\text{Re},P_s)}$$  \hspace{1cm} (12.13)

For different values of Reynolds number and shape factors (see Table 12.4), the function $f(\text{Re},P_s)$ can be calculated and plotted. Such a plot is shown in Fig. 12.7 using data from the work of Fitch and Roberts [3]. Thus for different values of $e$, obtained from Eq. (12.12), the hindrance factor, $H$, can be estimated.

3. Estimation of shape factors:
The shape factors of selected minerals are given in Table 12.4.

Table 12.4
Typical shape factors of selected minerals [3].

<table>
<thead>
<tr>
<th>Particles</th>
<th>Shape factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>spheres</td>
<td>1.0</td>
</tr>
<tr>
<td>cubes</td>
<td>0.93</td>
</tr>
<tr>
<td>sand</td>
<td>0.9</td>
</tr>
<tr>
<td>crushed galena</td>
<td>0.7</td>
</tr>
<tr>
<td>crushed dolomite/pyrite</td>
<td>0.67</td>
</tr>
<tr>
<td>crushed quartz</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Eq. (12.8) can be now be used to compute the pool area for a given volume of overflow, $Q_{VLO}$. Example 12.1 illustrates the method of sizing pool area of gravity settling classifiers.

---

**Example 12.1**

A slurry containing 50% solids (quartz) is to be classified at a rate of 100 tph at a separation size of 250 microns in a rake classifier. The density of the solids is 2650 kg/m$^3$ and the size analysis given in the table below. The water recovery to the overflow is 95% at an areal efficiency of 0.5. Estimate the pool area.

<table>
<thead>
<tr>
<th>Particle size, microns</th>
<th>Cum. Mass % retained</th>
</tr>
</thead>
<tbody>
<tr>
<td>710</td>
<td>10</td>
</tr>
<tr>
<td>355</td>
<td>25</td>
</tr>
<tr>
<td>180</td>
<td>45</td>
</tr>
<tr>
<td>90</td>
<td>60</td>
</tr>
<tr>
<td>45</td>
<td>75</td>
</tr>
<tr>
<td>75</td>
<td>100</td>
</tr>
</tbody>
</table>

Data: Viscosity of water equals 0.001 Pa.s, density of water, 1000 kg/m$^3$ and density of solid, 2650 kg/m$^3$.

**Solution**

Step 1
To determine the pool area, use Eq. (12.8) and determine each parameter.

The velocity parameter can be determined using Eq. (12.9). Substituting data we have: the velocity parameter $v$ as:
\[
\nu = \left[ 9.81 \left( \frac{2650 - 1000}{1000} \right) \left( \frac{0.001}{1000} \right) \right]^{1/3} = 0.0253 \text{ m/s}
\]

Step 2

The reduced Reynolds number, \( \text{Re}_R \), is obtained by using Eq. (12.10).

\[
\text{Re}_R = \left[ \frac{0.00025 \times 0.0253 \times 1000}{0.001} \right] = 6.325
\]

From Fig. 12.6 at a \( \text{Re}_R \) value of 6.325, the value of \( \frac{\nu_s}{\nu_T} = 1.4 \),
that is, \( \nu_s = 1.4 \nu_T = 1.4 \times 0.0253 = 0.0354 \text{ m/s} \)

Step 3

Feed volume dilution, \( \nu_f = \frac{(100 \text{ -- solid in feed}) \rho_s}{(\% \text{ solid in feed}) \rho_f} = \frac{(100-50) \times 2650}{50 \times 1000} = 2.65 \)

From the data, the feed solid is 65% minus 250 microns. That is, the mass fraction of solids less than the separation size is \( \alpha = 0.65 \) which is also the volume fraction assuming that all solids have the same density.

Step 4

Next the void fraction, \( \varepsilon \), is determined by using Eq. (12.11). Substituting values:

\[
\varepsilon = \frac{1}{\left(1 + \frac{0.65}{2.65}\right)} = 0.803
\]

Step 5

To determine the hindrance factor \( H \), the Reynolds number has to be estimated.

From data: Reynolds number = \( \text{Re}_R \) \( \times \) \( \frac{\nu_s}{\nu_T} \) = 6.325 \( \times \) 1.4 = 8.86, and from Fig. 12.7 the corresponding exponent \( f(\text{Re}_R, \rho_s) = 4.3 \).

Substituting these in equation in Eq. (12.11) we have:

\[
H = 0.803^{4.3} = 0.389
\]

Step 6

As the suspension is quartz, its shape factor can be taken as 0.5 (Table 12.4). We may now substitute the values of \( \nu_s, H, \rho_s \) and \( A_{\text{EFF}} \) in Eq. (12.8) to determine area \( A \) if \( Q_{\text{VL}0} \) is known. Otherwise:

\[
\frac{Q_{\text{VL}0}}{A} = \nu_s H \rho_s A_{\text{EFF}} = 0.0354 \times 0.389 \times 0.5 \times 0.5 = 0.0034 \text{ m}^3/\text{s/m}^2
\]
Step 7
From the available data,
Water in the feed = 100 x (100 - %solids)/% solids = 100 x (100 - 50)/50 = 100 t/h
Water in the overflow = 100 x 0.95 = 95 t/h = 95,000 kg/h
(Q_{VL(O)}) = 95000/100 = 95 m^3/h = 0.0264 m^3/s

Thus, the classifier area, A = 0.0264/0.0034 = 7.76 m^2/100 t/h feed
= 7.76/100 = 0.077 m^2/t/h

Also, the solids in the overflow = 100 x 0.65 = 65 t/h

And hence the % solids in the overflow = 65 x 100/(65 + 95) = 40.6%

12.3. Design Features of Centrifugal Classifiers

12.3.1. Hydrocyclone classifiers
Rapid settling and classification is achieved by increasing the force acting on the particles by replacing the gravitational force by centrifugal forces. Several types of equipment based on this principle are used for the purpose, like the hydrocyclone, Dyna Whirlpool and basket centrifuges. The hydrocyclone is the simplest and is the only one discussed here. The hydrocyclone has no moving parts and is the easiest to operate. Fig. 12.8 is a sketch of a typical hydrocyclone. The feed entry is either tangential to the centre line of entry or forms an involuted entry. The cross-section of the entry pipe is usually circular, oval or rectangular; each of which provide a different velocity profile inside the feed chamber and the cyclone cone. The top of the feed chamber is closed with a plate through which a pipe known as a vortex finder passes. The bottom of the vortex finder protrudes below the feed chamber. Below the feed chamber the body of a cyclone is shaped like an inverted cone, which converges to a smaller cone, which serves as the outlet of the coarser size fractions in the feed. The feed chamber and the cones are lined inside with rubber or synthetic linings due to abrasive nature of most metallurgical slurries. The lining material is hard rubber, neoprene or urethane. In some cases, the protective lining is sprayed inside forming a hard monolithic bond with the base metal. The apex is sometimes fitted with a concentric, hardwearing synthetic rubber inner sleeve, which can be squeezed hydraulically or pneumatically to alter the diameter of the opening.

Hydrocyclones are occasionally provided with nozzles just above the apex for injecting water to compensate for water loss and loss of fines [5]. However constant effort is made to improve on the design, aimed at improvement of the flow dynamics of the slurry inside the cyclone.

Krebs [6] has introduced the SpinTop hydrocyclones with circular inlet forming a well defined involute feed entrance, parabolic body to provide a smooth transition between the cylindrical and conical sections, bell shaped vortex finder increases rotational acceleration to give a sharper separation and solid centre core in the vortex finder to replace the air core, stabilising the rotational flow [7]. For coarse size separation the “Flat bottom cyclones” has been introduced [8]. The fully flat bottom instead of a conical section increases the separation size by a factor of 2. The flat bottom hydrocyclone produces a very clean underflow by
forcing a large amount of coarse and fine solids to the overflow. Cyclones with 90-degree cone angles are also available.

The actual dimensions of most models for metallurgical operations have been derived from experimental results. Suggested relations between design variables are given in Tables 12.5 and 12.6. Experience has shown that the dimensions of an hydrocyclone acting as a classifier and a dewatering tool are slightly different. These differences are also indicated in Table 12.5.
Popularly used symbols for describing different parts of an hydrocyclone are shown in Fig. 12.9 and used in the tables.

Table 12.5.
Dimensions of Hydrocyclones [9,10].

<table>
<thead>
<tr>
<th>Hydrocyclone (Dewatering)</th>
<th>Hydrocyclone (Classifier)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet diameter (D_1 = \frac{D_c}{4})</td>
<td>Inlet diameter (D_1 = \frac{D_c}{7})</td>
</tr>
<tr>
<td>Vortex finder diameter, (D_0 = \frac{D_c}{3})</td>
<td>Vortex finder diameter, (D_0 = \frac{D_c}{5})</td>
</tr>
<tr>
<td>Length or height, (L = 5D_c)</td>
<td>Diameter of underflow = (D_c/15)</td>
</tr>
<tr>
<td>Length of vortex finder, (L_v = 0.4D_c)</td>
<td>Length of vortex finder, (L_v = 0.4D_c)</td>
</tr>
</tbody>
</table>

Table 12.6
Standard cyclone as defined by different authors.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross-sectional area of feed pipe at point of entry</td>
<td>6-8% of the cross-sectional area of the feed chamber</td>
<td>((0.015-0.02)\pi D_c^2)</td>
</tr>
<tr>
<td>Vortex finder diameter, (D_0)</td>
<td>35-40% of (D_c)</td>
<td>0.35 (D_c)</td>
</tr>
<tr>
<td>Cone Angle</td>
<td>(12^\circ) for (D_c &lt; 250) mm</td>
<td>(12^\circ) for (D_c &lt; 250) mm</td>
</tr>
<tr>
<td></td>
<td>(20^\circ) for (D_c &gt; 250) mm</td>
<td>(20^\circ) for (D_c &gt; 250) mm</td>
</tr>
<tr>
<td>Apex diameter</td>
<td>&gt; 0.25 (D_0)</td>
<td>&gt; 0.10 (D_0)</td>
</tr>
</tbody>
</table>

However, as a general rule:

1. the inlet cross sectional area is roughly 70% of the cross sectional area of the feed chamber,
2. the diameter of the vortex finder is about 25-40% of the cyclone diameter, and
3. the diameter of the apex is 25% of the vortex finder.

The apex diameter is selected to discharge the maximum possible density of slurry, avoiding the roping condition of the discharge stream.

Tarr [13] presented graphical relationships between the cyclone dimensions for optimum operating conditions. These relationships are shown in Figs. 12.10 and 12.11.

Presently the largest hydrocyclone in use has a diameter of 2.3 m (90 inch) and the least cone angle about 10.5° in contrast to the usual cone angles of 20° [14]. The lower cone angle produces a finer separation.

The following general observations can be made for designing:

1. Rectangular sections of the inlet is probably better than other sections.
2. Increased inlet area permits increased input and therefore imparts increased tangential velocity to the slurry inside the cyclone.
3. Larger diameter cyclones are more suitable for coarse size separations as acceleration in the feed chamber is less. (Mular and Jull, [11] suggest that the acceleration of slurries in similar but small diameter cyclone could be 40 times less).
Fig. 12.10. Approximate relationship between cyclone diameter and feed inlet [13].

Fig. 12.11. Approximate relationship between cyclone diameter and vortex finder diameter [13].
4. Longer cylindrical sections tend to yield high underflow recoveries.
5. Shorter cylindrical sections yield coarse separations [15].
6. Smaller cone angles are suitable for finer separations.
7. Larger cone angles are suitable for producing sharper and coarser separations.
8. Apex diameter should have the flexibility so that it may be adjusted and be just larger
   than that at which roping occurs. [Roping is a condition of discharge through the apex
   when the discharge slurry appears like rope and is not flared or spread out].
9. If the pressure drop is greater than 70 kPa the ratio \((D_o/D_y)\) should be less than 3.5-4.0.
   If greater than this then the air core diameter will be greater than the apex leading to
   unstable and inefficient operation [13,15].

12.4. Operation of Mechanical Classifiers

The feed to the mechanical classifier with a rectangular cross-section is spread along the
width and is usually directed towards the top end. On entry, the solids in the slurry
commence to settle, the coarser and denser particles settling at a faster rate than the others.
Particles settling to the bottom form a layer (region 5 in Fig. 12.12), which is least disturbed
by the blades of the rakes or spirals and possibly serves to protect the base of the tank. Region
4 is the zone of moving sands dragged into the underflow by the raking mechanism. Above
the bottom layers is the zone marked 3 in Fig. 12.12 where hindered settling occurs. A
continuously changing concentration gradient is set up in this layer, the upper portion being
least concentrated and the lower end having the maximum concentration of particles. The
mechanical rakes or spirals continuously stir this zone, breaking up agglomerated particles
generally accelerating the separation process. The layer marked zone 2 is where
maximum agitation takes place, the lighter and smaller particles are separated here where they
join with the overflow stream and are carried over to the overflow launder. The heavier
particles settle by gravity to zone 3 forming the thick bottom layer. The surface of the top
layer 1 is at the same level as the weir allowing the light particles to flow over to the overflow
launder.

Separation of solids in classifiers has been the study of workers like Fitch [16], Stewart
and Restarick [17], Reid [18], Schubert and Neesse [19] and Fitch and Roberts [3]. While
Stewart and Restarick recognised four zones, others like Schubert and Neesse considered that
the slurry was divided into two layers at a particular height and diffusion and sedimentation

---

**Fig. 12.12.** Slurry movement and zones of particle separations in an operating classifier.
velocities were significant at this level. Reid [18] also considered particle movement as two streams that travelled as plug flow with *intense radial mixing*. Reid proposed that the recovery $R$, of a size $d_i$, was given by the expression:

$$R = 1 - e^{-0.693 \left( \frac{d_i}{\sigma} \right)^{12}} \quad (12.14)$$

where $d_i$ = the mean of the size interval $i$ in the sieve analysis of the feed.

Plitt [20] examined the equation and stated that the value of $s$ varied from 1 to 3.8. Fitch and Roberts made a much simpler approach. They considered the mass balance of water in a classifier and expressed it as:

volume rate of water, $Q_{VL(F)}$ = volume rate of water, $Q_{VL(U)}$ + volume rate of water, $Q_{VL(O)}$

in the feed in the underflow in the overflow

They also considered that:

1. the fraction of size of particles that travelled to the overflow depended on the settling velocity,
2. the settling rate was affected by turbulence,
3. the ratio of any size, $d_i$, to the settling rate of the separating particle will remain effectively constant.

Taking $Q_{VL(F)}$ and $Q_{VL(O)}$ as the volume rates of flow of feed and overflow water they derived an expression for the fraction of particles of size, $d_i$, that was removed and separated into the overflow, $E_i$, as:

$$E_i = \frac{Q_{VL(O)}}{Q_{VL(F)}} K [1 - F_i] \quad (12.15)$$

where $F_i$ = the settling factor described as the ratio of the settling rates of particles of size $d_i$ and $d_{50}$ (the separation size)

$K$ = a factor taking into account the change in concentration of particles of size, $d_i$, and is represented by the ratio of the volume fraction of size $i$ in the overflow to size $i$ in the feed. $K$ is always greater than unity.

For gravity pool classifiers, Eq. (12.15) can be simplified to:

$$E = \frac{Q_{VL(O)}}{Q_{VL(F)}} [1 - F] \quad (12.16)$$

Fitch and Roberts determined the settling factors for different ratios of particle size $d_i$ and $d_{50}$ where $d_i$ was the lower size of size interval $i$. The results obtained are indicated for four selected sizes in Fig. 12.13 where the $x$-axis is a root 2 series of numbers.
To determine $E_i$, it is necessary to know $Q_{VL(O)}$. It was suggested by Fitch and Roberts that $Q_{VL(O)}$ may be eliminated from the equation indirectly by using the water balance in the following manner.

The method is summarised below:

Let $Q_{MS(F)} = \text{the mass flowrate of solids in the feed stream of any size interval i}$, $Q_{MS(U)} = \text{the mass flowrate of solids in the underflow stream}$.

The mass balance of water may be written as:

$$Q_{VL(O)} = Q_{VL(F)} - Q_{VL(U)}$$  \hfill (12.17)

Substituting the value of $Q_{VL(O)}$ in Eq. (12.16):

$$E = \left[ \frac{Q_{VL(F)} - Q_{VL(U)}}{Q_{VL(F)}} \right] (1 - F)$$ \hfill (12.18)

But, $E = \frac{Q_{MS(F)} - Q_{MS(U)}}{Q_{MS(F)}} = \left[ \frac{Q_{VL(F)} - Q_{VL(U)}}{Q_{VL(F)}} \right] (1 - F)$ \hfill (12.19)

By substituting and simplifying, the mass of solids in the size interval in the underflow would be:
Experience has shown that about 51% of the void space in the underflow is occupied by slurry of the overflow stream consistency. Hence while estimating the characteristics of the underflow stream this factor has to be taken into account.

Examples 12.2 illustrates the method advocated by Fitch and Roberts [3] for computing the performance of a gravity classifier, like a Rake Classifier.

Example 12.2

A quartz slurry, made of 45% solids in water is fed to a rake classifier. The size distribution of the dry quartz is given in the table below. The classifier was commissioned to classify and cut at 500 microns. Estimate the underflow and overflow particle size distribution. The S.G. of quartz is 2.54 and the density of water is 1.0.

Feed size distribution of quartz

<table>
<thead>
<tr>
<th>Particle Size, microns</th>
<th>Cum. mass % passing</th>
</tr>
</thead>
<tbody>
<tr>
<td>4000</td>
<td>85</td>
</tr>
<tr>
<td>2000</td>
<td>80</td>
</tr>
<tr>
<td>1000</td>
<td>62</td>
</tr>
<tr>
<td>600</td>
<td>52</td>
</tr>
<tr>
<td>300</td>
<td>45</td>
</tr>
<tr>
<td>150</td>
<td>25</td>
</tr>
<tr>
<td>75</td>
<td>10</td>
</tr>
</tbody>
</table>

Solution

A log-log plot of the size distribution is seen in the figure below.

To calculate the stream characteristics, consider each stream separately. For convenience, assume a feed rate of 100 mass units (g, kg or t). The procedure to follow is to determine the lower sieve size fraction of each size interval and then to determine the mass of solid in each size fraction of the feed. The calculations are illustrated in tabulated form for ease of understanding.

Step 1
In the table below:

| Column 1 | A root 2 series of numbers represented by the x-axis, d/d_{50}, in Fig. 12.13 |
| Column 2 | A root 2 series of screens based on the separation size as the top size (500 μm) obtained by multiplying column (1) by the separation size. |
| Column 3 | Feed size distribution (cumulative % passing), obtained from the table or figure above. |
Size distribution of the feed quartz

<table>
<thead>
<tr>
<th>Size i</th>
<th>√2 series</th>
<th>Sieve sizes</th>
<th>ΣP_i</th>
<th>R_i</th>
<th>F**</th>
<th>M_{S(i)}</th>
<th>M_{S(0)}</th>
<th>M(t) %</th>
<th>Cum % O/F</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.707</td>
<td>354</td>
<td>48</td>
<td>2.0</td>
<td>0.78</td>
<td>1.56</td>
<td>0.44</td>
<td>1.10</td>
<td>97.40</td>
</tr>
<tr>
<td>2</td>
<td>0.50</td>
<td>250</td>
<td>40</td>
<td>8.0</td>
<td>0.465</td>
<td>3.72</td>
<td>4.28</td>
<td>10.71</td>
<td>86.69</td>
</tr>
<tr>
<td>3</td>
<td>0.35</td>
<td>175</td>
<td>30</td>
<td>10.0</td>
<td>0.275</td>
<td>2.75</td>
<td>7.25</td>
<td>18.14</td>
<td>68.56</td>
</tr>
<tr>
<td>4</td>
<td>0.25</td>
<td>125</td>
<td>20</td>
<td>10.0</td>
<td>0.165</td>
<td>1.65</td>
<td>8.35</td>
<td>20.89</td>
<td>47.67</td>
</tr>
<tr>
<td>5</td>
<td>0.18</td>
<td>90</td>
<td>13</td>
<td>7.0</td>
<td>0.098</td>
<td>0.69</td>
<td>6.31</td>
<td>15.79</td>
<td>31.87</td>
</tr>
<tr>
<td>6</td>
<td>0.13</td>
<td>65</td>
<td>8.4</td>
<td>4.6</td>
<td>0.056</td>
<td>0.26</td>
<td>4.34</td>
<td>10.86</td>
<td>21.01</td>
</tr>
<tr>
<td>0.00</td>
<td>0</td>
<td>0</td>
<td>8.4</td>
<td>0</td>
<td>0</td>
<td>8.40</td>
<td>21.01</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* lower end of the particle size in a size interval. ** take nearest sizes from Fig. 12.13.

ΣP_i = cumulative % passing; R_i = mass % retained; F = settling factor; M_{S(i)} = mass settled; M_{S(0)} = mass not settled; M_{S(t)} = mass % overflow.

Column 4 The mass % retained (or actual mass, based on the 100 unit feed mass assumed) obtained by subtract the cumulative mass% in size fraction i from size fraction i+1, column (3).

Column 5 Take the settling factor, F, from the nearest size in Fig. 12.13.

Column 6 Multiply column (4) by column (5). This gives the mass distribution of particles that settles. For the separation size interval (top size), an empirical figure of 0.6 is subtracted for rake, spiral classifiers to account for misplaced material.
Step 2
To determine the size distribution in the overflow stream.

Column 7 Subtract the mass of solids that settle from the feed mass in each size fraction to give the mass of particles that doesn’t settle, column (4) – column (6).

Summing column (7) will give the total mass of solids in the overflow.

Column 8 size distribution of the overflow, column (7) x 100/\Sigma(7)
Column 9 Cumulative percent in overflow, \Sigma(8)

Step 3
To determine the cumulative percent underflow product

According to Fitch and Roberts [6], the mass of solids reporting to the overflow, column (7) in the table above, is accompanied with all of the feed water. A portion of the overflow pulp is entrained with the underflow solids and this adds fines to the underflow, hence affecting the underflow size distribution as well as providing the underflow water.

The proportion of overflow solids entrained in the underflow is taken as equal to the water split to the underflow, Q_{VL(U)}/Q_{VL(F)}.

1. determine the ratio of Q_{VL(U)}/Q_{VL(F)}

Q_{VL(U)} can be estimated from the volume of solids in the feed (or the percent solids in the slurry).

\[
\frac{Q_{VL(F)}}{Q_{VS(F)}} = V_F = \frac{(100-\% \text{ Solid in feed}) \cdot \rho_S}{\% \text{ Solid in feed} \cdot \rho_W}
\]

From the given data:

\[
V_F = \frac{(100 - 45) \cdot 2.54}{45 \times 1.0} = 3.104
\]

2. the volume dilution in the overflow, \( V_O = Q_{VL(O)}/Q_{VS(O)} = Q_{VL(F)}/Q_{VS(O)} \) assuming that all the feed water initially goes to the overflow.

Thus, \( V_O = \frac{Q_{VL(F)}}{Q_{VS(O)}} = \frac{Q_{VL(F)}}{Q_{VS(O)}} \cdot \frac{Q_{VS(O)}}{Q_{MS(O)}} = V_F \cdot \frac{Q_{MS(O)}}{Q_{MS(O)}} = V_F \cdot \frac{100}{Q_{MS(O)}} \) for a 100 mass feed

\[
V_O = 3.104 \times 100/39.98 = 7.76
\]

3. Fitch and Roberts [3] estimate that on average the underflow solids entrain approximately 51% of overflow pulp by volume.
That is, \( \frac{Q_{\text{VOP(U)}}}{Q_{\text{VOP(U)}} + Q_{\text{VS(U)}}} = 0.51 \) and \( \frac{Q_{\text{VS(U)}}}{Q_{\text{VOP(U)}} + Q_{\text{VS(U)}}} = 1-0.51 = 0.49 \)

where \( Q_{\text{VOP(U)}} \) = volume of entrained O/F pulp in the U/F, 
\( Q_{\text{VS(U)}} \) = volume of settled solids in the U/F.

Therefore \( \frac{Q_{\text{VOP(U)}}}{Q_{\text{VS(U)}}} = \frac{0.51}{0.49} = 1.04 \)

and \( Q_{\text{VOP(U)}} = 1.04Q_{\text{VS(U)}} = 1.04 \frac{Q_{\text{MS(U)}}}{\rho_s} = 1.04 \left( \frac{Q_{\text{MS(F)}} - Q_{\text{MS(O)}}}{\rho_s} \right) = 1.04 \left( \frac{100 - Q_{\text{MS(O)}}}{\rho_s} \right) \)

4. Since the volume ratio of water to solid in the entrained pulp in the U/F is the same as in the overflow:

\[
V_0 = \frac{Q_{\text{VLO(O)}}}{Q_{\text{VLO(U)}}} = \frac{Q_{\text{VOL(U)}}}{Q_{\text{VOS(U)}}}
\]

where \( Q_{\text{VOL(U)}}, Q_{\text{VOS(U)}} = \) volume of entrained O/F water and solids in the U/F respectively.

Then \( (1+V_0) = \frac{Q_{\text{VOL(U)}} + Q_{\text{VOS(U)}}}{Q_{\text{VOS(U)}}} = \frac{Q_{\text{VOP(U)}}}{Q_{\text{VOS(U)}}} \)

and \( \frac{V_0}{(1+V_0)} = \frac{Q_{\text{VOL(U)}}}{Q_{\text{VOP(U)}}} \)

Since the water in the U/F is assumed to be made up entirely of the water entrained from the O/F, \( Q_{\text{VOL(U)}} = Q_{\text{VLO(U)}} \) hence:

\[
Q_{\text{VLO(U)}} = \left[ \frac{V_0}{(1+V_0)} \right] Q_{\text{VOP(U)}} = \left[ \frac{V_0}{(1+V_0)} \right] 1.04 \left( \frac{100 - Q_{\text{MS(O)}}}{\rho_s} \right) \text{ per 100 units of solid.}
\]

Since the volume of water in the feed, \( Q_{\text{VLO(F)}} = \frac{100(100-\%\text{solid in feed})}{\%\text{solids in feed} \times \rho_w} \)

then \( \frac{Q_{\text{VLO(U)}}}{Q_{\text{VLO(F)}}} = 1.04 \left( \frac{100 - Q_{\text{MS(O)}}}{\rho_s} \frac{V_0}{1+V_0} \right) \frac{\%S_F \cdot \rho_w}{100(100-\%S_F)} \)

where \( \%S_F = \% \text{ solids in the feed.} \)

Substituting values from the given data:
\[
\frac{Q_{VL(\text{U})}}{Q_{VL(\text{F})}} = \frac{1.04(100 - 39.98)}{2.54} \left[ \frac{7.75}{1 + 7.75} \right] \frac{45 \times 1.0}{100(100 - 45)} = 0.18
\]

That is, 18% of each size fraction in the overflow is entrained in the underflow.

**Step 4**

We can now construct the following table to obtain the underflow particle size distribution. The column numbers follow in sequence from the previous table.

<table>
<thead>
<tr>
<th>Size interval</th>
<th>Mass of entrained O/F (7) x 0.18</th>
<th>Mass of underflow (6) + (10)</th>
<th>Mass % underflow (11)/Σ(11)</th>
<th>Cum. % underflow Σ(12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10)</td>
<td>(11)</td>
<td>(12)</td>
<td>(13)</td>
<td></td>
</tr>
<tr>
<td>0.11</td>
<td>49.51</td>
<td>73.73</td>
<td>26.27</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.08</td>
<td>1.64</td>
<td>2.44</td>
<td>23.83</td>
</tr>
<tr>
<td>2</td>
<td>0.76</td>
<td>4.48</td>
<td>6.68</td>
<td>17.16</td>
</tr>
<tr>
<td>3</td>
<td>1.29</td>
<td>4.04</td>
<td>6.02</td>
<td>11.14</td>
</tr>
<tr>
<td>4</td>
<td>1.49</td>
<td>3.14</td>
<td>4.67</td>
<td>6.46</td>
</tr>
<tr>
<td>5</td>
<td>1.13</td>
<td>1.81</td>
<td>2.70</td>
<td>3.77</td>
</tr>
<tr>
<td>6</td>
<td>0.77</td>
<td>1.03</td>
<td>1.54</td>
<td>2.23</td>
</tr>
<tr>
<td>0.15</td>
<td>1.50</td>
<td>2.23</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Σ67.15</td>
<td>Σ100</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Column 10 = Column (7) x (Q_{VL(U)}/Q_{VL(F)})

Column 11 = Mass settled (6) + entrained fines (10)

Thus column (9) and column (13) provide the required particle size distribution in the two streams.

During the operation of the mechanical classifiers, slurry is fed evenly along the width of the classifier and at a distance of about two-thirds of the length of the tank measured from the bottom weir. The feed slurry normally carries 70% – 80% solids. Water is added so that the solids in the slurry can easily settle. Too much dilution or too little water addition, affects the particle size distribution of the overflow stream. Hence an optimum amount of water has to be determined and maintained.

Separations in such mechanical classifiers are achieved for particles of 600 microns down to about 75 microns. The baffle positions and its depth below the surface controls the velocities and particle size of the overflow stream. Lowering the baffle level obviously promotes coarser particles in the overflow.

One of the greatest problems in the operation of mechanical classifiers is the surging of the slurry. To counter this, water additions and maintaining relatively constant slurry characteristics help. Clayey matter promoting slimes and thixotropic slurries could be an added source of trouble in operation.
12.5. Capacity of Mechanical Classifiers

Usually the capacities are recommended in manufacturer’s literature. Overflow capacity is normally the limiting design capacity of mechanical classifiers. The overflow volume can be expressed as [21]:

\[ Q_{V(O)} = W H v \]  

(12.21)

where

- \( Q_{V(O)} \) = overflow volume, m\(^3\)/s
- \( W \) = weir width, m
- \( H \) = weir height, m
- \( v \) = flow velocity from the feed to the overflow, m/s

or \( Q_{V(O)} = \frac{1}{2} A v_T \)

where

- \( A \) = pool area
- \( v_T \) = terminal velocity, m/s

For spiral classifiers, the overflow solids capacity may be given by [21]:

\[ Q_{MS(O)} = n k_2 k_3 (3.92 D^2 + 0.67 D) \] for low weir pools  

(12.22)

and \( Q_{MS(O)} = n k_2 k_3 (3.12 D^2 + 0.42 D) \) for high weir pools  

(12.23)

where

- \( Q_{MS(O)} \) = overflow solids capacity, t/h
- \( n \) = the number of spirals
- \( k_1 \) = factor from Table 12.7
- \( k_2 \) = solids density correction factor, Table 12.7

In some cases the sand raking capacity is the determining factor in sizing mechanical classifiers. Hill [22] describes an empirical equation for the raking capacity:

\[ Q_{MS(U)} = 0.035 WP \rho_B (D - 0.75 W) \]  

(12.24)

where

- \( Q_{MS(U)} \) = raking capacity, t/h/spiral revolutions/min
- \( W \) = flight width, m
- \( P \) = flight pitch, m
- \( \rho_B \) = bulk density of the solids in the underflow, kg/m\(^3\)
- \( D \) = diameter of the spiral, m

The effect of spiral diameter on the raking capacity is given by (Hill, 1982):

\[ Q_{U2} = Q_{UI} \left( \frac{D_2}{D_1} \right)^3 \]  

(12.25)
Table 12.7
Factors $k_1$ and $k_2$ for spiral classifiers capacity, [21].

<table>
<thead>
<tr>
<th>Cut size, $\mu$m</th>
<th>400</th>
<th>300</th>
<th>200</th>
<th>150</th>
<th>100</th>
<th>74</th>
<th>53</th>
<th>44</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pulp/tof O/F</td>
<td>1.8</td>
<td>2.0</td>
<td>2.33</td>
<td>4.0</td>
<td>4.5</td>
<td>5.7</td>
<td>6.0</td>
<td>7.5</td>
</tr>
<tr>
<td>$k_1$ (low weir)</td>
<td>1.95</td>
<td>1.7</td>
<td>1.46</td>
<td>1.00</td>
<td>0.66</td>
<td>0.46</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$k_1$ (high weir)</td>
<td>-</td>
<td>-</td>
<td>2.9</td>
<td>2.2</td>
<td>1.60</td>
<td>1.00</td>
<td>0.57</td>
<td>0.36</td>
</tr>
<tr>
<td>SG of solids</td>
<td>2.0</td>
<td>2.5</td>
<td>3.0</td>
<td>3.5</td>
<td>4.0</td>
<td>4.5</td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td>$k_2$</td>
<td>0.75</td>
<td>0.92</td>
<td>1.08</td>
<td>1.25</td>
<td>1.42</td>
<td>1.58</td>
<td>1.75</td>
<td></td>
</tr>
</tbody>
</table>

12.6. Operation of Centrifugal Classifiers

In the minerals industry cyclones are normally operated under wet conditions and seldom as dry classifier. The feed, in the form of a slurry, on entering the feed chamber is divided into two streams as a result of the inlet pressure of the slurry and the swirling action inside the feed chamber and the conical section of the hydrocyclone. The denser particles which settle faster are forced down by the combined gravity and centrifugal forces while the less dense and lighter particles remain near the central axis of the cyclone and exit through the vortex finder. Some lighter particles however are entrapped in the heavier particle stream and are lost through the apex while some heavier particles are similarly lost to the overflow stream.

The hydrocyclone is a classifier with no moving parts and its operation depends on:

1. the characteristics of the feed stream and
2. the geometry of the cyclone.

The characteristics of the feed stream includes:

1. size and size distribution of solids in the feed stream,
2. pulp density (percent solids in the slurry) and pulp viscosity, and
3. inlet pressure (pressure differential between inlet and vortex finder outlet).

The geometry of the cyclone involves:

1. inlet shape and inlet area,
2. cyclone dimensions (length of cylindrical section, total overall length and cone angle),
3. inlet, vortex finder and apex diameters.

The feed size varies from coarse (150 microns and more) down to fines. In open circuit operation the solid content of the slurry is about 30% and in closed circuits, it could be as high as 60% [23]. For most operations the feed pressure ranges between 345 kPa to 700 kPa and in actual practice depends on cyclone diameter. The minimum pressure for a stable air core is around 30–35 kPa [21]. The feed velocity is about 3.7–6.1 m/s [15] and its acceleration in the feed chamber is inversely proportional to the hydrocyclone diameter, [11].

12.6.1. Efficiency of separation in hydrocyclones

By convention the efficiency of operation and separation of hydrocyclones are determined by the sharpness of separation and the $d_{50}$ value. Less conventional but also widely used is the
the size at which 95% of the particles have the probability of reporting to the underflow. To determine the efficiency of separation of a sample of known size distribution, pulp density and flow rate, a hydrocyclone of known geometry, including the inlet, overflow and underflow diameters, is operated in closed circuit until a steady state is reached. Simultaneous samples of the feed, overflow and underflow streams are collected dried and analysed for size distribution. The calculations involved to determine the efficiency are best understood by the following example.

Let us assume that a hydrocyclone is fed with slurry and at steady state the operating conditions are:

1. Feed rate = 55.0% solids at 206.5 t/h
2. Overflow rate = 19.6% solid at 29.4 t/h
3. Underflow rate = 78.2% solids at 177.1 t/h

and the size analysis of samples from each stream are given in Table 12.8.

Table 12.8

<table>
<thead>
<tr>
<th>Size (\mu m)</th>
<th>Geom. mean size (\sqrt{d_d_{d_{i+1}}})</th>
<th>Feed mass, t</th>
<th>Overflow mass, t</th>
<th>Underflow mass, t</th>
<th>Calculated feed, t ((4)+(5))</th>
<th>Partition coefficient ((5)\times 100/(6))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-600+425</td>
<td>505.0</td>
<td>120.0</td>
<td>0</td>
<td>121.0</td>
<td>121.0</td>
<td>100.0</td>
</tr>
<tr>
<td>-425+300</td>
<td>357.1</td>
<td>26.0</td>
<td>0.6</td>
<td>24.0</td>
<td>24.6</td>
<td>97.6</td>
</tr>
<tr>
<td>-300+250</td>
<td>273.9</td>
<td>13.0</td>
<td>2.0</td>
<td>11.0</td>
<td>13.0</td>
<td>84.6</td>
</tr>
<tr>
<td>-250+150</td>
<td>193.6</td>
<td>12.0</td>
<td>4.7</td>
<td>8.2</td>
<td>12.9</td>
<td>63.6</td>
</tr>
<tr>
<td>-150+106</td>
<td>126.1</td>
<td>9.0</td>
<td>4.6</td>
<td>4.2</td>
<td>8.8</td>
<td>47.7</td>
</tr>
<tr>
<td>-106+75</td>
<td>89.2</td>
<td>5.0</td>
<td>3.2</td>
<td>2.2</td>
<td>5.4</td>
<td>40.7</td>
</tr>
<tr>
<td>-75</td>
<td>-</td>
<td>21.5</td>
<td>14.3</td>
<td>6.5</td>
<td>20.8</td>
<td>31.3</td>
</tr>
</tbody>
</table>

The partition coefficient is the recovery of particles in each size fraction to either the underflow or the overflow (see Tromp curve, Chapter 11).

The distribution of water in the different streams may be determined as:

- Water in feed = 100–55.0 = 45.0%
- Water in overflow = 100–19.6 = 80.4%
- Water in underflow = 100–78.2 = 21.8%

Hence:

- Mass of water in feed = \(206.5 \times \frac{45}{55} = 169.0\) t/h
- Mass water in overflow = \(29.4 \times \frac{80.4}{19.6} = 120.6\) t/h
Fig. 12.14. Typical performance curves of a hydrocyclone (1-actual, 2-corrected).

mass water in underflow = $177.1 \times \frac{21.8}{78.2} = 49.4$ t/h

A plot of mean particle size against the partition coefficient (column (2) vs column (7), Table 12.8) yields the partition curve 1 shown in Fig. 12.14.

Fig. 12.14 is a typical distribution curve for a hydrocyclone underflow stream. The curve shows that the cyclone cut size, separation size or $d_{50}$ is 135 microns. A similar curve can be drawn for the overflow stream which in effect will be a mirror image of the underflow curve.

Note that the curve does not pass through the origin. It has been suggested [9,24,25] that this is due to a fraction of the slurry bypassing the cyclone and not being classified. Thus if 5% of the feed slurry bypassed the unit then only 95% of the slurry would be subjected to the classification process. Thus the $d_{50}$ calculated by the above method has to be corrected. Kelsall [5] suggested that the fraction of solids in each size fraction that is bypassed from the feed to the underflow is in the same ratio as the fraction of feed water that reported to the underflow. This is not necessarily true, according to Austin et al [26], however, Kelsall’s assumption is simple and widely accepted as it yields a reasonably accurate correction for the true $d_{50}$ value. The usual symbol for the corrected cut size is $d_{50C}$. Using Kelsall’s concept, the manner of evaluating the $d_{50C}$ value is illustrated in Table 12.9 and details of the calculation are shown below:

<table>
<thead>
<tr>
<th>Mean Size</th>
<th>Corrected partition coefficient, $E_C$ (% recovery to U/F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>505</td>
<td>$\frac{121-(0.292 \times 121)}{121-(0.292 \times 121)} \times 100 = 100$</td>
</tr>
<tr>
<td>357</td>
<td>$\frac{24-(0.292 \times 24.6)}{24.6-(0.292 \times 24.6)} \times 100 = 96.6$</td>
</tr>
</tbody>
</table>
381

\[
\frac{11 - (0.292 \times 13)}{13 - (0.292 \times 13)} \times 100 = 78.3
\]

\[
: \quad \frac{m_{ui} - (w \ m_{s})}{m_{uf} - (w \ m_{f})} \times 100 \quad \text{or} \quad \frac{E - w}{1 - w} \times 100
\]

where \( d_i \) = mean size of screen interval \( i \)
\( m_{ui}, m_{uf} \) = mass in size interval \( i \) in the underflow and feed respectively
\( w \) = fraction of feed water in the underflow

A plot of the corrected percent recoveries (column (3), Table 12.9) against the mean particle size (column (1), Table 12.9) gives the corrected partition curve (curve 2 in Fig. 12.14). From the curves it can be seen that in this specific case, the \( d_{50} \) value is 135 \( \mu m \) and the corrected \( d_{50c} \) value is 198 \( \mu m \).

Table 12.9
Correction of partition coefficient

<table>
<thead>
<tr>
<th>Geom. Mean size, ( \mu m )</th>
<th>Partition coefficient</th>
<th>Corrected partition coeff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>505.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>357.1</td>
<td>97.6</td>
<td>96.6</td>
</tr>
<tr>
<td>273.9</td>
<td>84.6</td>
<td>78.3</td>
</tr>
<tr>
<td>193.6</td>
<td>63.6</td>
<td>48.5</td>
</tr>
<tr>
<td>126.1</td>
<td>48.9</td>
<td>27.8</td>
</tr>
<tr>
<td>89.2</td>
<td>38.5</td>
<td>13.1</td>
</tr>
<tr>
<td>8.7</td>
<td>31.3</td>
<td>2.9</td>
</tr>
</tbody>
</table>

It can be easily seen that the corrected curve represents the efficiency of separation of that portion of the slurry that is subjected to classification. The sharpness and separation efficiency values can be quantified by reading the values of \( d_{25}, d_{75} \) and \( d_{50} \) from the graph and are calculated in the same manner as described for screen classifiers.

<table>
<thead>
<tr>
<th>Uncorrected</th>
<th>Corrected</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_{25} )</td>
<td>235</td>
</tr>
<tr>
<td>( d_{25} )</td>
<td>-</td>
</tr>
<tr>
<td>( d_{50} )</td>
<td>135</td>
</tr>
<tr>
<td>( d_{65} )</td>
<td>200</td>
</tr>
<tr>
<td>( d_{35} )</td>
<td>45</td>
</tr>
</tbody>
</table>

Imperfection = \( (d_{75} - d_{25})/2d_{50} \)

75% partition error = \( (d_{75}/d_{50}) \)

Sharpness Index = \( d_{35}/d_{75} [21] \)

= \( d_{35}/d_{65} \) (high bypass)
The cyclone Imperfection ranges from 0.2 – 0.6 with an average of around 0.3 [21].

The water split between the feed and the underflow will depend on the diameter of the apex (Du) and the vortex finder (D0). From limited experimental data, Lynch [21] observed that the water split bears a linear relationship with the apex diameter. For all particle sizes data, Lynch derived the equation:

\[ W_s = -1.61 + \frac{193(D_u - 1.41)}{Q_{ML(F)}} \]  \hspace{1cm} (12.26)

where

- \( W_s \) = Water split, \( Q_{ML(O)}/Q_{ML(F)} \),
- \( D_u \) = Apex diameter, m,
- \( Q_{ML(O)} \) = Mass flow rate of water in the overflow, t/h and,
- \( Q_{ML(F)} \) = Mass flow rate of water in the feed, t/h.

The corrected efficiency curve derived after correcting for the water split is specific for the specific slurry and cyclone geometry. To apply the method in a wider context, such as different flow rates, slurry percent solids, diameters of vortex finder and apex, Lynch and Rao [24] normalised the curve by dividing each particle size, \( d \), by \( d_{50C} \). Plotting \( d/d_{50C} \) against the fraction to underflow they obtained a series of curves which described the performance of a hydrocyclone independent of operating conditions and hydrocyclone size. Lynch and Rao tested the curves for four cyclone diameters (10.2, 15.2, 24.5 and 28.1 cm) and obtained similar curves. Such plots are illustrated as reduced efficiency curves. Using the above data a typical curve is plotted in Fig. 12.15.

The advantage of plotting in this manner is that the results can be translated to any larger size cyclone.

![Fig. 12.15. Reduced efficiency curve.](image-url)
It must be emphasised that the reduced efficiency curves for different minerals of different density and shape are different but as the size $d$ is simply divided by a constant, the nature of the curve remains unaltered.

Attempts have been made to derive the equation of the reduced efficiency curve [18,24,27-29]. The derivation by Lynch [24] is now widely used and is represented by the equation:

$$E_C = \frac{e^{\alpha\left(\frac{d}{d_{50}}\right)} - 1}{e^{\alpha\left(\frac{d}{d_{50}}\right)} + e^{\alpha} - 2}$$

(12.27)

where $E_C = \text{the corrected partition coefficient and}$

$\alpha = \text{the efficiency parameter}$

The value of $\alpha$ is typically 3 – 4 for a single stage cyclone but can be as high as 6. A closed circuit grinding operation can have values around 2.5. Fig. 12.16 illustrates typical efficiency curves drawn for three arbitrarily selected values of $\alpha$. It can be seen that with increasing values of $\alpha$, the curves are steeper indicating a greater efficiency of classification and sharpness of split.

As this method of representing classification is independent of cyclone geometry, it is successfully used to scale up laboratory results to full-scale industrial units. It has been found that results accurately predict industrial scale operations.

It is important to note that in practice, the value of $\alpha$ cannot be determined directly from the $d/d_{50}$ value. It has to be determined by a trial and error method. However, Han and Chen [30] obtained an empirical correlation, based on a similarity principle, for $\alpha$ as:

![Fig. 12.16. The effect of efficiency parameter, $\alpha$ on the shape of the performance curve.](image-url)
\[
\alpha = 6.11 \left( \frac{10D_{vu}}{D_o} \right)^{-0.02} \left( \frac{d_f \rho_c (\rho_s - \rho_c) / \mu^2}{C_{MS/FJ}} \right)^{0.16} \left( \frac{10000d_{50C}}{D_c} \right)^{-0.79}
\]

(12.28)

where \( d_f \) = the 63.2% passing size of the feed.

Each bracketed term is dimensionless so that the units have to be consistent. That is, if \( d_f \) is in meters then all other diameters are also in meters, \( \rho \) is in kg/m³, \( g \) is m/s² and viscosity, \( \mu \), is in Pa s (or N/m²).

12.6.2. Effect of cyclone variables on operation

As the operation of hydrocyclones depend on large number of interdependent variables, attempts have been made by a number of workers to determine the extent of the effect of the individual variables [30,34-36]. A survey of the literature indicates the following general conclusions. These conclusions were drawn by varying a single parameter while keeping others constant.

A. Cyclone geometry:

1. \( d_{50C} \) will increase with increasing vortex finder diameter,
2. \( d_{50C} \) will increase with decreasing spigot diameter,
3. \( d_{50C} \) will increase with increasing inlet diameter,
4. \( d_{50C} \) will decrease with increasing length.

B. Slurry characteristics:

1. Finer the feed size the smaller the \( d_{50C} \) value,
2. Increased feed rate decreases the \( d_{50C} \) value,
3. Increased SG of the feed solids decreases the \( d_{50C} \) value.

These general relations were quantified by using regression analysis by several workers starting as early as 1949 and 1954 by Dalhstrom. The relations established later by Lynch and Rao [24], Plitt [28] and Arterburn [12], are now more generally accepted.

In deriving the models it is obvious that only non-roping conditions were applicable. A roping discharge condition can be seen simply by observing the nature of the discharge stream. For instance, Fig. 12.17 shows a normal condition of flow (A) where the stream is flared like a fishtail and a rope discharge (B) where the underflow discharges as a continuous stream resembling a rope. The normal spray discharge has a cone angle of 20-30° with a hollow center [12].

To prevent a roping condition, the underflow density must be kept below a limiting value. The roping conditions have been quantified by Laguitton [31] who stated that the limiting underflow and feed conditions for roping is:

\[
V_{SU} < 0.56 + 0.20 (V_{SF} - 0.20)
\]

(12.29)

and by Mular and Jull [11] as:
\[ V_{S(U)} < 0.5385 V_{S(O)} + 0.4911 \]  \hspace{1cm} (12.30)

where \( V_{S(U)} \) is the volume fraction of the solids in the underflow and \( V_{S(F)} \) is the volume fraction of the solids in the feed stream.

For values of \( V_{S(U)} \) greater than the right hand side of Eqs. (12.29) and (12.30), roping is likely to occur. Eq. (12.30) suggests that a higher underflow density can be achieved, without the risk of roping, if the cyclone is operated with a high overflow density. A higher solid density will also allow a higher underflow density before roping occurs. For example, for an overflow of 30% solids and a solid S.G. of 2.7 the underflow will start to rope at approximately 78% solids by mass whereas for a solid S.G. of 3.7, the underflow density can be increased to around 82% solids before roping occurs [11].

Plitt et al [32] indicate that the particle size of the underflow is the controlling factor for changing from a normal spray to roping discharge but Bustamante [33] asserts that the ratio of the underflow to overflow discharge diameters are the governing factors. Concha et al [34] has quantified this ratio in relation to roping conditions. These authors state that roping will occur if the air core diameter is greater than the spigot diameter. Since the air core diameter depends on the surface tension, viscosity and overflow and underflow diameters, the ratio \( D_U/D_O \) will be a determining variable. Table 12.10 gives some limiting values.

Table 12.10
Transition from spray to roping discharge.

<table>
<thead>
<tr>
<th>( D_U/D_O )</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bustamante [33]</td>
<td></td>
</tr>
<tr>
<td>&lt; 0.34</td>
<td>Roping discharge</td>
</tr>
<tr>
<td>0.34 – 0.5</td>
<td>Roping or spray</td>
</tr>
<tr>
<td>&gt; 0.5</td>
<td>Spray discharge</td>
</tr>
<tr>
<td>Concha et al [34]</td>
<td></td>
</tr>
<tr>
<td>&lt; 0.45</td>
<td>Roping</td>
</tr>
<tr>
<td>0.45 – 0.56</td>
<td>Roping or spray</td>
</tr>
<tr>
<td>&gt; 0.56</td>
<td>Spray discharge</td>
</tr>
</tbody>
</table>

Fig. 12.17. Hydrocyclone discharge. A - Normal spray discharge, B - rope discharge
For efficient hydrocyclone operation it is necessary to operate as close to roping conditions as possible, so that maximum coarse particles are removed.

### 12.7. Hydrocyclone Models

The operation of hydrocyclones depends on a number of interdependent variables. Attempts to inter-relate them with performance has been made by several workers \([27,35-37]\). Most workers used crushed quartz or limestone slurries as the medium in their laboratory studies. Lynch used real sulphide ores (copper, lead) in his investigation.

The model developed by Lynch and Rao \([24]\) was obtained as a product of individual (quantitative) relationships of each variable with the \(d_{50}\). Using a Krebs hydrocyclone, 508 mm in diameter they found that the \(d_{50}\) was a function of particle size and cyclone geometry. They determined three different equations corresponding to arbitrarily defined coarse, medium and fine particle sizes. However their final model encompassed this variation and is now written as:

\[
\log d_{50} = 4.18 D_o - 5.43 D_u + 3.04 D_i + 0.0319 C_{MS(F)} - 3.6 Q_{VF} - 0.0042 (\%{+420}) + 0.0004 (\%-53) \tag{12.31}
\]

where

- \(C_{MS(F)} =\) % solids by mass in the feed,
- \(Q_{VF} =\) volume flowrate of feed, \(m^3/s\),
- \(C_{+420} =\) % + 420 \(\mu m\) in the feed,
- \(C_{-53} =\) % - 53 \(\mu m\) in the feed,
- \(D_o, D_i, D_u =\) diameters of the overflow, inlet and underflow respectively, \(m\),
- \(d_{50} =\) cut size in microns.

The constants strictly apply for a Krebs cyclone and limestone slurry, but is widely used for most slurries with fair accuracy. For minerals of different densities to limestone, a correction may be applied, such as the one given in Eq. (12.32).

\[
\left(\frac{d_{50C}}{d_{50L}}\right)_2 = \left(\frac{\rho_{L} - \rho_{S}}{\rho_{S2} - \rho_{L}}\right) \tag{12.32}
\]

Lynch and Rao's model has been subsequently modified by Nageswararao \([38]\) who included the cone angle of the cyclone and hindered settling conditions. The hindered settling factor was taken as the ratio of free settling to hindered settling. \(H_s\), and written as:

\[
H_s = \frac{10^{1.824V_{SF}}} {8.05 \left[1 - V_{SF}\right]^{1/3}} \tag{12.33}
\]

where \(V_{SF} =\) volume fraction of solids in the feed slurry.

The final model translates, with slight modification by JKTech \([39]\), to:

\[
\frac{d_{50C}}{D_c} = K_{dc} \left(\frac{D_o}{D_c}\right)^{0.52} \left(\frac{D_u}{D_c}\right)^{-0.47} \left(\frac{P}{\rho_{SL} g D_c}\right)^{-0.22} \left(\frac{D_i}{D_c}\right)^{-0.59} \left(\frac{L_{cyl}}{D_c}\right)^{0.15} \theta^{0.15} D_c^{-0.63} H_s^{0.95} \tag{12.34}
\]
where $P$ = feed Pressure, kPa,
$g$ = acceleration due to gravity,
$\theta$ = cone angle, degrees,
$H_s$ = hindered settling factor,
$K_{DO}$ = material constant depending on the SG and size of particles in the feed,
$L_{CYL}$ = length of the cylindrical section, m,
$D_C$ = diameter of the cylindrical section, m,
$\rho_{SL}$ = feed slurry density, t/m$^3$,
$d_{SOC}$ = cut size in microns.

To evaluate Eq. (12.34), $K_{DO}$ has to be determined for each case. As this is not possible, it is estimated in a laboratory using a laboratory size hydrocyclone and scaled to suit a particular condition. This model has been applied with considerable success.

Using pure silica suspensions, Plitt [28], Plitt et al [40] and later Arterburn [12], developed mathematical models relating the operational variables and the cut point. Both these models were derived empirically from experimental data obtained in laboratory size hydrocyclones. According to Plitt:

$$d_{SOC} = \frac{k_1 \cdot 2689.2 \cdot D_C^{0.46} \cdot D_I^{0.6} \cdot D_O^{1.21} \cdot \mu^{0.5} \cdot \exp(0.063 \cdot C_{VS(F)})}{D_U^{0.71} \cdot LVF^{0.34} \cdot Q_{VF(F)}^{0.45} \cdot (\rho_S - \rho_L)^{0.5}}$$

where
$LVF$ = free vortex height (distance from end of vortex finder to apex), m
$D_C$ = cylindrical diameter, m
$D_U$, $D_I$, $D_O$ = underflow, inlet and overflow diameters, m
$Q_{VF}$ = volumetric flowrate of the feed, m$^3$/s
$C_{VS(F)}$ = % solids by volume in the feed
$d_{SOC}$ = corrected cut size, microns
$\mu$ = liquid viscosity, mPa.s
$\rho_S$, $\rho_L$ = density of solid and liquid respectively, kg/m$^3$, and
$k_1$ = a calibration factor (taken as 1.0 when no data is available)

Austin et al [26] state that the models advocated by Lynch and Rao and Plitt yield $d_{SOC}$ values that depend on the conditions of determination and were more suited for dilute slurries. Despite this, the expressions are extensively used to design and operate industrial size cyclones.

Arterburn [12] derived a simpler relation, which is also used extensively but mostly for the designing of hydrocyclones. According to Arterburn, for a typical Krebs hydrocyclone:

$$d_{SOC} = \frac{8253.5 \cdot D_C^{0.67}}{\Delta P^{0.28} \cdot (\rho_S - \rho_L)^{0.5} \cdot \left[1 - (1.9 \cdot F_{SF})\right]^{0.43}}$$

where $D_C$ is in meters, $\Delta P$ in kPa, $\rho_S$ and $\rho_L$ in kg/m$^3$ and $d_{SOC}$ in microns.

An alternative empirical approach for hydrocyclone models has recently been attempted by Han and Chen [30] using the similarity principle. According to Han and Chen:
Han and Chen used a 50 mm diameter cyclone with a cone angle of 12° and a quartz slurry with particle size distributions in coarse, medium and fines ranges between 250 μm and 10 μm. A coefficient of correlation in excess of 0.95 is claimed.

Bradley [9] as early as 1965 and later Klimpel [41] and Austin et al [26] have indicated that the viscosity of the slurry also affects the efficiency curve and therefore the \( d_{50} \) value. As viscosity generally decreases with an increase in temperature, it is likely that the cut point will also depend on temperature. Work in this area has been reported by Gupta and Eren [42].

### 12.8. Hydrocyclone Capacity

Assessing the capacity of hydrocyclones has been the study of several workers. The generally acceptable relation for capacity, \( Q_{VF} \), is given by:

\[
Q_{VF} = k \Delta P^{0.5} \tag{12.38}
\]

Empirically, the exponent for pressure drop has been found to range from 0.44 – 0.56. The constant, \( k \), is a function of cyclone dimensions and the pressure and flow characteristics of the slurry entering the feed chamber. According to Dahlstrom, the capacity is also proportional to the square of the cyclone diameter and is given by the relation:

\[
Q_{VF} = k \times 10^{3} \Delta P^{0.5} D_{C}^{2} \tag{12.39}
\]

where
- \( Q_{VF} \) = the volumetric flow rate of pulp in the feed, \( m^{3}/h \),
- \( \Delta P \) = the feed pressure, kPa,
- \( D_{C} \) = the diameter of the cyclone, m.

Tarr [13] developed a graphical solution relating capacity and diameter of the cyclone. The relation is reproduced in Fig. 12.18 where the mean values of feed capacities are plotted against cyclone diameter.

For more appropriate values of capacities Tarr states that the capacities have to be adjusted according to the percent solids in the slurry and the feed pressure of the slurry. Such adjustments, described as correction factors, are shown in Fig. 12.19 and Fig. 12.20.

A further correction factor was introduced by Tarr to account for the differences in specific gravity of minerals as the original expression was derived using a quartz slurry. This correction factor is given in Eq. (12.40).

\[
\text{Correction factor} = \sqrt[0.5]{\frac{2650 - 1000}{\rho_{S} - 1000}} \tag{12.40}
\]

The derivation of Tarr’s method was based on “typical” hydrocyclone dimensions.

An application of the method is given in Example 12.3.
Fig. 12.18. Capacity of typical hydrocyclone of varying diameters [13].

Fig. 12.19. Correction factor for different solids content in the feed [13].
Example 12.3

A 35% pulp (by volume) had to be classified in a 100 mm diameter hydrocyclone at an inlet pressure of 100 kPa. Determine the cyclone capacity under the following operating conditions:

Specific gravity of solid = 2650 kg/m³

Solution

From Fig. 12.18 the capacity corresponding to a 100 mm diameter cyclone is 0.1 m³/min.

Correction factor for 34% solids in the feed (Fig. 12.19) = 1.32

Correction factor for pressure (Fig. 12.20) = 1.2

Hence Capacity = 0.1 x 1.32 x 1.2 = 0.16 m³/min.

Fitch and Roberts [3] considered the diameter of the vortex finder, D₀, the inlet diameter, Dᵢ, and the input pressure, ΔP, to calculate the capacity of cyclones. The hydrocyclone capacity, Qᵥ(Fᵢ), is given by:
The unit of \( Q_{V(F)} \) is \( m^3/min \) with \( D_0 \) and \( D_1 \) in meters and pressure in kPa.

Nageswararao (1995) considered all the variables in a hydrocyclone and derived the relation between the hydrocyclone variables, feed mass flow rate and geometry of the hydrocyclones as:

\[
Q_{V(F)} = K_{Q0} \frac{D_c^{0.50}}{\theta^{0.1}} \left[ \frac{\Delta P}{\rho_{SL}} \right]^{0.59} \left[ \frac{D_o}{D_c} \right]^{0.67} \left[ \frac{D_1}{D_c} \right]^{0.45} \left[ \frac{L_c}{D_c} \right]^{0.2} \tag{12.42}
\]

where \( K_{Q0} \) = a constant depending on the feed solids and determined experimentally in a laboratory size cyclone of known parameters and scaled to the size for a commercial cyclone.

The throughput through the hydrocyclone can also be measured in terms of flow through the vortex finder, or apex. However, the sensitivity of the split \( (d_{50}) \) is largely dependant on the throughput of the underflow.

Plitt [28] developed a series of models to describe the behaviour of a cyclone. These models estimate the \( d_{90C} \), pressure drop, the sharpness of separation and the flow split.

The Plitt equation for the flow split is:

\[
S = \frac{Q_{V(U)}}{Q_{V(O)}} = \frac{k_2 \cdot 3.79 \left( \frac{D_U}{D_O} \right)^{3.31} L_{VF}^{0.54} \left( D_U^2 + D_O^2 \right)^{0.36} \exp(0.0054 C_{VS(F)})}{H^{0.24} D_c^{1.11}} \tag{12.43}
\]

or

\[
S = \frac{Q_{V(U)}}{Q_{V(O)}} = \frac{k_2 \cdot 6.56 \left( \frac{D_U}{D_O} \right)^{3.31} \rho_{SL}^{0.24} L_{VF}^{0.54} \left( D_U^2 + D_O^2 \right)^{0.36} \exp(0.0054 C_{VS(F)})}{P^{0.24} D_c^{1.11}}
\]

where \( H \) = pressure head in meters of slurry,
\( P \) = pressure drop in Pa,
\( Q_{V(U)}, Q_{V(O)} \) = volume flow rate in underflow and overflow respectively, \( m^3/h \),
\( C_{VS(F)} \) = % solids by volume in the feed,
\( D, L \) = dimensions in meters,
\( \rho_{SL} \) = slurry density in kg/m\(^3\),
\( k_2 \) = a calibration factor (taken as 1.0 when no data is available).
The other Plitt models are:

\[
P = \frac{k_3 0.0651 Q_{VF}^{1.8} \exp(0.0055 C_{VF})}{D_c^{0.37} D_i^{0.94} L_{VF}^{0.28} (D_i + D_o)^{0.07}}
\]

(12.44)

where \(Q_{VF}\) = the volume flowrate of the feed, m³/h, and

\[
m = k_4 10.10 \exp(-1.58R_v) \left( \frac{D_c^2 L_{VF}}{Q_{VF}} \right)^{0.15}
\]

(12.45)

where

- \(k_3, k_4\) = calibration factors (taken as 1.0 when no data is available)
- \(R_v\) = recovery of feed volume to the underflow
- \(m\) = sharpness of separation

Using the similarity principle, Han and Chen [30] obtained the expression for the throughput for a 50 mm cyclone as:

\[
Q_{VF} = 0.14 \left( \frac{10 D_o}{D_c} \right)^{0.9} \left( \frac{d_{50} \times 10^4}{D_c} \right)^{0.68} D_c^{-2.5}
\]

(12.46)

12.9. Hydrocyclone Circuits

Almost all crushing and grinding circuits include hydrocyclones in close circuit to yield a product of the required size distribution. Hydrocyclones are generally installed at an elevated position above the grinding unit so that the coarse underflow product can flow by gravity back to the grinding unit for further size reduction. The configurations adopted in practice are varied. Three typical set ups are illustrated in Fig. 12.21.

For a better control of the product size, hydrocyclones are connected in series (Fig. 12.22), while for greater throughput cyclones are connected in parallel.

While operating in series, the underflow from the first cyclone forms the feed to the second cyclone. Trawinski [43] suggests that the second cyclone should be operated at as near to roping conditions as possible. Dahlstrom and Wai-Ping Kam [35] suggest that in addition to metallurgical advantages, two stage classification leads to energy savings. It can be easily seen that when one cyclone is in operation a mass balance illustrates the distribution of products between the oversize and undersize. Such a mass balance is illustrated in Fig. 12.23. The distribution of a particular size \(i\) in the feed between overflow and underflow can therefore be determined.

Defining the fraction that goes selectively to the coarser stream as selectivity \(E_i\), the selectivity (partition coefficient) can be determined using the expression indicated by Austin et al [26].

\[
E_i = \frac{\text{mass fraction of size } i \text{ in underflow}}{\text{mass of underflow stream}} \times \frac{\text{mass of underflow stream}}{\text{mass fraction of size } i \text{ in feed}} \times \frac{\text{mass of feed}}{\text{mass of underflow stream}}
\]

(12.47)
\[ E_i = \frac{Uu_i}{Ff_i} \]  \hspace{1cm} (12.48)

where \( F, O, U \) = the flow rates of feed, overflow and underflow and,
\( f_i, o_i, u_i \) = the mass fraction of the size \( i \) in the respective streams.

Fig. 12.21. Hydrocyclones in closed circuits with grinding mills.

Fig. 12.22. Hydrocyclones connected in series, two stage classification.
However, since most hydrocyclone circuits operate in closed circuit, the recirculating load or ratio also needs to be included in the Eq. (12.48). Describing each stream symbolically as in Fig. 12.23 the mass balance, at steady state, may be re-written as:

\[
E_i = \frac{C u_i}{(1+C)p_i} \tag{12.49}
\]

where \( C \) = the circulation ratio = \( \frac{\text{Mass flow rate of underflow stream}}{\text{Mass flow rate of overflow stream}} \),

\( p_i \) = mass fraction of size \( i \) in the new cyclone feed.

So when two cyclones are involved, say in series with the second stage retreating the coarse stream (Fig. 12.22), then the overall partition will be a product of each \( E_i \), say \( E_{ii} \) and \( E_{i2} \) for particle size \( d_i \). Thus the overall partition of size \( i \), \( E_{ii} \), will be:

\[
E_{ii} = E_{i1} E_{i2} \tag{12.50}
\]

The \( E_{ii} \) value will depend on the manner of the hook up of the cyclones. For two cyclones in series with the overflow retreated in the second stage, Luckie and Austin [29] proposed the expression:

\[
1 - E_{ii} = (1 - E_{i1})(1 - E_{i2}) \tag{12.51}
\]
12.10. Problems

12.1
An alkaline slurry from a bauxite grinding mill was scheduled to be classified using a spiral classifier at the underflow rate of 1100 t/day. The width of the classifier flight was 1.3 m and the outside diameter of the spiral flights was 1.2 m. Estimate the pitch of the spirals if the spiral speed is 5 rev/min and the bulk density of the underflow solids is 2000 kg/m$^3$.

12.2
A Krebs D-6B (6 inch) hydrocyclone was placed in open circuit to classify a predominantly silicious ore. The dimensions of the hydrocyclone were:

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet pipe diameter</td>
<td>40 mm</td>
</tr>
<tr>
<td>Overflow pipe diameter</td>
<td>45 mm</td>
</tr>
<tr>
<td>Apex diameter</td>
<td>13 mm</td>
</tr>
<tr>
<td>Feed density</td>
<td>62% solids</td>
</tr>
<tr>
<td>SG of the solids</td>
<td>2.65</td>
</tr>
<tr>
<td>SG of the slurry</td>
<td>1.629</td>
</tr>
</tbody>
</table>

Determine:

1. The feed rate to obtain a cut point at 150 microns
2. The cut point if the dilution is halved due to faulty operation

Take $L_V = 3D_C$ and all pipe dimensions are internal.

12.3
A hydrocyclone had the following dimensions:

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spigot diameter</td>
<td>2.5 cm</td>
</tr>
<tr>
<td>Cyclone diameter</td>
<td>12.5 cm</td>
</tr>
<tr>
<td>Vortex finder</td>
<td>5.0 cm</td>
</tr>
<tr>
<td>Inlet diameter</td>
<td>3.1 cm</td>
</tr>
<tr>
<td>Included angle</td>
<td>12°</td>
</tr>
</tbody>
</table>

A pulp slurry of S.G. 1.24, containing solids of S.G. 2.9 was fed at a rate of 200L/min. Trial runs indicated a $d_{50}$ of 100 microns. Determine:

1. The pressure differential,
2. The underflow flow rate.

12.4
The diameter of a typical hydrocyclone was 30.5 cm. The apex was fitted with a rubber sleeve 12 cm in length and 8.0 cm in internal diameter. A quartz suspension at a density of 1.33 was fed to the cyclone at the rate of 1000L/min. The underflow measured 75% solids. The apex diameter was reduced by 10 % twice. Estimate:
1. The change in the cut point after each setting of the apex
2. The roping conditions for the cyclone operation.

12.5

The volume flow rate of pulp fed to a hydrocyclone was 129 L/min. Its solid content was held at 32% by volume. Samples of the feed, under flow and over flow streams were taken simultaneously, dried and a size analysis carried out. The results obtained were:

Underflow rate = 30 L/min
Overflow rate = 99 L/min

<table>
<thead>
<tr>
<th>Particle size, μm</th>
<th>Feed mass%</th>
<th>Underflow mass%</th>
<th>Overflow mass%</th>
</tr>
</thead>
<tbody>
<tr>
<td>+212</td>
<td>2.6</td>
<td>6.24</td>
<td>0.0</td>
</tr>
<tr>
<td>-212+150</td>
<td>8.9</td>
<td>18.77</td>
<td>0.4</td>
</tr>
<tr>
<td>-150+106</td>
<td>22.1</td>
<td>42.59</td>
<td>2.8</td>
</tr>
<tr>
<td>-106+90</td>
<td>10.8</td>
<td>13.99</td>
<td>6.8</td>
</tr>
<tr>
<td>-90+75</td>
<td>7.9</td>
<td>7.26</td>
<td>8.3</td>
</tr>
<tr>
<td>-75+63</td>
<td>7.8</td>
<td>4.77</td>
<td>9.4</td>
</tr>
<tr>
<td>-63+53</td>
<td>5.8</td>
<td>2.57</td>
<td>6.0</td>
</tr>
<tr>
<td>-53+45</td>
<td>4.8</td>
<td>1.22</td>
<td>5.7</td>
</tr>
<tr>
<td>-45+34</td>
<td>6.5</td>
<td>0.83</td>
<td>10.4</td>
</tr>
<tr>
<td>-34</td>
<td>22.80</td>
<td>1.75</td>
<td>50.23</td>
</tr>
<tr>
<td>Total</td>
<td>100.00</td>
<td>100.0</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Draw a partition curve and from it determine:

1. Cut point (d_{50}),
2. Water split,
3. Corrected cut point (d_{50c}),
4. Imperfection.

12.6

An hydrocyclone is to be installed in a closed circuit grinding circuit with a mill discharge containing 30% solids by volume. The solid density is 2800 kg/m³ and the density of water is 1000 kg/m³. Given that the maximum pressure deferential between the inlet and overflow was 50 kPa and the throughput from the mill was 800 t/h, estimate:

1. The dimensions of a suitable hydrocyclone if there are two operating in parallel,
2. The cut point.

12.7

A hydrocyclone classifier is fed with quartz slurry at the rate of 20.8 t/h from a grinding mill. The underflow is recirculated. The screen analysis of each stream were determined with the following results:
No. | Size, μm | Feed mass % | Overflow mass % | Underflow mass %
--- | --- | --- | --- | ---
1 | +300 | 25 | 0 | 33.2
2 | -300-250 | 15.3 | 0 | 20.3
3 | -250-150 | 11.6 | 0.2 | 15.4
4 | -150-106 | 12.8 | 5.4 | 15.2
5 | -106+75 | 9.6 | 18.3 | 6.7
6 | -75 | 25.7 | 76.1 | 9.2
Total | | 100 | 100 | 100

Determine:

1. The circulation ratio
2. The efficiency of the cyclone

12.8

The input and output streams of an operating cyclone were sampled simultaneously for the same period of time. The dried samples were analysed for size distribution and the mass per cent retained on each size fraction was determined with the following results:

No. | Size, μm | Feed mass % | Overflow mass % | Underflow mass %
--- | --- | --- | --- | ---
1 | +425 | 1.5 | 0 | 2
2 | -425+300 | 3.8 | 0 | 6.3
3 | -300+212 | 6.2 | 0.3 | 12.9
4 | -212+150 | 10.7 | 1.8 | 21.2
5 | -150+106 | 16 | 15.2 | 28
6 | -106+75 | 23 | 26.2 | 10
7 | -75+53 | 28 | 38.4 | 5
8 | -53 | 10.8 | 18.1 | 14.6
Total | | 100 | 100 | 100

Data:

% Solids in Feed slurry = 35%
% Solids in Overflow = 17.2%
% Solids in Underflow = 70.2%
Feed capacity (dry solids) = 25 t/h
Solid density = 2650 kg/m³
Inlet pressure = 35 kPa
Apex diameter = 6.0 cm
Vortex finder diameter = 14.2 cm
Mass split to the underflow = 39.4%

After a steady state operation the solid content of feed slurry was increased by 20% while all other conditions remained the same. Determine the size distribution of each stream under the altered condition.
12.9
If a second cyclone is added in series to the cyclone in problem 12.8, what is the effect of the overall efficiency of the classification. What will be the size distribution of the cyclone U/F of the second stage? The partition coefficient of the second stage cyclone is given as:

<table>
<thead>
<tr>
<th>No.</th>
<th>Size, μm</th>
<th>E_{ij}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+425</td>
<td>1.000</td>
</tr>
<tr>
<td>2</td>
<td>-425+300</td>
<td>0.996</td>
</tr>
<tr>
<td>3</td>
<td>-300+212</td>
<td>0.887</td>
</tr>
<tr>
<td>4</td>
<td>-212+150</td>
<td>0.520</td>
</tr>
<tr>
<td>5</td>
<td>-150+106</td>
<td>0.223</td>
</tr>
<tr>
<td>6</td>
<td>-106+75</td>
<td>0.077</td>
</tr>
<tr>
<td>7</td>
<td>-75+53</td>
<td>0.021</td>
</tr>
<tr>
<td>8</td>
<td>-53</td>
<td>0.010</td>
</tr>
</tbody>
</table>

12.10
A rod mill discharge is to be classified in a straight sided, single pitch screw classifier. The classifier feed has the following size distribution:

<table>
<thead>
<tr>
<th>Particle size, μm</th>
<th>Feed mass%</th>
</tr>
</thead>
<tbody>
<tr>
<td>+710</td>
<td>2.6</td>
</tr>
<tr>
<td>-710+425</td>
<td>8.9</td>
</tr>
<tr>
<td>-425+250</td>
<td>22.1</td>
</tr>
<tr>
<td>-250+125</td>
<td>10.8</td>
</tr>
<tr>
<td>-125+75</td>
<td>7.9</td>
</tr>
<tr>
<td>-75</td>
<td>7.8</td>
</tr>
<tr>
<td>Total</td>
<td>100.00</td>
</tr>
</tbody>
</table>

If the classifier is to separate the feed at 200 microns estimate the classifier area and screw diameter if the feed capacity required is 100 t/h.

Data:
- Solid density = 2740 kg/m³
- Water density = 1100 kg/m³
- Water viscosity = 0.001 Pa.s
- Classifier feed = 40% solids (by mass)
- Overflow = 35.1% solids (by mass)
- Spiral speed = 5 rpm
- Areal efficiency = 0.45
Assume pitch is 0.5 x spiral diameter.
REFERENCES

Chapter 13. Solid – Liquid Separation

13. INTRODUCTION

The process of grinding and classification involves the use of large quantities of water. In the gold industry for instance, the rule of thumb is a tonne of water for a tonne of ore. This bulk of water has to be separated or reduced for downstream treatment for recovery of the mineral in the ore. The separation of solids from liquids is usually achieved by gravity sedimentation in thickeners. For fine particles this is a slow process. In general 75-80% of the water can be separated and removed by thickeners. For further water removal, filters are used where in excess of 90% of the water can be removed. The thickener operation can be a batch or continuous process, with either co-current or counter-current flow of underflow and overflow slurries. The filtering operation may also be batch or continuous.

For rapid solid-liquid separations, centrifugal forces are used and equipment similar to those described under classification are employed. In this chapter, we shall deal mostly with thickeners working under gravitational forces.

13.1. Design Features of Thickeners

Thickeners are essentially clarifiers producing a clearer over flow. The design considerations are based on the settling rates of the slowest settling particles and conditions for minimum disturbance of the medium (water) through which the solid particles are allowed to settle. To achieve these objectives cylindrical tanks with conical or flat bottoms are used and the velocity of the feed slurry entering the settling tank is minimised to reduce turbulence in the settling tank. A schematic diagram of a typical thickener is shown in Fig. 13.1. The feed in the form of slurry is generally guided by a launder, which is laid at a slope just sufficient for the slurry to flow without depositing any solids. The feed launder terminates in a feed well located at the centre of the tank. The feed well is designed to break the fall of the slurry and dissipate the energy.

The feed well is concentric with the rake driving shaft. The rakes are bolted or welded on to this drive shaft and for long and large rakes they have additional support from cables. Usually four rakes are employed of which two may be short and two long. Attached to the rakes and below them are spikes, particularly in situations where the sludge is thick. The spikes help to break up the sludge and render it more suitable for pumping. The rakes are driven by a motor which is mounted on a plate above the well. An alternative is to mount the drive motor on a track running along the rim of the tank. A bridge usually runs from the periphery to the centre of the tank. It is supported by the wall of the feed well and the rim of the tank. The bridge serves as a walkway and also carries an open launder (or pipe), which carries the slurry to the feed well. In some designs the bridge spans the entire length of the tank. As in clarifiers, the bottom of most tanks slope towards the centre where the thickened underflow sludge accumulates. When a flat bottomed tank is designed, the settled sludge builds up to form its own slope depending on the angle of repose of the material thus forming an artificial sloping tank bottom. The sludge collected at the bottom is discharged through an outlet shaped like a cone with steep cone angle. Alternately, the thickened slurry is swept
towards a trench at the bottom of the tank. Usually a scraper is installed for smooth delivery of sludge from the discharging cone or trough. A slush or centrifugal pump subsequently removes the sludge.

The thickener tanks are usually fabricated using steel sheets. But tanks with concrete sides are quite common. Some small tanks (usually < 30 m in diameter) are made of plastics. The whole assembly is installed either above ground sitting on pillars or at ground level with the discharge well below the ground level. In the latter case, an access tunnel is provided where the discharging pump is located. In some installations the discharge pump is located above the tank; in such cases, a suction pipe runs down the centre column to the bottom well. Alternatively, a submerged motor pumps the under flow slurry to the top of the tank discharging its contents to a holding tank.

Several variations are known to exist. For instance the rakes are either supported by cross beams or truss above the tank or supported by the central column and cables. The cables are also connected to torque meters.

Fig. 13.1 is a sketch of a bridge thickener where the bridge runs across the thickener tank. The bridge support the rakes and the motor rotating the rakes sit on a platform in the centre of the tank. The rakes are bolted to the central column which is rotated by the motor. The Bridge thickeners have a maximum diameter of about 30 meters.

When the rakes are supported entirely by the central pillar, the access bridge usually runs half way on the tank surface terminating on the central pier. The centre pier thickeners are considerably larger than the Bridge type. The diameter of the tank ranges from about 35 - 180 meters.

A variation is the tray thickener where trays or compartments are placed one on top of the other. Each tray acts as a thickener and the assembly operates in parallel with a common pier or shaft where the rakes are fixed. Clarification takes place in series operation, that is, the thickener underflow from the top compartment serves as feed to the lower compartment. Ultimately the underflow from say, a six tray thickener, form the final thickened underflow. Similarly all the overflow from each tray combine forming the final overflow slurry. Fig. 13.2 is a schematic diagram of a 3-compartment clarifier. Up to seven compartments are available.

---

**Fig. 13.1.** Sketch of a thickener showing the access bridge, feed well, rakes supported by the central column and cables and the underflow discharge.
The thickening process is accelerated by the addition of flocculants. *Hi-Capacity thickeners* allow a mixing arrangement in the feed box where the flocculant is intimately mixed. The other design features of the Hi-Capacity thickeners are similar to the Bridge thickeners.

While installing the feed pipe or launder to thickeners, the slope is held at 1 to 1.5. King [2] suggests that this slope provides minimum turbulence of the settling slurry in the tank. The feed is actually made to enter about a meter below the surface of the tank level thus helping to minimise turbulence.

The feed well diameters are between 1 and 1.2 m with lengths of 1.2 to 5 m. Tank sizes vary according to feed characteristics and the sedimentation time. Manufacturers such as Dorr-Oliver-Eimco, [3] have suggested that the water depth should be between 3.0 and 3.6 m and the feed well size about 25% of the basin area.

The rake-drives in bridge clarifiers are either centre driven (as shown in the Fig. 13.1) where the motor is mounted on a support plate or are peripheral driven. When the sludge is too thick and the rakes struggle to move or in extreme cases cease altogether, the rakes are designed to rise either mechanically or pneumatically. Usually the torque on the rakes is monitored and the rakes rise automatically at a fixed torque level. This precautionary procedure is generally attached to thickeners of diameter greater than 10 meters. The allowable torque is about 5-30 times greater than normal operating torque [1].

A recent innovation is the Dorr-Oliver Eimco E-Cat thickeners which has dispensed with the rakes and introduced *clarifying cylinders* through which the suspension passes to produce the clear overflow (Fig. 13.3). These thickeners are designed for rapid sedimentation by the use of flocculants. The clarified slurry then passes through filters producing a clear overflow.
13.2. Thickener Design-Batch Process

Thickeners have been designed using the basic laws of sedimentation. Empirical methods devised by manufacturers are also used for rapid work. For designing, the chief criterion is to determine the relation between the settling velocity and the dimensions of the vessel to be used for each particular slurry. The settling velocity for a particular slurry can be easily determined in the laboratory by using small-scale tests. The tests consist of determining the downward movement of the boundary of the clear liquid and the suspension. It has been found that this rate is initially constant but the rate decreased as the particles slowly settled to the bottom and the interface met the sludge zone. This can easily be visualised from Fig. 13.4 where the progressively increasing concentration with depth is shown. It is obvious that the deeper the vessel and longer the time given for settling, the clearer will be the supernatant liquid and the thicker will be the sludge.

The decrease in the settling rate is due to hindrance by increased crowding of the particles as they settle and collect at the bottom of the vessel. At the sludge-forming layer, the particles pack down by displacing the liquid in between. In so doing, the clear liquid level rises. These considerations apply both to batch and continuous processes, with the difference that in the continuous process a balance between the flow rate of the overflow stream and the removal rate of the sludge has to be maintained.
These considerations originally used by Coe and Clevenger [5] are now in use extensively.

The quantitative basis for designing the thickener area assumes that:

1. Settling rate was a function of concentration,
2. The volume rate of discharge of the clear supernatant liquid was equal to the difference of the rate of feed of the slurry minus the rate of removal of the thickened layer.

For determining the thickener area, Coe and Clevenger assumed that the liquid moving upwards is always greater than the movement downwards. The mass of liquid flowing upwards is given by:

\[ (F - D) \frac{Q_{MF}}{h} \text{ t/h} \]  

where

- \( F \) = the feed mass ratio (liquid/solids, also known as the feed dilution),
- \( D \) = discharge mass ratio (liquid/solid) and
- \( Q_{MF} \) = Feed capacity by mass, t/h

At equilibrium, the upward velocity of liquid equals the downward velocity of the solids. Thus if \( v_s \) is the velocity of sedimentation, \( A \) the cross-sectional area of the tank, in m\(^2\), and \( \rho_L \) the specific gravity of the liquid, then at equilibrium:

\[ \left( \frac{F - D}{A \rho_L} \right) Q_{MF} = v_s \]  

(13.2)

hence,

\[ A = \left( \frac{F - D}{v_s \rho_L} \right) Q_{MF} \]  

(13.3)

In practice, to determine the design value of the thickener area, a number of laboratory sedimentation tests are run using 2 litre cylinders and determining the value of \( v_s \) for a range
of $F$ values. The maximum value of $A$ is taken as the design cross-sectional area of the thickener tank.

Dahlstrom and Fitch [2] has analysed each of the settling zones and arrived at a practical expression similar to the expression of Coe and Clevenger for sizing a thickener. Considering that the flow rate in the clear zone should be less than the settling rate of the smallest particle that has to be removed by settling, they derived the velocity of sedimentation as:

$$v_s = \frac{[F - D]}{\rho} Q \quad (13.4)$$

This equation is similar to Eq. (13.2) by Coe and Clevenger. Dahlstrom and Fitch [2] suggested that the actual sedimentation rate must be multiplied by the areal efficiency factor, $A_{EF}$ to obtain a realistic value. The areal factor is a function of the tank dimensions (height and diameter) and ranges between 0.20 and 0.25.

Eqs. (13.3) and (13.4) are extensively used to determine the cross-sectional areas of tanks. The laboratory estimations are performed at different concentrations of $F$ and $D$ and the largest value of $A$ is taken as the designed size of the tank as in the Coe and Clevenger method. For practical purposes they suggest a scale-up factor of 1.25 – 1.5 for thickener units less than 15.2 m in diameter and 1.3 – 1.5 for units greater than 15.2 m in diameter.

13.3. Thickener Design-Continuous Thickeners

For designing continuous thickeners, the three most important parameters that need to be established are:

1. Cross sectional area of the tank
2. Depth of thickened layer
3. Depth of the clarifying zone

Other factors include discharge slurry properties, such as liquid/solid ratio, viscosity and the characteristics of pumping.

13.3.1. Estimation of Cross-Sectional Area of Tank

Coe and Clevenger's equation fails to accurately estimate the cross-sectional area of the tank when the slurry is treated with a flocculating agent. In such cases the mathematical approach of Kynch [6] as applied by Talmage and Fitch [7] is more suitable. A particular advantage is that while several determinations of settling velocities, $v_s$, are required by Coe and Clevenger's method, a single estimation is sufficient when analysis of the sedimentation curve is made. To apply Kynch's method the following assumptions are made:

1. The concentration of particles in any horizontal plane is uniform,
2. Differential settling due to differences in shape, size or composition of mineral particles do not take place,
3. The sedimentation velocity is a function of concentration and tends to zero at a concentration equivalent to the sediment layer at the bottom of the container,
4. The wall effect is negligible.

A single laboratory test therefore involves the suspension of a slurry in a 2 litre tall transparent cylinder and measuring the clear fluid interface with the slurry at different times...
till the level falls and all particles settle at the bottom as sludge. Where the sedimentation rate is very slow or the supernatant liquid remain turbid and unclear, flocculants are added. If required, rakes are introduced to break up agglomerated particles. A typical sedimentation curve indicating the height of the interface with time and the structure of the slurry in the cylinder is shown in Fig. 13.5. It can be seen that at the initial stages, the rate of fall of the interface is nearly constant. When the settling rate of the bulk of the slurry diminishes, (as seen in cylinder 4), the clear zone-sludge interface merges and the curve then flattens out. At this stage, further lowering of the clear level interface can take place by the expulsion of water between the particles in the sludge. Fig.13.5 shows that at time \( t = 0 \), the height of the interface is \( H_0 \). As it is assumed that the concentration of slurry is uniform across the cross-section of the tube, at any height, \( H_1 \), the concentration of the sludge will be the same across the settling tube.

For a dispersed slurry, the solids start settling at a uniform velocity which is a function of the local solids concentration \([5,6]\). As the settled solids build up at the bottom of the container, the boundary between the settling solids and slurry of the initial concentration starts to rise in the slurry as indicated in Fig. 13.5. Zones of intermediate concentration between the initial and final concentrations will move upwards from the bottom at a rate related to the concentration of solids in that zone. When the rising and settling zones meet, the settling slows and is controlled by the extraction of retained water from the solids as it goes through compaction.

The rise velocity of the zone of concentration \( C \), from the bottom of the cylinder to the interface of the settling mudline, \( v_R \), given by:

\[
\frac{dH}{dt} = -\frac{d\psi}{dC}
\]  

and is represented by the line OY in Fig. 13.5. \( \psi \) is the settling flux, kg/m²/s.

Fig. 13.5. Settling curve – Kynch’s interpretation.
If line OY represents the initial uniform concentration \( C_0 \) then higher concentrations resulting from settling solids at the bottom of the cylinder are represented by lines of lower slope, OY\(_1\) (for intermediate concentration) until the maximum concentration of the settled solids is reached and represented by \( C_{\text{MAX}} \) and line OY\(_{\text{MAX}}\). Any line parallel to these will represent the rise velocity of zones of the same concentration, \( C \), so line H\(_1\)Y\(_1\) will represent a zone of concentration \( C_0 \) which originates from height \( H_1 \) in the slurry propagating upwards and reaching the mudline interface at \( Y_1 \) after time \( t_1 \).

Since the sedimentation rate is dependant on concentration only, until the zone of initial concentration from the cylinder bottom reaches the interface, the sedimentation rate of the interface will be constant and hence the rate \( v_{\text{SO}} \) will be represented by a straight line, \( H_0Y \).

According to Kynch, if a tangent is drawn to the settling curve at point \( Y_t \), the slope, \( \alpha \), corresponds to the settling velocity, \( v_{\text{st}} \), of the layer or zone of concentration \( C_t \) just below the settling interface. The intercept of the tangent on the Y-axis, \( H_t \), corresponds to the height of slurry of uniform concentration equal to \( C_t \). Then by a mass balance:

\[
H_t C_t = H_0 C_0
\]

for a cylinder of constant cross-sectional area. Consequently, a plot of settling rate versus concentration can be constructed from a single settling curve.

Kynch's theory has been tested experimentally on many occasions and found to hold for the batch settling of equi-sized rigid spheres in water but deviates for flocculated suspensions that form compressive sediments [8].

Yalcin [9] reported the sedimentation curves of a copper-nickel tailings for several initial percent solids. By constructing tangents to the low density pulp curve at different higher percent solids, using the Kynch construction, estimates of the settling rates can be compared to the actual measured sedimentation rates of these slurries. Fig. 13.6 shows such a construction on the settling curve of an unflocculated slurry having an initial concentration of 5% solids. The estimates of the settling rates of the higher % solids are obtained from the tangents to the 5% sedimentation curve, intersecting the Y-axis at the mudline heights corresponding to 15, 25, 35 and 45% solids. Fig. 13.7 shows the measured sedimentation velocities versus the Kynch estimates from the slope of the tangents. The plot for the estimates from the 5% solids curve shows considerable difference from actual measured values being higher than the estimates according to the Kynch theory. If the estimates are constructed from the 15% solids curve for slurries of higher densities, Fig. 13.7 shows a closer correlation between the estimates and real sedimentation velocities. The estimates constructed from the 25% solids curve are similar to that obtained from the 15% solids curve.

Figs. 13.8 and 13.9 show similar constructions for a flocculated gold tailings at 20, 30 and 40% solids. In this case, the Kynch estimates of the settling velocities are in close agreement with the actual measured velocities.

Although the Kynch theory is not considered suitable for all mineral slurries, especially flocculated slurries, nevertheless it can give satisfactory results as indicated in Fig. 13.9. It is still used for thickener design calculations [8].

Talmage and Fitch [7] showed that the settling velocity was related to the concentration. For a point on the settling curve of time \( t \) and height \( H_t \), the equation is:

\[
C_t = \frac{C_0 H_0}{H_t + v_{\text{st}} t}
\]  

(13.7)
Fig. 13.6. Cu-Ni tailing sedimentation data replotted from Yalcin [9] with Kynch construction on the 5% solids curve.

Fig. 13.7. Kynch estimated sedimentation rates compared to measured rates for different % solid slurries, (data from [9]).
Fig. 13.8. Sedimentation curves of a flocculated gold tailing with Kynch construction on the 20% solids curve.

Fig. 13.9. Kynch estimated sedimentation rates compared to measured rates for a flocculated gold tailing.
In a batch settling test, the mass of solids in the test cylinder is given by \( C_0H_0A \). If the time taken for all the solids to settle past a layer of concentration \( C \) is \( t_U \) then \( C_0H_0A/t_U \) represents the quantity of solids that can be brought through the concentration layer per unit time. The area of thickener required to settle 1 tonne of solid per unit time is then given by:

\[
A = \frac{t_U}{C_0H_0} \text{ m}^2/\text{t/h} \quad (13.8)
\]

The time \( t_U \) is obtained by drawing a line from mudline height \( H \), corresponding to the concentration \( C \), at a tangent to the settling curve. The intersection of this tangent with the mudline corresponding to the underflow concentration is the value \( t_U \) on the time axis. This is illustrated in Fig. 13.10.

The maximum thickener area requirement will occur when the tangent is drawn through the compression point on the sedimentation curve since this tangent will give the highest value of \( t_U \) in the free settling range which, according to Talmage and Fitch [6] is the zone determining the unit area. When the line, corresponding to \( H_0 \), intersects the settling curve above the compression point, the value of \( t_U \) corresponding to the maximum thickener area will be the point of intersection with the settling curve, shown as \( t_U(1) \) in Fig. 13.10.

Fig. 13.5 shows that a near steady concentration is reached at about \( Y_{MAX} \). Assuming this to be an equilibrium state, a material balance of solid and liquid can be made. Svarovsky [10] expressed the area of the tank in terms of the overflow rate. From a material balance, the

---

**Fig. 13.10.** Talmage and Fitch construction for determination of \( t_U \); \( t_U(1) \) is the value where \( H_0 \) lies above the critical point and \( t_U(2) \) is the value where \( H_0 \) lies below the critical point.
ratio of the overflow rate to feed rate can be determined in terms of feed and underflow concentrations as:

\[
\frac{Q_{V(O)}}{Q_{V(F)}} = \frac{C_U - C_F}{C_U}
\]  

(13.9)

where \( Q_{V(O)}, Q_{V(F)} \) = volumetric overflow rate and feed rate, m\(^3\)/s, \( C_U, C_F \) = concentrations of the underflow and feed respectively, expressed as the mass of solid/volume of slurry, kg/m\(^3\).

The overflow rate can be easily measured providing the velocity of the underflow is measured and is constant. The overflow rate for the system is related to the liquid rise velocity by the equation:

\[
Q_{V(O)} = A v_{(O)}
\]  

(13.10)

where \( v_{(O)} \) = liquid rise velocity or overflow velocity.

Substituting this value in Eq. (13.9) and simplifying, the area of the tank may be expressed as:

\[
A = \frac{Q_{V(F)}}{v_0} \left[ \frac{C_U - C_F}{C_U} \right]
\]  

(13.11)

Eq. (13.11) gives the area of the cross-section of the tank at a known feed rate, known concentrations of feed and underflow and liquid rise velocity.

13.3.2. Determination of Critical Point

The Talmage and Fitch and other methods of thickener design require the determination of the critical point on the sedimentation curve. As the solids settle they pass from free settling to hindered settling to compression conditions. At each of these transitions there is a discontinuity in the sedimentation curve. In the free settling region, the settling rate is constant and representative of the initial solids concentration. When the solids concentration increases to the point where the near neighbours start to influence the settling rate of the particles, the settling rate slows and is affected by the concentration of nearest neighbours. The slurry is in a hindered settling condition and the decrease in settling rate is referred to as the first falling rate. The settling behaviour becomes non-linear, inversely proportional to the solids concentration. When the solid concentration increases to the extent where the solids touch, settling ceases and further consolidation of the solids occurs by compression. This further drop in sedimentation rate is referred to as the second falling rate and a second discontinuity in the settling curve will occur (Fig. 13.11). The end of hindered settling and the start of the compression zone is referred to as the compression or critical point. This discontinuity in the settling curve is not always readily discernable and some procedures have been suggested to try and locate the compression point on the settling curve.

These procedures try to replot the data to accentuate the discontinuity in the settling behaviour, making some assumption as to the shape of the curved sections:
1. replot on log-log axes. The upper and lower sections of the curve generally approximate straight lines which intersect at the critical point.

2. draw tangents to the sedimentation curve at both ends and bisect the angle formed. The line bisecting the angle often intersects the sedimentation curve close to the critical point. This however will change as the scale on the axes changes [11]. See Fig. 13.12.

3. The mudline height at the critical point, Hc, can be obtained from a plot of dH/dt versus time as indicated by Mondal and Majumdar [12], Fig. 13.13. Again, the change in slope at the critical point should be evident. Barnea [13] also plots the differential:

\[ \frac{H_{n-1} - H_{n+1}}{t_{n+1} - t_{n-1}} \text{ versus } \frac{H_{s} - H_{e}}{H_{0} - H_{e}} \]

where n is one data reading on the sedimentation curve. Hn is defined as the mean of Hn-1 and Hn+1. See Fig. 13.14.

4. Plot the distance log (H - Ho) versus time where Hn is the final (equilibrium) sedimentation height (at infinite time). If the curved sections of the sedimentation curve are represented by an inverse exponential function, then plotting the log of the height vs. linear time will give a straight line and a change in slope will occur at the sedimentation discontinuity [14]. See Fig. 13.15.

5. Dahlstrom and Fitch [1] assumes the start of the compression of the sediment takes place at a mudline height half-way between the initial height and the final height of the sediment; i.e., Hc = H0 - He. For the sedimentation curve in Fig. 13.11, this gives an estimate of the critical point at 56 s. This point appears to be the start of the hindered settling zone rather than the compression zone. This method and the bisected angle method are not recommended.

From the sedimentation curve given in Fig. 13.11, the critical point estimation by the various methods is given in Table 13.1

Table 13.1
Critical point estimates, tc, from the sedimentation data in Fig. 13.11 by various methods.

<table>
<thead>
<tr>
<th>Method</th>
<th>Hindered settling, s</th>
<th>Critical point, s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log-log plot</td>
<td>~58</td>
<td>~410</td>
</tr>
<tr>
<td>Bisected angle</td>
<td>-</td>
<td>145</td>
</tr>
<tr>
<td>Roberts</td>
<td>60</td>
<td>590</td>
</tr>
<tr>
<td>Mondal and Majumdar</td>
<td>80</td>
<td>580</td>
</tr>
<tr>
<td>Barnea</td>
<td>40</td>
<td>460</td>
</tr>
<tr>
<td>Dahlstrom and Fitch</td>
<td>-</td>
<td>56</td>
</tr>
</tbody>
</table>

The Roberts, Barnea and Mondal methods appear to give similar estimates. The log-log plot gives little deviation from a straight line and the critical point is less easily identified for this data. On the Barnea plot, for this data, it is also difficult to identify the critical point.
Fig. 13.11  Batch settling tests showing discontinuities at the transition from free settling to hindered settling and to compression.

Fig. 13.12. Rough location of the critical point by bisecting the angle formed by two tangents to the extremities of the sedimentation curve [11]. Critical point at 145 s.
Fig. 13.13. Plot of change in slope versus time [12]. Critical point at 580 s.

Fig. 13.14. Barnea (1977) plot where \((dH/dt)_n = \frac{H_{n-1} - H_{n+1}}{t_{n+1} - t_{n-1}}\) and \(dH_n = \frac{H_a - H_\infty}{H_o - H_\infty}\).
13.3.3. Determination of Settling Flux

Instead of using the concentrations of streams it is more convenient to express Eq. (13.10) in terms of the mass of sedimentation per unit area, known as the settling flux ($\psi$) and given by:

$$\psi = C_t \nu_s$$  \hspace{1cm} (13.12)

Substituting the value of $C_t$ from Eq. (13.6):

$$\psi = \frac{C_t H_o}{H_t} \cdot \nu_s$$ \hspace{1cm} (13.13)

Thus on the sedimentation curve, if tangents are drawn at several points, then the slopes and intercept with the $H$ axis gives the corresponding flux–concentration curve as shown in Fig. 13.16.

The settling flux curve can only be reconstructed from the sedimentation curve for the conditions where the gradient to the flux curve is decreasing with increasing concentration. That is, conditions which are found in the normal sedimentation test and are represented by concentrations higher than the point of inflection on the flux curve.

A non-graphical approach was proposed by Yalcin [9] and is dependant on a power law relationship between the slurry % solids and sedimentation time being established. For the 20% solids slurry in Fig. 13.8 the underflow % solids corresponding to each mudline height is plotted against time in Fig. 13.17. From Fig. 13.17:

$$%S = k t^n = 31.06 t^{0.1961}$$ \hspace{1cm} (13.14)
Fig. 13.16. Settling flux calculated from tangents to the 20% solids pulp curve in Fig. 13.8.

Fig. 13.17. Relationship between underflow % solids and sedimentation time for a gold tailing at 20% solids initial concentration.
At any mudline height, \( H \) and time \( t \) on the sedimentation curve (Fig. 13.5), the % solids (%S) corresponding to the slurry near the interface having a settling rate of \( v_s \) is given by the intercept of the tangent to the curve, \( H_t \). The % solids at \( H_t \) is given by [9]:

\[
\frac{1}{\%S_t} = \frac{1}{\%S_0} - \frac{(H_0 - H_t)p_w}{100C_0H_0}
\]

where \( \%S_t = \) % solids at time \( t \) and \( \%S_0 = \) initial % solids (at \( t = 0 \)).

Substituting for %S from Eq. (13.14) into Eq. (13.15) and rearranging gives:

\[
H = \frac{100H_0C_0}{\rho_wk} - n + H_0\left(1 - \frac{100C_0}{\%S_0\rho_w}\right)
\]

Then, differentiating Eq. (13.16) with respect to time:

\[
\frac{dH}{dt} = v_s = -\frac{100nH_0C_0}{\rho_w k} t^{-(n+1)}
\]

Thus for any sedimentation time, \( t_s \), the value of \( H \) is obtained from Eq. (13.16), \( v_s \) from Eq. (13.17) and \( H_t \) from Fig. 13.5:

\[
H_t = H + v_s t_s
\]

The % solids below \( H_t \) (%S_t) is then obtained from Eq. (13.15). Thus a series of \( v_s \) values can be obtained for different underflow % solids, which are related to the solids concentration by the equation:

\[
C = \frac{\%S}{\%S + \frac{100 - \%S}{\rho_s} \rho_w} \text{ kg/m}^3
\]

Points on the settling flux curve can thus be evaluated as shown in Fig. 13.16 for a gold tailing sample. Good agreement is found between the calculated flux and the graphically estimated flux values.

A second graphical method was advocated by Jernqvist [15,16] and described by Kelly and Spottiswood [17] and is based on the laboratory determination of batch settling in a tall cylinder. The time axis of the height-time curve is reversed and the following steps taken:

Step 1: The Y-axis is extended to form the Y-axis of the solid flux-concentrate curve.

Step 2: Draw a horizontal line to form the x-axis of the flux curve.

Step 3: Let the initial concentration of the slurry be \( C_0 \) located on the C axis. Through \( C_0 \) draw a vertical line.
Step 4: Through $H_O$ draw a horizontal line, indicating concentration (could be the C axis).

Step 5: Draw a tangent to the sedimentation curve, say $\alpha_1$. Through the origin of the flux curve draw line $OF$ parallel to the tangent $\alpha_1$.

Step 6: From the intersection of $\alpha_1$ with the y-axis, draw a horizontal line to cut the vertical line through $C_O$ at $H_1$. Join origin $O'$ to $H_1$ and proceed to cut the x-axis at $C_1$. Draw a vertical line through $C_1$ to cut the OF line at 1. This point of intersection is a point on the flux-concentration curve.

Step 7: Repeating steps 5 and 6, several points can be obtained which on connecting, provide the $\psi$ - concentrate curve.

The construction of the ($\psi$-C) is illustrated in Fig. 13.18 and compares well with other techniques. The scale used for the flux axis must be consistent with the dimensions of the other 3 axes. This procedure also assumes that the sediment is not compressible.

Fig. 13.18. Jernqvist method for construction of the flux curve from a sedimentation curve [15,16].
The thickener area is then obtained from the settling flux value according to the equation:

\[ A = \frac{Q_{M(t)}}{\Psi} \]  \hspace{1cm} (13.20)

For smooth operation of a thickener and to achieve the required properties of the product streams it is imperative to know:

1. the maximum allowable concentration of the underflow,
2. the optimum conditions of the overflow and the concentration of the feed slurry.

This information is derived from the flux-concentration and flux-time curves.

In continuous sedimentation process two forces are simultaneously in operation:

1. Sedimentation flux (\(\psi_S\)),
2. Withdrawal flux (\(\psi_W\)).

Thus the total flux is given by:

\[ \Psi_T = \psi_S + \psi_W \]  \hspace{1cm} (13.21)

The two flux-concentration curves and the total flux curve resulting from the combination of each set of data are shown in Fig. 13.19. The combined flux curve shows a minimum value at some critical concentration, \(C_{\text{CRIT}}\). The corresponding minimum flux, \(\psi_{\text{CRIT}}\), is the maximum that the thickener can handle. At a concentration less than \(C_{\text{CRIT}}\), solids enter the sludge layer faster than can leave via the underflow and hence the concentration, \(C\), increases up to \(C_{\text{CRIT}}\). At concentrations greater than \(C_{\text{CRIT}}\), solids leave the underflow faster than is entering sludge layer and hence \(C\) drops to \(C_{\text{CRIT}}\).

Coe and Clevenger [5] suggested that at this critical concentration, the flux of solids to the underflow of a continuous thickener would be a minimum and the critical flux, \(\psi_{\text{CRIT}}\), is the maximum flux that can flow through the thickener into the underflow at steady state. This critical flux is rate determining and will determine the thickener area for a given feed rate and underflow density, according to Eq. (13.20).

Yoshioka et al. [18] obtained the critical flux from the settling flux-concentration curve by constructing a tangent to the curve passing through the underflow concentration \(C_U\) on the \(x\)-axis (Fig. 13.19). The tangent is called the Operating Line and the intercept on the flux axis is the critical flux, and the thickener area is:

\[ A = \frac{Q_{M(t)}}{\psi_{\text{CRIT}}} = \frac{Q_{V(t)} C_O}{\psi_{\text{CRIT}}} \]  \hspace{1cm} (13.22)

It should be noted that if \(\psi_O > \psi_{\text{CRIT}}\) then the thickener is overloaded and corrective action is necessary.

Oltmann [19] suggested a simple empirical approach to the critical flux determination and hence thickener area. In cases where the settling rate at the beginning of the sedimentation test is non-linear due to turbulence resulting from mixing, an extrapolation of the linear settling rate section to the horizontal extension of \(H_0\) will give the start time \(t_a\) (Fig. 13.20).
Fig. 13.19. Flux curves for a continuous thickener [17].

Fig. 13.20. Oltmann construction to determine the critical settling flux.
A line drawn from the point \((t_a, H_0)\) through the critical point on the settling curve to intersect the underflow mudline \(H_u\) will determine the time \(t_u\). The detention time is then given by \(t_D = (t_u - t_a)\) and the critical flux is given by:

\[
\psi_{crit} = \frac{C_0 H_0}{t_D 1.2}
\]  

(13.23)

The factor 1.2 is used to give a 20% safety factor. The thickener area is then given by Eq. (13.22).

13.3.4. Long Tube Method for Estimating Thickener Dimensions

Some pulps containing fines or materials of colloidal dimension do not settle with a clear interface. Some do not tend to settle at all. For such suspensions the long tube method of determining the dimension of thickeners or clarifiers are useful. The test consists of determining the rate of rise of a fluid with different detention times. The rate of rise is related to the concentration of solids in the overflow.

The test is carried out in a long plastic or glass tube about 3-4 meters in length (height) and about 0.075 m in diameter. Along its length are a number of sampling points. The first sampling point is about 100 mm from the top of the tube with the remaining sampling points every 200-300 mm. The slurry mixed with an appropriate flocculant is charged and the top most level is established at the top most outlet by opening the value to allow any excess slurry to drain off. The pulp is allowed to settle for a time till a visibly clear supernatant liquid level is seen. Pulp samples are then rapidly taken from all the sampling points, starting from the top and the solid concentrations in each sample is determined. The procedure is repeated 4-5 times for different settling times depending on the expected operating conditions of the thickener/clarifier. The results are tabulated for cumulative depth (H) and solid concentration.

The test results indicate that the clarification zone clarity is a function of overflow rate \((v_0)\) and the detention time. The overflow rate is given by:

\[
v_0 = \frac{H}{t} = \frac{Q_{V(O)}}{A}
\]  

(13.24)

where

- \(H\) = the cumulative height (or depth),
- \(t\) = time,
- \(Q_{V(O)}\) = volumetric flowrate in the overflow and
- \(A\) = area.

The detention time is related to the feed rate, depth and area as:

\[
t_D = \frac{AH}{Q_{V(F)}}
\]  

(13.25)

where \(Q_{V(F)}\) = the volumetric feed rate.
The overflow rate determined by the long tube test is the ideal overflow rate, \( v_{O(i)} \), that is required for a certain feed concentration and therefore Eq. (13.24) can be rewritten as

\[
v_{O(i)} = \frac{H}{t}
\]  

(13.26)

In the operation of a thickener or clarifier, generally a volume overflow rate, \( Q_{V(O)} \), is specified (or chosen) for a particular feed solid concentration, \( C_F \). At this overflow a maximum solid concentration \( C_0 \) is tolerated. From the data it can be seen that:

The pool area, \( A = \frac{\text{overflow rate required}}{\text{ideal overflow}} = \frac{Q_{V(O)}}{v_{O(i)}} \)  

(13.27)

The pool volume \( V = \text{Overflow rate required} \times \text{time} = Q_{V(O)} \times t \), and

The pool depth \( H = \frac{\text{pool volume}}{\text{pool area}} = \frac{V}{A} \)  

(13.28)

(13.29)

The units for \( Q_{V(O)} \) is \( m^3/h \), \( v_{O(i)} \) is \( m/h \), \( t \) is \( h \) and the overflow solids concentration tolerable is in ppm.

In practice the overflow concentration is less than that obtained by the long tube experiments. To account for this discrepancy, Perry and Chilton [20] plot suspended solids concentration \( C \) against the rise rate and graphically integrate between \( C=0 \) and \( C=C_C \). The resulting value is then subtracted from the observed suspended solids concentration at the chosen rise rate.

Osborne [21] however suggests the use of a suspensoid factor, \( f(s) \), to correct the error. The corrected pool area would be given by:

The pool area \( A = \frac{Q_{V(O)}}{v_{O(i)}} \times \frac{1}{f(s)} \)  

(13.30)

where \( f(s) = 0.7 \).

13.3.5. Estimating Height (depth) of the Compression Layer
The approximate depth of the thickened sludge layer can be readily determined by the method outlined by Osborne [21]. The height, \( H \), of the layer would depend on the total volume of the solids and liquid in the compression zone and inversely as the area of the vessel. That is:

\[
H = \frac{V_c}{A}
\]  

(13.31)

where \( V_c \) = the total volume of the liquid and the solids in the compression layer.

\( V_c \) can be determined if the average concentration of solids in the layer is known. Hence, if the average concentration of solids in the compression zone is \( C_c \) (mass solid/volume of the
compression zone), then the mass of liquid in the zone per unit volume will be \((\rho_p - C_c)\) and the liquid/solid ratio in the compression zone will be \((\rho_p - C_c)/C_c\). The depth of the compression zone will depend on the amount of sludge deposited and that would depend on the retention time. The retention time is a function of the rate of discharge of the underflow and the underflow concentration. Thus if the feed rate is expressed as \(Q_{v(F)}\), the feed concentration as \(C_F\) and the retention time in the compression zone as \(t_D\), then the volume of solids plus the volume of liquid in the compression zone would be:

\[
V_c = \frac{Q_{v(F)} C_F t_D}{\rho_s} + \frac{Q_{v(F)} C_F t_D}{\rho_L} \left( \frac{\rho_p - C_c}{C_c} \right) \quad (13.32)
\]

Thus if \(A\) is the cross-sectional area of the thickener, then the height of the compression zone, \(H_c\), will be:

\[
H_c = \frac{Q_{v(F)} C_F t_D}{A} \left( 1 + \frac{\rho_s}{\rho_L} \left( \frac{\rho_p - C_c}{C_c} \right) \right) \quad (13.33)
\]

The units are: \(Q_{v(F)} = \text{m}^3/\text{s}\), \(t_D = \text{s}\), \(A = \text{m}^2\), \(C = \text{kg/m}^3\) and \(\rho = \text{kg/m}^3\).

**Dahlstrom Method**

Dahlstrom and Fitch [1] obtained the compression zone volume from the settling curve as shown in Fig. 13.21.

---

![Fig. 13.21. Settling curve showing Dahlstrom's construction for compression zone height [1].](image-url)
The settling test is carried out in a 2 L cylinder with a picket rake and run for 24 hours to obtain the final sediment height, $H_{oc}$. It is assumed that the start of the compression of the sediment occurs at the point B, which is located at the height $(H_0 - H_x)/2$ and time $t_1$. The height corresponding to the underflow solids concentration is given by $H_u$ which intersects the sedimentation curve at time $t_2$. The detention time, $t_d$ (residence time) in the compression zone required to achieve the desired underflow density is then given by $t_2 - t_1$.

The compression zone volume is calculated from the average solids concentration in the compression zone obtained by integrating the area under the curve from $t_1$ to $t_2$. Alternatively, a line bisecting the area under the curve from $t_1$ to $t_2$ will intersect the sedimentation curve at $B'$. The sediment height at this point can be converted to obtain the average concentration, $C_C$ in kg of solid/m$^3$ of pulp. The compression zone volume is then obtained from:

$$V_C = \frac{Q_{M(F)}}{C_C} t_d SF$$  \hspace{1cm}(13.34)$$

where $SF = a$ scale factor, normally taken as 1.75 and $Q_{M(F)} = the$ mass flowrate in the thickener feed.

The compression zone height is then obtained by dividing the compression volume by the thickener area. Empirically it is found that if the calculated compression height is greater than 1 meter then the thickener area should be increased or the thickener underflow density reduced to maintain a maximum compression height of 1 meter [1,22].

13.3.6. Estimating the Depth of the Clarifying Zone

For estimating the depth of the clarifying zone, a tall cylinder is again taken and filled with slurry. Sample points are inserted every 200 mm and the clear level recording started immediately and continued at regular intervals. If a clear level is not obtained, a flocculant is added and a height-concentration curve drawn. The overflow rate, $v_O$, is determined as [11]:

$$v_O = \frac{H_{of}}{t} = \frac{Q_{v(O)}}{A}$$

That is: $H_{of} = \frac{Q_{v(O)}}{A} t$  \hspace{1cm}(13.35)$$

where $v_O = overflow$ rate or overflow volume flux expressed as m/s and $H_{of} = height$ of the clarification zone.

13.3.7. Estimating the Retention Time

Dahlstrom and Fitch [1] described a procedure for determining the retention time of continuously operating thickeners. The overflow rate was related to the retention time by the relation:

$$t_d = \frac{AH_{of}}{Q_{v(O)} A_{EF}}$$  \hspace{1cm}(13.36)$$
where \( Q_{v(o)} \) = overflow rate, \( m^3/h \),
\( A \) = cross sectional area of the tank, \( m^2 \), and
\( A_{EF} \) = areal efficiency factor.

The overflow flow rate can be measured by determining the overflow velocity, \( v_o \), and using the relationship:

\[
v_o = \frac{Q_{v(o)}}{A_{EF}}
\]  

(13.37)

Dahlstrom and Fitch reported areal efficiency values between 0.1 and 0.6, depending on the height to diameter of the thickener and feed well, with typical values of 0.20 – 0.25. The overflow velocity must be less than the settling velocity of the smallest particle to be removed. This maximum velocity for thickeners is generally 0.00034 – 0.0020 m/s.

Examples 13.1 – 13.2 illustrate the use of the above concepts.

---

**Example 13.1**

The volume rate of flow of slurry from a dust catcher was 3.7 \( m^3/min \). The concentration of slurry (by mass) was 10% and the specific gravity of the solid is 2.75. The slurry is to be thickened to produce a sludge containing 47% minimum solids by mass in a continuous thickener. Settling tests on the sample gave the following data:

<table>
<thead>
<tr>
<th>Rate of settling, m/min</th>
<th>0.72</th>
<th>0.36</th>
<th>0.24</th>
<th>0.051</th>
<th>0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concentration, % solids by mass</td>
<td>10</td>
<td>15</td>
<td>25</td>
<td>35</td>
<td>45</td>
</tr>
</tbody>
</table>

Estimate the cross-sectional area that will separate 1000 tonnes of solids per hour.

**Solution**

Take as the basis, 1 kg solid and time in seconds.

Step 1
Solid to be separated/min = 1000 x 1000/60 = 16,667 kg = 277.8 kg/s
The underflow contains 47% solids and 53% water, hence the underflow water/solids ratio = 53/47 = 1.12.

Step 2
Estimate the water to underflow from the given data as shown in the table below.
The table shows that the maximum flowrate in the overflow stream, per unit sedimentation rate = 858.2 s/m.
According to the given conditions, the sludge contains 277.8 kg/s \((Q_{MF})\). Assuming a density of water = 1000 kg/m\(^3\), the thickener area, from Eq. (13.3) = 858.2 x 277.8/1000 = 238.4 m\(^2\).
Hence the diameter of the thickener = \( \left( \frac{238.4}{3.14} \right)^{0.5} \times 2 = 17.4 \text{ m.} \)

### Example 13.2

A slurry containing 300 kg solid per cubic meter of slurry is to be dewatered in a thickener such that the underflow will contain 750 kg/m\(^3\). The feed rate to the thickener was expected to be 0.5 m\(^3\)/min. A batch settling test of the slurry gave the following results:

<table>
<thead>
<tr>
<th>Solid concentration, C, kg/m(^3)</th>
<th>Settling velocity, (v_s), mm/min</th>
</tr>
</thead>
<tbody>
<tr>
<td>300.0</td>
<td>26.667</td>
</tr>
<tr>
<td>362.3</td>
<td>15.588</td>
</tr>
<tr>
<td>497.4</td>
<td>7.148</td>
</tr>
<tr>
<td>774.2</td>
<td>1.610</td>
</tr>
<tr>
<td>960.0</td>
<td>0.455</td>
</tr>
<tr>
<td>1010.5</td>
<td>0.271</td>
</tr>
<tr>
<td>1078.7</td>
<td>0.111</td>
</tr>
<tr>
<td>1128.1</td>
<td>0.068</td>
</tr>
</tbody>
</table>

### Solution

**Step 1**

Determine \( \psi \) from \( v_s \) and C values using the expression:

\[
\psi = v_s \cdot C \quad \text{kg/m}^2 \cdot \text{s}
\]

This can be determined for different velocities as illustrated in the table below:

<table>
<thead>
<tr>
<th>Settling velocity, (v_s), mm/min</th>
<th>Solid concentration, C, kg/m(^3)</th>
<th>Settling Flux, (\psi = v_s \cdot C), kg/m(^2) \cdot \text{s}</th>
</tr>
</thead>
</table>
Step 2
Plot settling velocity against solid concentration as in Fig. 13.22. Since the underflow has to be 750 kg/m³, draw a line, tangent to the curve and passing through 750 kg/m³ on the x-axis. This line cuts the y-axis at $\psi_{CRIT} = 0.17$ kg/m²/s.

Step 3
As a first approximation, using Eq. (13.22):

$$area \ A \ = \ (Q_{VP} \ C_O) / \psi_{CRIT} = 0.5 \times 300 / (0.17 \times 60) \ m^2$$

$$= 14.7 \ m^2$$

and    diameter = 4.3 m

Considering a safety factor of 1.5, the practical diameter = 6.5 m

In the discussions and computations explored in examples 13.1 and 13.2, the effect of different particle sizes (and possibly density) has not been considered. The velocity of descent of different sized particles will obviously be different. In such a case the sedimentation profile will consist of more than three zones (Fig. 13.23) due to the upward
Fig. 13.23. Sedimentation layers resulting from particles of different size and density

flow of the displaced liquid by the movement of the different size of particles. The lines of
demarcation between these zones are not well defined and flux determinations are difficult.

Due to such difficulties, adjustments to experimentally computed design parameters have
been published from time to time to yield realistic approximations of the different parameters
[1,20]. These modifications are summarised in Table 13.2.

Table 13.2
Multiplying factors for different thickener parameters [20].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Multiplying factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tank size</td>
<td>0.5 – 0.7 to rise rate</td>
</tr>
<tr>
<td>Sedimentation time</td>
<td>ratio of static detention time/volume</td>
</tr>
<tr>
<td>Cross-section area of tank</td>
<td>1.2 for diameter &gt; 30 m</td>
</tr>
<tr>
<td></td>
<td>1.5 for diameter &lt; 4.6 m</td>
</tr>
<tr>
<td></td>
<td>1.1-1.25 for safety</td>
</tr>
<tr>
<td>Transition zone depth</td>
<td>Add about 2 m</td>
</tr>
<tr>
<td>Compression zone depth</td>
<td>1.75</td>
</tr>
</tbody>
</table>

The estimated area of the tank is increased by multiplying by a factor of 1.2 for tank
diameters greater than 30 meters and a factor of 1.25 for tanks with estimated diameters less
than 5 meters [20].

Often the thickener area and depth are calculated by manufacturers from standard tables
established from a large number of field operations. However, no two circumstances are the
same and the following method adopted by Eimco [3] is of interest for rapid estimates and
may be accepted with reservations. The effective clarification area is obtained from:

\[
A_k = \frac{\text{Average daily flow rate}}{\text{Specified overflow rate}}
\]  

(13.38)
where \( A_E = \text{the effective clarification area} = \text{tank area} - \text{feedwell area} \).

The average daily flow rate is in gallons per day or \( m^3/h \) and the specified overflow rate is in gallons per day per \( ft^2 \) or \( m^3/h.m^2 \). The relation between the effective clarification area and the diameter of tank is given in Fig. 13.24.

To calculate the required area for a thickener, the recommended expression is:

\[
A = \frac{\text{Daily solid load in kilograms}}{\text{Floor Loading Rate}}
\]

(13.39)

where the solid load is in kg/day and the Floor Loading Rate is in kg/m\(^2\)/day, obtained from Table 13.3.

Table 13.3
Thickener floor loading [3].

<table>
<thead>
<tr>
<th>% Sludge in feed</th>
<th>Floor Loading kg/m(^2)/day</th>
<th>Typical % Solids in underflow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>107.51</td>
<td>10</td>
</tr>
<tr>
<td>25</td>
<td>73.30</td>
<td>6</td>
</tr>
<tr>
<td>35-50</td>
<td>48.87</td>
<td>5</td>
</tr>
<tr>
<td>75</td>
<td>29.32</td>
<td>3</td>
</tr>
<tr>
<td>100</td>
<td>19.55</td>
<td>2</td>
</tr>
</tbody>
</table>
13.4. Operation of thickeners
The operation of thickeners involves a delicate balance of the feed rate, the overflow rate and the underflow withdrawal rate and is dependant on the concentration of the feed, overflow and underflow streams.

The feed stream generally enters the feed well at a speed of about 15 m/min but this would depend on its characteristics, such as concentration (liquid/solid ratio), particle size, particle shape and viscosity. The characteristics of the overflow and underflow streams depend on the sedimentation time and particle properties like, specific gravity, shape, size and wettability. If the particles are very small, the associated surface charge or zeta-potential is of importance. Flocculants play an important role in affecting the surface charge on particles and help to accelerate or reduce the rate of sedimentation by dispersion or agglomeration.

Rakes help to increase the sedimentation rate and also break up large agglomerates. The rakes are operated between 8-18 m/min. To prevent damage to the rakes and torque meters the recommended operation is to discharge the sludge at regular intervals at predetermined set conditions. It is necessary for the operator to detect the build up on the rakes and operate to avoid the jamming and seizure of the rakes. Usually the built-up mud tends to form islands which grows and develops a moment that could easily damage the rake mechanism. During normal operation the rise rate varies from about 0.01–0.03 m/min/m of cross-sectional area and the detention time is between 2–5 hours.

Some common operating parameters and cross-section of tank sizes for selected metallurgical operations are given in Table 13.4.

<table>
<thead>
<tr>
<th>Material</th>
<th>Feed % solids</th>
<th>Underflow % solids</th>
<th>Area m²/tonne/day</th>
<th>Overflow rate m³/h/m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper concentrate</td>
<td>14-50</td>
<td>40-75</td>
<td>0.2-2.0</td>
<td>-</td>
</tr>
<tr>
<td>Iron ore (concentrate, coarse)</td>
<td>25-40</td>
<td>60-75</td>
<td>0.02-0.1</td>
<td>-</td>
</tr>
<tr>
<td>Iron ore (concentrate, fine)</td>
<td>15-30</td>
<td>60-70</td>
<td>0.15-0.4</td>
<td>-</td>
</tr>
<tr>
<td>Lead concentrate</td>
<td>20-25</td>
<td>60-80</td>
<td>0.5-1.0</td>
<td>-</td>
</tr>
<tr>
<td>Nickel carbonate ore (acid leach residue)</td>
<td>15-25</td>
<td>45-60</td>
<td>0.3-0.5</td>
<td>-</td>
</tr>
<tr>
<td>Uranium (acid leach residue)</td>
<td>10-30</td>
<td>25-65</td>
<td>0.02-1.0</td>
<td>-</td>
</tr>
<tr>
<td>Iron making blast furnace flue dust</td>
<td>0.2-2.0</td>
<td>40-60</td>
<td>-</td>
<td>1.5-3.7</td>
</tr>
<tr>
<td>Steel making BOF flue dust</td>
<td>0.2-2.0</td>
<td>30-70</td>
<td>1.0-3.7</td>
<td>-</td>
</tr>
</tbody>
</table>

13.5. Thickeners in Circuits
Thickeners used to produce low solid overflows (eg. about 1% solids), may be referred to as clarifiers. Both thickeners and clarifiers are extensively used in metallurgical operations for dewatering purposes. In processing gold, nickel, iron, copper ores etc. thickeners are used to produce overflows suitable for use as process water in circuits such as flotation. The clear overflow water is used for re-pulping the flue dusts or fine dust from precipitators. Therefore the feed to thickeners vary considerably. A common arrangement is illustrated in Fig. 13.25.
Thickeners serve as classifiers when a near clear overflow is required. For example, clarifiers used in iron blast furnace dust cleaning plant or electrostatic precipitator circuits are required to produce clean overflows as the water is for reuse and the sludge is for secondary use. In such cases the sludge is washed continuously by counter current decantation, where the underflow from a thickener/clarifier is pumped to the next thickener/clarifier (connected in series) forming the feed to the second tank. A typical set up is illustrated in Fig. 13.13 consisting of three units of thickeners/clarifiers.

Such setups are structured so that the overflow from one clarifier/thickener flows by gravity to the adjacent clarifier. The sludge is usually pumped to the next clarifier. Make up water is added at the third thickener.

Fig. 13.25. Thickener arrangement.

Fig. 13.26. Thickeners in a counter-current decantation (CCD) arrangement.
13.6. Problems

13.1.
Settling tests in a cylindrical tube were performed on a slurry containing 300 ppm solids. After a detention time of 80 minutes, the overflow fluid was found to contain 10 ppm solids. The overflow from the test data was found to be 8.2 m/h. The classifier was required to achieve an overflow rate of 120 m$^3$/h. Estimate:

1. The pool volume,
2. Pool area,
3. Pool depth,
4. Pool diameter.

13.2.
Laboratory tests on a sample of slurry showed the heights of the clear interface with time as:

<table>
<thead>
<tr>
<th>Height, H, mm</th>
<th>600</th>
<th>516</th>
<th>434</th>
<th>285</th>
<th>176</th>
<th>147</th>
<th>128</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time, t, sec</td>
<td>0</td>
<td>100</td>
<td>200</td>
<td>400</td>
<td>650</td>
<td>800</td>
<td>1000</td>
</tr>
</tbody>
</table>

The slurry containing 15% solids (by volume) was required to feed a continuous thickener to produce an overflow containing no more than 1% solids (by volume). Specific gravity of solids was 2.65 and water 1.0. If the feed rate is 75 t/h and the desired underflow density is 75% solids by mass, determine:

1. The settling velocity at each time interval,
2. The concentration of solids corresponding to each settling velocity,
3. The flux-concentration curve and
4. The area of the thickener.

13.3.
Using the data of problem 13.6.2, determine:

1. The volume of sludge in the underflow and hence the compression zone height
2. The height of clarification zone.

13.4.
A batch settling test on a flotation tailing gave the following results.

<table>
<thead>
<tr>
<th>time (min)</th>
<th>mud height (mm)</th>
<th>time (min)</th>
<th>mud height (mm)</th>
<th>time (min)</th>
<th>mud height (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>340</td>
<td>8</td>
<td>140</td>
<td>25</td>
<td>63</td>
</tr>
<tr>
<td>1</td>
<td>290</td>
<td>9</td>
<td>125</td>
<td>30</td>
<td>60</td>
</tr>
<tr>
<td>3</td>
<td>236</td>
<td>10</td>
<td>120</td>
<td>40</td>
<td>58</td>
</tr>
<tr>
<td>5</td>
<td>189</td>
<td>11</td>
<td>107</td>
<td>50</td>
<td>55</td>
</tr>
<tr>
<td>6</td>
<td>175</td>
<td>15</td>
<td>81</td>
<td>60</td>
<td>55</td>
</tr>
<tr>
<td>7</td>
<td>150</td>
<td>20</td>
<td>68</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
13.5.
A settling curve of a 15% solids (by mass) copper concentrate pulp is shown in the graph below. Estimate the thickener diameter required to dewater this material to an underflow of 55% solids (by mass) at a rate of 1000 t/h. Density of the solid is 4100 kg/m$^3$ and water is 1000 kg/m$^3$.

13.6.
A flocculated pulp settles according to the settling curve in the graph below. From these results, and a desired 1000 kL of clarified process water per hour from a feed slurry of 300 tph at 20% solids;

1. locate the critical point on the plot,
2. What is the initial concentration of solids ($C_o$) and the underflow concentration ($C_u$) in kg solid/m$^3$ of pulp,
3. estimate the size of thickener required using the method of Talmage and Fitch.

Data: Solids density = 2800 kg/m$^3$  water density = 1000 kg/m$^3$
A slurry of 20% solids (by mass) is to be dewatered to produce a product of 8% moisture at 75 tph. A settling test is carried out on the slurry. The critical point of the settling curve occurs at a mudline height of 80 mm and 250 seconds. The initial mudline height in the test cylinder is 300 mm. Solid density is 2500 kg/m$^3$ and the water density is 1000 kg/m$^3$.

1. If the mudline height corresponding to the thickener discharge is 70 mm, what would be the thickener discharge % solids?

2. What method could be used to calculate the thickener area requirement for this slurry? Calculate the thickener diameter using this method.

A slurry of 20% solids (by mass) is to be dewatered to produce a product of 50% solids (by mass) at 75 tph (solid). A settling test is carried out on the slurry at 20%, 30% and 40% solids. The initial settling rates of the slurries are recorded below. Calculate the thickener diameter requirement for this slurry.

<table>
<thead>
<tr>
<th>Slurry</th>
<th>R (mm/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20%</td>
<td>0.7796</td>
</tr>
<tr>
<td>30%</td>
<td>0.0780</td>
</tr>
<tr>
<td>40%</td>
<td>0.0242</td>
</tr>
</tbody>
</table>
According to the Kynch theory, the settling velocity of a slurry of a concentration given by the settling interface, is given by the slope of a tangent to the settling curve. If the slope to the settling curve of a flocculated copper flotation tail is 0.4 mm/s at the critical point which occurs at a point (67s, 115mm) on the settling curve, calculate the thickener area requirement to treat this slurry if the desired underflow is 65% solids (mass), the feed density is 22% solids (mass) and the initial mudline height in the settling test is 250 mm.

**Throughput**  =  150 tph  
**Density of solid** = 2750 kg/m³  
**Density of water** = 1000 kg/m³

REFERENCES

Chapter 14. Solid Liquid Separation - Filtration

14. INTRODUCTION

The separation of solids from liquid by gravity can be easily done by batch or continuous sedimentation processes. The underflow, however still contains appreciable amounts of liquid and the overflow contains some amount of solids. Further removal of liquid is necessary for some downstream operations. The removal of this liquid is usually possible by passing the suspension through a semi-permeable membrane which is designed to hold the solids and permit the liquid to pass through. In effect the membrane forms a screen. In the early stages of separation across this membrane the solids deposit forming a second semi-permeable medium or cake. These two layers then form the filtering medium for the remainder of the slurry. The structure of the filtering cake changes continuously as more particles deposit with time. The main changes relate to permeability and porosity of the filtering zone. The permeability of the cake depends on the particle size, shape, thickness (depth) of solids and on the liquid properties, such as viscosity. The filtration rate is affected by differential pressure that is applied on the membrane to improve performance. Once a thick cake is formed the permeability decreases to the extent that the process is stopped. Filtration can continue by changing both the membrane and removing the deposited solids. The process of filtration is therefore essentially batch or continuous. Fig. 14.1A shows the function of a typical filtering medium. Fig.14.1B is an enlargement of the semi-permeable medium. The figures show the mechanism of filtration where particles larger than the pore size are held back while the fluid passes through. Particles smaller than the pore space are also liable to pass through, but small particles existing away from the membrane surface may not be separated unless brought in contact with the membrane surface. Once the cake begins to build up, further filtration is continued through the deposited layer of solids as well as the medium. Therefore the permeability of both the filter cake and the medium is of paramount importance.

Most filter cakes can be compressed to varying degrees by pressure. In some cases, like a siliceous cake, limited packing can be achieved but in others like clayey deposits, compressibility may be high and application of pressure may result in appreciable reduction of permeability of the entire bed. The process of filtration is predominantly carried out at conditions of either constant pressure or constant volume flow rate.

The filtering process is completed when nearly all the liquid has been removed from the pulp and the filter cake is removed from the filtering medium. Before removing the cake, it can be washed to remove the adhering fluid, the fluid that is retained in the pore spaces in the cake and any solute in the feed that is entrapped within the cake.

The structure of the supporting base of the filtering medium is a guide to the nomenclature of filters in industry. Thus when the filtering medium is between grooved plates the filter press is known as plate filters, when it is in the form of disc they are known as disc filter, when in the form of a drum or continuous belt they are known as drum filters and horizontal belt filters. The method of application of pressure also contributes to the nomenclature, thus industrially they are known as pressure or vacuum filters. Several combinations of these options are practiced including constant rate or constant pressure filtration.
14.1. Design Features of Filters

The semi-permeable membrane that permits the passage of liquids and prevents the solids to permeate is one of the main components of filters. The membrane consists of a large number of capillaries which forms tortuous channels most of which are continuous. With the assumption that the passage of fluid is streamlined, it is reasonable to assume that Poiseuille’s law of fluid flow through capillaries is applicable to both the medium and the filter cake. Poiseuille’s law states that the rate of filtration per unit area of the filter bed equals the ratio of the driving force to the product of the total resistive forces and the viscosity of the fluid. The total resistive force, $R_T$, is given by the sum of the resistive forces of the medium, $R_M$, and of the cake, $R_C$. That is:

$$R_T = R_C + R_M$$  \hspace{1cm} (14.1)

Providing the structure of the cake does not change with pressure, that is, it cannot be compressed, the resistance of the cake will be proportional to the mass of dry cake deposited during filtration. Thus the resistance of the cake is given by:

$$R_C = \frac{\alpha M_C}{A}$$  \hspace{1cm} (14.2)

where $M_C = \text{mass of dry cake, kg}$, $A = \text{area, m}^2$, $\alpha = \text{specific cake resistance (resistance per unit mass per unit area), m/kg.}$
If \( V \) is the volume of filtrate produced in time \( t \) by the application of a differential pressure \( \Delta P \), then according to Poiseuille’s law:

\[
Q_v = \frac{dV}{dt} = \frac{A \Delta P}{\mu R_T} = \frac{A \Delta P}{\mu [R_C + R_M]} = \frac{A \Delta P}{\mu [\frac{M_C}{A} \alpha + R_M]}
\]

where \( Q_v \) is the volume flow rate.

This is the fundamental equation on which the process of filtration is based. This equation assumes that the flow through the capillaries in the porous medium is streamlined and that \( R_M \) is constant, which in practice, may not always be true. Further the resistance of the cake is assumed to be uniform and constant. This also is not always true, particularly for cakes that are compressible. However, \( R_C \) is a function of porosity, \( \varepsilon \), diameter of the pores \( d_p \), and the specific surface area of the particles, \( S \), forming the cake.

The specific surface area of the particles is equal to \( 6/d_p \), where \( d_p \) is the diameter of the particles. The porosity, \( \varepsilon \), is defined as the ratio of the void volume to the total bed volume. That is:

\[
\varepsilon = \frac{\text{Void volume}}{\text{Total volume of bed}}
\]

Working with porous media, Darcy [1] established that the pressure drop, \( \Delta P \), of a fluid flowing through a porous medium was proportional to the thickness of the bed, \( L \), volume rate of flow, \( Q \), viscosity of the fluid, \( \mu \), and inversely proportional to the cross-sectional area, \( A \). That is:

\[
\Delta P = \frac{LQ \mu}{K A}
\]

where \( K \) is the proportionality constant and is defined as the permeability of the porous medium.

The permeability is related to the specific cake resistance by the following expression, derived from the comparison of Eqs. (14.3) and (14.5). That is:

\[
K = \frac{1}{\alpha (1-\varepsilon) \rho_s}
\]

The rate of flow of fluid in time \( t \) can be obtained by rearranging Eq. (14.5):

\[
Q = \frac{dV}{dt} = \frac{\Delta P K A}{L \mu}
\]
The flow of fluid through the pores is subjected to frictional resistance and therefore cannot be compared directly with flow through the pores in a smooth pipe where the friction is low. Thus the flow rate through a porous medium is considered as a superficial velocity.

Eqs. (14.3) and (14.7) are the basis of the mathematical models that describe the filtration process under different operating conditions.

Examination of these equations indicates that they can be integrated in terms of constant pressure and constant volume. We shall see later in this chapter how the equations help to understand the filtration processes under different methods of operation.

In industry the entire filtering process consists of a cycle of four main steps:

1. Filtering,
2. Washing of cake,
3. Drying of cake and,

The designs of filters therefore depend on the filtering process and the cycle adopted. Roughly the different processes used in metallurgical operations may be summarized as shown in Fig. 14.2.

---

**Fig. 14.2. Classification of filtration units.**

14.1.1. *Batch Processes of Filtration*

Two types of batch filters are commonly used in the metallurgical industry. The basis of designs of these filters depend whether gravity forces or external forces are applied to achieve the separation.

**Gravity Filters**

Gravity filters consist of a circular or rectangular vessels with a semi-permeable membrane forming the base. The membrane is usually laid horizontally. A receptacle is placed below the
membrane to receive the filtrate. The membrane forms the medium of filtration. The feed, in the form of slurry, is charged above the medium and allowed to stand. The liquid component of the slurry is forced to permeate through the membrane by gravitational force and the hydrostatic head of the fluid.

As filtration proceeds the solid deposit builds up with time and the filtrate collected. The filters are operated till the rate of filtration diminishes appreciably. The assembly is then dismantled and the filtrate and deposited cake removed. Usually scrapers are used to dislodge the cake. If the cake is sticky they could be dried before using the scraper. The filtering medium is then replaced, and the operation repeated.

These filters are essentially slow operating and seldom used for metallurgical operation. They are however, used extensively in small scale laboratory work for metallurgical purposes. On an industrial scale, gravity filters are often used for water purification operations.

**Plate and Frame Pressure Filter**

The simplest batch pressure filter is the plate filter where the slurry is placed between two vertical plates clamped together by an externally operated screw system or hydraulic ram (Fig. 14.3). A series of hollow frames separate the plates which are placed side by side and hung from two parallel rails on either side of the plates. The filtering medium is placed against the sides of the plates and the slurry is pumped between them. The slurry pressure presses the pulp against the medium forcing the liquid through the cloth and leaving the solids as a cake, on both surfaces of the frame.

The plates are usually square shaped with ribbed or studded surfaces. Circular plates are also available in industry. The size of plates varies from about 450 x 450 mm to 2000 x 2000 mm and frames from 10 mm to 202 mm in thickness. They are usually made of steel to withstand pressures in excess of 1800 kPa.

The usual number of plates in commercial practice varies between 25-50 but up to 100 plate filters are reported [2].

![Fig. 14.3. Section of a plate and frame filter press.](image)
In operation, feed in the form of slurry is pumped in through a common channel entering the filters through individual ports. This ensures uniform distribution of feed in each chamber. The feed can be charged either through top or bottom ports in the frame. The filtrate leaves the individual filter beds through ports to a common discharge channel. In some installations the discharge from individual filters can be controlled. This introduces the flexibility of accepting or rejecting the product from a particular plate which can be suspected to be faulty and probably discharging unclear or turbid filtrate into the main product stream. Production of turbid and unclear filtrate often occurs due to a tear or bursting of the filter medium and needs to be isolated.

The normal and often used medium is woven cotton or plastics which permits filtering rates ranging between 0.1 and 0.6 m$^3$/h/m$^2$. Non-woven plastics with various apertures are also used.

For washing the cake the same hook-up for feed and discharge pipes are used to supply the wash water and to discharge the effluent. The cake is recovered by dismantling the entire unit. Steam is used in some cases to assist in drying the filter cake. The Larox RT Filters have provision for multistage washing, vibration and hot air drying of cakes [3].

Merrill Filters working on these principles have been used in the gold industry for filtering gold cyanide solutions and zinc dust precipitates.

**Chamber Filters**

The Chamber filters are improved plate and frame filters (Fig. 14.4). These filters use recessed plates which when clamped together form chambers. The recess can be up to 25 mm. By recessing the plate, it forms its own frame and permits a thicker cake than the plate-and-frame filters. The feed usually enters through a central port in the plate. The filtrate escapes through a manifold at the top. The other features of the filter plate are essentially similar to the plate and frame filters.

![Sketch of a Chamber Press filter.](image-url)
Chamber filters are usually designed to operate with a maximum of 153 plates with surface areas varying between 0.2 to 2.6 m² per chamber [4]. The plates are connected to water lines for washing and to steam line for hot drying of the deposited cake. The cake is released from the medium by reversing the clamping device which is hydraulic or mechanical. Recessed plate filters are preferred where the cake is not very permeable, e.g., cakes produced on filtering slurries with excessive fine clays, or metallurgical slurries like in iron and alumina industries where the hydroxides have to be filtered.

While designing plate filters it is important to remember that the entire filter surface is not effective. For example, the available filtering surface of a filter of size 1450 mm x 1450 mm will be about 12% less than the maximum 2.10 m². For smaller filter sizes of say 250 mm x 250 mm, the filtering surface could be less by about 30% [5]. A rough relationship indicating the availability of filtering surface for different sizes of filters is illustrated in Fig. 14.5.

**Leaf Filter**

These filters, originally known as Dorr-Oliver Kelly filters [6], were in use in Australian gold operations but are now seldom used in this industry. The batch pressure leaf filters consists of a number of leafs, mostly rectangular in shape, but they can also be circular. The leaves are grooved plates over which filtering medium made of knitted cloth like, hessian, canvas, woollen sheets or synthetic polymer material is fitted. Both sides of the plate serves as filtering surfaces. A number of such leaves are supported on one or more common rails and placed inside a tank that can be closed (Fig. 14.6). The spacing between leaves usually vary depending on the cake thickness, but 30 mm to 152 mm is common.

Filters are designed to take several leaves. For example, a 914 mm diameter filter can take between 6-13 filters while a 1828 mm diameter filter can take 20 leaves. Filtering areas of 100 - 300 m² are common operating at pressures of up to 600 kPa [7].

![Fig. 14.5. Effective filter area loss for a plate filter; data from Svarovsky [5].](image)
On locking and sealing the chamber, slurry is pumped in through a common main under the tank and hydrostatic pressure applied. Filters are operated either at constant pressure or constant volume rate. When filtering at constant pressure, filtration is stopped when the filtrate flow is low or negligible. When filtering at constant flow rate, the pressure drops, and the filtration rate falls indicating the end of the operation. When the filtration is carried out under vacuum, it is applied through the discharge manifold. After filtration the leaves are sometimes removed to a second tank for washing and subsequently for drying, if necessary. Several manufacturers have patented methods for removing and washing the cake. For example, Dorr-Oliver (Sweetland) filters are designed to open the bottom of the tank and the cake is discharged by blowing compressed air from inside. In Niagara filters, the entire nest of leaves, riding on rails, are withdrawn out of the pressure tank, air pressure applied, the cake blown out and dried. Once dried, they are blown off the medium by air. When a wet cake is required, the cake is subjected to a high pressure water hose and scraped off. Most filter leaves in pressure units are stationary, but the circular filter leaves in Vallez filters are designed to rotate. The rotation of the leaves yields a more uniform thickness of cake. Solid-liquid pressure filter are also manufactured with the horizontal plates. In these, the horizontal plates are stacked one below the other. Filtration takes place on one side of the filter plate. These filters are mostly used in chemical and pharmaceutical industries [8,9]. They are not used much in the mineral industry.
14.1.2 Continuous Vacuum Filtration

Types of continuous vacuum filters common in metallurgical operations are the *rotating drum filter*, *rotating disc filter* and *belt filter*.

**Rotating Drum Filter**

Rotating drum continuous filters consists of a horizontal drum with its bottom one-third section immersed in a tank of slurry that has to be filtered. The drum shell is perforated and covered with shallow compartments which serve as a drainage grid about 22 mm in depth. The grid is covered with metal gauze which in turn is covered with the filtering cloth. The ends of the drum are either open or are closed with a spider through which the trunnion passes (Fig. 14.7). Each sector of the drum is connected from inside to a centrally located complex valve system. The valve has ports connected to vacuum, compressed air and water lines. Two of these rotate with the drum while the others are stationary. The valve acts in a manner such that the one third portion of the drum that is immersed in the slurry is under vacuum. The adjacent half of the drum is also under vacuum, but could be switched to dry air pressure. The remaining portion of the drum is under positive pressure which helps to dislodge the cake from the drum surface.

In the first stage of the filtering cycle the filtrate is drawn into the drum leaving a cake of solids adhering to the medium surface. When the drum continues to rotate, the cake in the first segment emerges from the slurry and is exposed to the atmosphere. It can then be washed under vacuum to rinse the adhering solids. The drum then enters the drying section where the cake is dried by drawing air through it. On further rotation the drum enters the final zone where the cake is blown out using reversed air pressure and discharged.

Several methods of discharging the cake have been adopted. The most common is to place a knife or scraper against the cake along the entire width of the drum. Other devices include:

![Fig. 14.7. Sketch of a Rotating Drum Filter.](image)
1. Continuous string discharge,
2. Continuous belt discharge,
3. Roller discharge.

The principle of designing and operating the string or belt arrangement to dislodge the cake is the same. Strings are placed parallel to each other 8-10 mm apart and wrapped over the drum surface. Sedimentation of cake takes place on and above the strings during filtration. Fig. 14.8 shows a string or belt passing over two auxiliary rolls at the end of the cycle. When the vacuum is released at the end of the second sector, the strings help to lift the cake off the drum surface which is then discharged. As the drum continues to rotate further, the strings pass between the auxiliary rolls and are washed and returned to the drum. The strings are commonly made of synthetic material like polyester.

In the roller discharge type (Fig. 14.9), the scraper roll rotates in the opposite direction to the drum. The speed of the scraper roll is 5-10% greater than the speed of the rotating drum. The scraper roll is placed against the cake at a suitable distance to slightly compact and peel off the cake from the filtering cloth surface. The surface of the scraper roll is capped with a rubber material. A scraper knife keeps the scraper roll surface clean. This technique of cleaning the drum is particularly suited to sticky clayey cakes, like that of bentonite or kaolinite clay. They are also suitable for highly alkaline red-mud slurry produced in the alumina industry’s Bayer process.

A variation of the drum filter design is made for slurries which are unstable and settle rapidly. In such cases, the feed is on the top-end of the drum and as filtration is rapid, arrangements are made to press the filter cake to de-water it. The vacuum system aids the process. The principle is illustrated in Fig. 14.10.

**Rotating Disc Filter**

The basic design characteristics of the rotating disc filter, like filtering under vacuum, washing the cake under vacuum and removing the cake by blowing the cake off the filter is the same as in the drum filter. Instead of one drum, a number of discs are placed in parallel.

![Rotating Disc Filter](image-url)

Fig. 14.8. Sketch of a continuous string discharge drum filter.
Fig. 14.9. Sketch of a continuous roller discharge drum filter.

Fig. 14.10. Schematic diagram of top charged drum filter.
The lower end of each disc is attached to a common horizontal pipe which passes through the centre of all the discs in the unit. The central pipe is designed to form the trunnion of the unit and serves as a conduit for the vacuum and pressure lines. The distance between the filters are fixed and this space is used to collect the cake off the filter surface. Each disk is designed to operate separately with its own slurry tank, thus more than one type of pulp can be filtered simultaneously if required. Fig. 14.11 shows a sketch of a disc filter unit.

The largest diameter of discs are about 5.6 m and the smallest available are of laboratory size. The disks have a number of sectors, usually 8–30 [2]. Frames of each sector are covered by a filtering medium in the form of a bag. The bags are made of strong fabric cotton (twill), or plastics to withstand a differential pressures of 0.6–0.8 atmospheres (60–80 kPa). The frames are constructed of either wood, synthetic material (e.g. polypropylene), fibreglass or stainless steel. Between 1–15 discs normally constitute a filtering unit. The filtering medium is chosen to provide porous cakes about 6.5–65 mm thick which translates to a deposition rate of 1.7–12 kg/m²/min depending on the specific gravity of the mineral. The permeability of the medium normally allows a filtering rate of 0.5–3.5 L/min. For disengaging the cake off the filter surface, the air pressure employed is about 20–250 kPa [10].

Manufacturers of disc filters offer options to the number of discs in a given length of unit. That is, the number of discs in a 1.8 m diameter filter could vary from 1 to 10, offering a filtration area of 4.3 m² per disc while for a 3.3 m diameter disc the number of discs could vary between 7 and 13 with a filtering area of 16.7 m² per disk (Fig. 14.12).

**Ceramic Disc Filters**

The Ceramec® filter is a unique rotary disc filter which uses a sintered alumina disc to dewater a slurry under low vacuum. The dewatering occurs by drawing water from the slurry by capillary action. This ensures that no air or particles are drawn into the filter medium to cause blockage. Fig. 14.13 shows a cross-section of the ceramic disc.
Fig. 14.12. Nominal filtration area available per disc [6].

Fig. 14.13. Cross-section of a 24 mm sintered alumina filtration disc.

The low vacuum used in the filter removes the filtrate from the internal passages of the discs while the small pressure differential across the disc causes cake formation. A reduction of up to 90% in energy consumption is possible.

**Horizontal Belt Vacuum Filter**

Flat horizontal belt and Pan filters have been designed for fast settling and fast filtering slurries like iron ore concentrates. The *Pan tilting* vacuum filters are gradually getting out of use and therefore are not considered here. The horizontal filters are in the form of a...
A continuous belt made of stainless or alloy steel and a medium in the form of a fabric network. The feed box is at one end of the belt which evenly spreads the slurry across the belt. The steel base of the travelling belt is covered by a rubber lining. The belt is stretched over two pulleys (Fig. 14.14). It is grooved so that the grooves are at right angles to the direction of movement. Between the pulleys the belt is flat and rectangular. The width of the belt of industrial units is usually 1–4 meter with a filtering area of up to 120 m² for a 4 m x 30 m belt. Under the belt and between the pulleys is a vacuum box. The vacuum box has compartments that are adjustable along the length. Filtration takes place in the first compartment under vacuum and the filtrate is withdrawn from the bottom. In the second compartment the cake is washed under vacuum by co-current and counter current recycled wash water. Fresh make-up water is added at the last section. The wash waters are withdrawn also from the bottom under vacuum and the washed cake then dewatered and dried. Receivers for filtrate and wash waters are positioned under each section of the vacuum box.

The belt speed is regulated usually between 5–100 mm per second. Cake thickness varies from 6–203 mm depending on the belt speed.

14.1.3. Design Rating of Filters.
Ratings of filters are based on the pore size of the medium. The chosen pore size would have to be smaller than the smallest particles in the pulp. The rating therefore indicates the minimum particle size that can be retained on the filter surface. The rating, Rₓ, is therefore defined as the ratio of the number of particles larger than the pore size of the medium and is given by:

\[
R_x = \frac{\text{No. of particles greater than } d_x}{\text{unit volume of feed}} / \frac{\text{unit volume of tails}}{\text{unit volume of tails}}
\]

where \( d_x \) = the size of the pores.
Thus an \( R_{10} \) value of 100 means that the filter is capable of retaining 99 out of 100 (99\%) of all particles greater than 10 \( \mu \)m. Again \( R_{200} > 200 \) means an efficiency in excess of 199/200 or 99.5\% relative to a particle size of 200 \( \mu \)m. The higher the \( R_X \) value the greater the amount of coarse particles retained on the filter medium.

In terms of \( R_X \) the efficiency \( E \) of separation by a filter is given by:

\[
E = \left( \frac{R_X - 1}{R_X} \right) \times 100
\]

where \( E \) is expressed in terms of per cent. The relation between \( R_X \) and \( E \) is illustrated in Fig. 14.15 [11].

### 14.2. Operation of Filters

The filtering process can be divided into two main operations which form a cycle:

1. Solid-liquid separation yielding solid cake and filtrate as products,
2. Treatment of the cake by dewatering, washing and drying.

In a batch process, once a cycle is completed, the assembly is dismantled, cleaned and re-assembled for the next cycle. The cycles are repeated until the entire volume of slurry has been filtered. The time taken for dismantling and re-assembly of the filter affects the total time of a filtering cycle.

![Fig. 14.15. Efficiency of filtration as a function of \( R_X \) [11].](image)
In the continuous process, this loss of time is almost minimal. The combined permeability of the medium plus the cake is the rate determining factor of the process. The permeability of the medium can be considered to be constant (unless the pores are clogged by particles smaller than the pores in the medium). The permeability of the cake may not be constant but would depend on the changing structure of the deposited layer which in turn depends on the concentration of particles, particle size, particle shape, particle size distribution in the feed slurry, porosity and thickness of the layer. The permeability of the cake is also affected by the pressure applied. This is specially true for soft compressible cakes.

Once the solid-liquid separation process ceases, the cake can be washed and dried. The final water (moisture) content in the cake is regulated by passing dry (cold or hot) air or gas through the cake. The time taken to wash and de-water the cake affects the time cycle and therefore the economics of the operation.

Experience has shown that dewatering of a cake is a complex phenomenon and there is always a residual water saturation remaining in a cake which cannot be removed easily by the application of pressure and prolonged air flow. Dahlstrom [2,12] described the minimum saturation as the _achievable moisture_ and ascribed it as a function of:

1. time of de-watering,
2. volume of air/gas through the cake,
3. pressure differential per mass of dry solids per unit area per cycle,
4. area of the filtering surface,
5. particle size distribution in the slurry,
6. shape of particles and
7. slurry density.

The cake moisture, \( m \), is expressed as:

\[
m = f(a, m_R, d)
\]  

(14.10)

where  
\( m_R \) = the equilibrium cake moisture if saturated air is forced through the cake at pressure \( \Delta P \),  
\( a \) = an approach factor indicating the rate of approach to \( m_R \), and  
\( d \) = a parameter incorporating particle size, shape and size distribution.

The parameter \( d \) is related to the specific surface area and a particle size distribution parameter such as the % passing 10 microns. The approach term, \( a \), is the major factor used to determine the optimum achievable cake moisture and can be expressed as:

\[
a = \frac{A \Delta P_D}{M_c} Q_{v(D)} t_{dw}
\]  

(14.11)

where  
\( \Delta P_D \) = differential pressure during dewatering,  
\( Q_{v(D)} \) = volume rate of flow of air through the cake as \( m^3/s/m^2 \) of filter area during the dewatering part of the cycle,  
\( t_{dw} \) = dewatering time,  
\( M_c \) = the mass of dry cake per cycle, and  
\( A \) = filter area.
It is found experimentally that if \( Q_{v(D)} \) is less than 0.102 mVs/m^2, then Eq. (14.11) can be simplified to:

\[
a = \frac{A t_{dw}}{M_c}
\]  

(14.12)

If the pressure changes then the term \( \Delta P_D \) must be included. Apart from predicting cake moisture, \( a \) is useful in determining vacuum pump energy requirements through \( \Delta P_D \) and \( Q_{v(D)} \).

The mechanism of washing and dewatering cake involves the penetration of the wash liquid in the pores and displacement of liquid from the medium and cake. The process is more effective when the viscosity of the wash water is less than that of the slurry liquid. The time of washing is a function of the flow of wash water and its ability to displace the liquid contained in the cake. That is:

\[
t_w = K t_c V_w
\]  

(14.13)

where \( t_w \) = time of cake washing per cycle, 
\( t_c \) = time of cake formation per cycle, and 
\( V_w \) = volume of wash liquid per unit volume of the water contained in the cake.

Eq. (14.11) holds with the assumption that the radius of particles may be neglected and that the pore size remains unaltered during the filtering process.

### 14.2.1. Constant Pressure Filtration

When pressure is applied for filtration, it takes some time to build up to the required level. During this period, some filtration takes place. Once the required pressure is attained it can be held constant for filtration to proceed at that pressure (Fig. 14.16). The mathematical expression depicting filtration at constant pressure condition can be obtained by integrating Eq. (14.3), taking \( \Delta P \) as constant. Eq. (14.3) may be re-written as:

\[
dV = \frac{A \Delta P}{\mu \left[ \frac{\alpha M_c}{A} + R_M \right]} \, dt
\]  

(14.14)

But \( M_c \) is sometimes written as:

\[
M_c = C_F V
\]  

(14.15)

where \( V \) = the cumulative volume of filtrate, and 
\( C_F \) = the feed solid concentration in mass of solid/volume of liquid.

However, since some of the feed water is retained in the cake as residual moisture, this equation will underestimate the mass of deposited cake. By using a mass balance on the moist cake, the true relationship between \( M_c \) and \( V \) can be derived as:
Fig. 14.16. Build up of pressure in a filter press.

\[
M_C = \frac{\rho_L m_F}{(1 - m m_F)} V = C_m V \tag{14.16}
\]

where \( m_F \) = mass fraction of solids in the feed,
\( m \) = cake moisture expressed as (mass of wet cake/mass of dry cake),
\( \rho_L \) = density of liquid (filtrate), and
\( C_m \) = solids concentration corrected for cake moisture, kg/m³

Eq. (14.16) can also be written in the form [5]:

\[
M_C = \left[ \frac{1}{C_F} - \frac{1}{\rho_S} - \frac{(m-1)}{\rho_L} \right] V = C_m V \tag{14.17}
\]

Substituting the value of \( M_C \) from Eq. (14.16) into Eq. (14.14) and rearranging:

\[
dV = \frac{\Lambda \Delta P}{\mu \left[ \alpha C_m \left( \frac{V}{\Lambda} \right) + R_M \right]} \, dt \tag{14.18}
\]

For mathematical convenience Eq. (14.18) may be written as:

\[
\frac{dt}{dV} = \frac{\mu \alpha C_m}{\Lambda \Delta P} \left[ \frac{V}{\Lambda} \right] + \frac{\mu R_M}{\Lambda \Delta P} \quad \text{or} \tag{14.19}
\]
\[ \frac{d \alpha}{A^2 \Delta P} + \frac{\mu R_m}{A \Delta P} \]

Eq. (14.20) can be integrated from \( V=0 \) to \( V=V \) at constant pressure to give:

\[ \frac{t}{V} = \frac{\mu \alpha C_m}{2A^2 \Delta P} V + \frac{\mu R_m}{A \Delta P} \] (14.21)

As each of the terms \( \frac{\mu \alpha C_m}{2A^2 \Delta P} \) and \( \frac{\mu R_m}{A \Delta P} \) are constants, Eq. (14.21) can be simplified and expressed as:

\[ \frac{t}{V} = \frac{K_1}{\Delta P} V + \frac{K_2}{\Delta P} \] (14.22)

where \( K_1 = \frac{\mu \alpha C_m}{2A^2} \) and \( K_2 = \frac{\mu R_m}{A} \)

Plotting \( t/V \) against \( V \) should give a straight line with a slope equal to \( K_1/\Delta P \) and intercept equal to \( K_2/\Delta P \), from which the specific cake resistance and the medium resistance can be determined.

It must be remembered that Eq. (14.22) represents that portion of the pressure–time curve where the pressure is constant (Fig. 14.16). If \( t_0 \) is the time at which constant pressure commenced and the operation continued to time \( t \) during which time filtrate volumes \( V_1 \) and \( V \) were obtained, then Eq. (14.20) can be integrated between the limiting values \( t_0 \), \( t \), and \( V_1 \), \( V \) to yield Eq. (14.23):

\[ (t - t_0) = \frac{K_1}{\Delta P} (V^2 - V_1^2) + \frac{K_2}{\Delta P} (V - V_1) \]

\[ = \frac{K_1}{\Delta P} (V - V_1)(V + V_1) + \frac{K_2}{\Delta P} (V - V_1) \]

\[ \frac{(t - t_0)}{(V - V_1)} = \frac{K_1}{\Delta P} (V + V_1) + \frac{K_2}{\Delta P} \] (14.23)

Substituting the values of \( K_1 \) and \( K_2 \) Eq. (14.23) can be re-written as:

\[ \frac{(t - t_0)}{(V - V_1)} = \left[ \frac{\mu \alpha C_m}{2A^2 \Delta P} \right] (V + V_1) + \frac{\mu R_m}{A \Delta P} \] (14.24)

In order to test Eq. (14.24) under industrial conditions it is necessary to ensure that the slurry feed rate, the feed tank levels and the pressure differential are constant.
In practice, it is sometimes observed that Eq. (14.24) breaks down. This has been attributed to possible distortion of the filtering medium, like woven cotton, which has a tendency to stretch.

Example 14.1 illustrates the use of the Eq. (14.24) to determine the cake resistance.

---

**Example 14.1**

A nickel mineral of specific gravity 3.091 kg/m$^3$ was pulped using dilute sulphuric acid. The pulp contained 28% solids. After dissolving the soluble salts the pulp was filtered through a thin ceramic medium in the shape of a circular disk of area 0.178 m$^2$. The initial applied pressure was 10 kPa. The filtrate was collected after a step-wise increase in pressure at known intervals which gave a cake thickness of 12 mm and 12% moisture. The collected data are tabulated below. The temperature of the filtrate was 25°C. Determine the specific resistance of the deposited cake and medium.

data:

<table>
<thead>
<tr>
<th>No</th>
<th>$\Delta P \times 10^3$ Pa</th>
<th>Time, s</th>
<th>Filtrate Vol., m$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
<td>72.4</td>
<td>0.072</td>
</tr>
<tr>
<td>2</td>
<td>0.4</td>
<td>130.0</td>
<td>0.079</td>
</tr>
<tr>
<td>3</td>
<td>0.6</td>
<td>823.0</td>
<td>0.112</td>
</tr>
<tr>
<td>4</td>
<td>0.8</td>
<td>1082.4</td>
<td>0.124</td>
</tr>
<tr>
<td>5</td>
<td>1.0</td>
<td>1370.0</td>
<td>0.138</td>
</tr>
<tr>
<td>6</td>
<td>1.2</td>
<td>1740.9</td>
<td>0.154</td>
</tr>
<tr>
<td>7</td>
<td>1.4</td>
<td>2400.0</td>
<td>0.180</td>
</tr>
<tr>
<td>8</td>
<td>1.4</td>
<td>3010.0</td>
<td>0.201</td>
</tr>
<tr>
<td>9</td>
<td>1.4</td>
<td>3640.0</td>
<td>0.221</td>
</tr>
<tr>
<td>10</td>
<td>1.4</td>
<td>4280.0</td>
<td>0.239</td>
</tr>
<tr>
<td>11</td>
<td>1.4</td>
<td>4820.0</td>
<td>0.253</td>
</tr>
<tr>
<td>12</td>
<td>1.4</td>
<td>5350.0</td>
<td>0.267</td>
</tr>
<tr>
<td>13</td>
<td>1.4</td>
<td>5800.0</td>
<td>0.278</td>
</tr>
<tr>
<td>14</td>
<td>1.4</td>
<td>6250.0</td>
<td>0.289</td>
</tr>
<tr>
<td>15</td>
<td>1.4</td>
<td>6700.0</td>
<td>0.299</td>
</tr>
<tr>
<td>16</td>
<td>1.4</td>
<td>7150.0</td>
<td>0.308</td>
</tr>
</tbody>
</table>

**Solution**

Step 1
From the table it can be seen that after 2400 seconds, constant pressure is reached and that the filtrate volume removed in that time was 0.18 m$^3$. Thus 2400 s is taken as $t_1$ and 0.18 m$^3$ as $V_1$.

Step 2
We can now determine $t-t_1$ and $V-V_1$ as in the following table:
Step 3
Plot \((t-t_i)/(V-V_1)\) against volume \(V\) as shown below. Draw the line of best fit through the points obtained under constant pressure conditions, i.e. from \(V_1\) onwards.

<table>
<thead>
<tr>
<th>No.</th>
<th>(t-t_i)</th>
<th>(V-V_1)</th>
<th>((t-t_i)/(V-V_1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-2327.6</td>
<td>-0.11</td>
<td>21500.0</td>
</tr>
<tr>
<td>2</td>
<td>-2270.0</td>
<td>-0.10</td>
<td>22500.0</td>
</tr>
<tr>
<td>3</td>
<td>-1577.0</td>
<td>-0.07</td>
<td>23100.0</td>
</tr>
<tr>
<td>4</td>
<td>-1317.6</td>
<td>-0.06</td>
<td>23600.0</td>
</tr>
<tr>
<td>5</td>
<td>-1030.0</td>
<td>-0.04</td>
<td>24600.0</td>
</tr>
<tr>
<td>6</td>
<td>-659.1</td>
<td>-0.03</td>
<td>25200.0</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>610.0</td>
<td>0.02</td>
<td>28760.0</td>
</tr>
<tr>
<td>9</td>
<td>1240.0</td>
<td>0.04</td>
<td>30243.9</td>
</tr>
<tr>
<td>10</td>
<td>1880.0</td>
<td>0.06</td>
<td>31864.4</td>
</tr>
<tr>
<td>11</td>
<td>2420.0</td>
<td>0.07</td>
<td>32970.0</td>
</tr>
<tr>
<td>12</td>
<td>2950.0</td>
<td>0.09</td>
<td>33908.0</td>
</tr>
<tr>
<td>13</td>
<td>3400.0</td>
<td>0.10</td>
<td>34700.0</td>
</tr>
<tr>
<td>14</td>
<td>3850.0</td>
<td>0.11</td>
<td>35321.1</td>
</tr>
<tr>
<td>15</td>
<td>4300.0</td>
<td>0.12</td>
<td>36134.5</td>
</tr>
<tr>
<td>16</td>
<td>4750.0</td>
<td>0.13</td>
<td>37010.0</td>
</tr>
</tbody>
</table>

Fig. 14.17. Time-Volume plot for Example 1 data.
In this particular case the equation of the line of best fit is:

\[
\frac{(t-t_0)}{(V-V_0)} = 75603.0 \frac{V}{V_0} + 13644.0
\]

That is, the slope of the line is 75603.0 (s/m^6) which is the value of \( (K_i/\Delta P) \) in Eq. (14.23), and the intercept equals 13644.0 (s/m^3) which equals \( [(K_2/\Delta P)+(V_1K_3/\Delta P)] \). Substituting these values we have:

\[75603.0 = \frac{K_i}{\Delta P}\]

Therefore \( K_i = 75603.0 \times 1.4 \times 10^6 \text{ Pa s/m}^6\)

and \( K_2 = (13644.0 - V_1 (75603.0)) \times \Delta P \)

\[= (13644.0 - 0.18 (75603.0)) \times 1.4 \times 10^6 = 4.964 \times 10^6 \text{ Pa s/m}^3\]

Step 4

From standard tables, the viscosity of sulphuric acid at 25°C is 1.33 mPa s. [13], and the density of 5% sulphuric acid is 1030 kg/m^3 [14].

For a cake moisture of 12%:

\[m = \frac{\text{mass of wet cake}}{\text{mass of dry cake}} = \frac{100}{100-12} = \frac{100}{88} = 1.136\]

\[\rho = \frac{\rho_m \rho_f}{(1-m \rho_f)} = \frac{1030 \times 0.28}{1-(1.136 \times 0.28)} = 423.0 \text{ kg/m}^3\]

From Eq. (14.22),

\[\alpha = \frac{2A^2K_1}{\mu C_m} = \frac{2 \times 0.178^2 \times 1.058 \times 10^{10}}{0.00133 \times 423.0} = 1.19 \times 10^9 \text{ m/kg}\]

and

\[R_m = \frac{K_2 A}{\mu} = \frac{4.964 \times 10^6 \times 0.178}{0.00133} = 6.64 \times 10^8 \text{ m}^{-1}\]

14.2.2. Constant Volume Filtration

In industrial situations a constant volume rate of flow of filtrate is often required to meet the demands of downstream operations, like flotation circuits. To maintain productivity, the filtering pressure has to be increased. It is therefore necessary to establish a relation between volume flow rate and pressure.
Considering \( Q_v \) as the volume rate of flow of filtrate, we can write:

\[
Q_v = \frac{V}{t} = \frac{dV}{dt} = \text{constant}
\]  (14.25)

Substituting the value of \( Q_v \) for \( dV/dt \) in Eq. (14.19) and re-arranging we can write:

\[
\Delta P = Q_v \left[ \frac{\mu \alpha V}{A^2} + \frac{\mu R_M}{A} \right]
\]  (14.26)

and from Eq. (14.25):

\[
\Delta P = \alpha \mu C_m \left[ \frac{Q_v^2}{A^2} \right] t + \frac{\mu R_M Q_v}{A} \]  (14.27)

In this case, \( \alpha \mu C_m A^3 \) and \( \mu R_M A \) are constants and can be simplified as \( 2K_1 \) and \( K_2 \), and Eq. (14.27) can now be written as:

\[
\Delta P = 2K_1 Q_v^2 t + K_2 Q_v
\]  (14.28)

It can be seen that this equation can be easily evaluated by plotting \( \Delta P \) against \( t \). The plot should be linear with the slope given by \( 2K_1 Q_v^2 \) and the intercept by \( K_2 Q_v \).

Eq. (14.27) is the basic equation for constant volume filtration. It provides the volume rate of filtration, \( Q_v \), and the required change in pressure with time to maintaining the steady flow rate of filtrate.

Example 14.2 illustrates the use of these equations for operating a filter at constant rate of filtration.

---

**Example 14.2**

A 20% pulp of a siliceous gold ore had to be filtered at constant rate to recover the gold. The filtering medium was cloth and the filtering surface area 0.09 m\(^2\). Pressure was gradually increased to maintain the filtering rate was at \( 1.8 \times 10^{-5} \) m\(^3\)/s. Estimate the resistances of the cake and the medium.

Data

<table>
<thead>
<tr>
<th>Filtrate recovery time, s</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>250</th>
</tr>
</thead>
<tbody>
<tr>
<td>Differential pressure, (x 10(^4) Pa)</td>
<td>1.2</td>
<td>1.52</td>
<td>2.08</td>
<td>2.50</td>
<td>3.0</td>
</tr>
</tbody>
</table>

Density of solid = 3845 kg/m\(^3\)  
Density of filtrate = 1000 kg/m\(^3\)  
Viscosity of filtrate at 25°C = 0.89 mPa s  
Cake moisture = 10%
Solution

Step 1
From the cake moisture of 10%:

\[
m = \frac{100}{100 - 10} = 1.11
\]

Concentration of feed, \( C_m = \frac{1000 \times 0.20}{(1 - (1.11 \times 0.20))} = 257.1 \text{ kg/m}^3 \)

Step 2
Plot the filtration data as shown in Fig. 14.18 and determining the line of best fit. The intercept and slope of the line equals 6.86 kPa and 0.0916 kPa/s respectively.

Step 3
From Eq. (14.27) we have:

\[
6.86 \times 1000 = \frac{\mu R_M Q_v}{A}
\]

Fig. 14.18. Increasing pressure with time for constant rate filtration.

Substituting values we therefore have \( R_M = \frac{6860 \times 0.09}{1.8 \times 10^{-3} \times 0.00089} = 3.85 \times 10^{10} \text{ m}^{-1} \)

Similarly:
14.2.3. Variable Pressure and Variable Volume Filtration

Various combinations of constant pressure and variable pressure filtration are practiced especially in plate filters. In such cases appropriate combinations of the mathematical models cited in sections 14.2.1 and 14.2.2 are applicable. For example in constant pressure followed by constant volume rate of filtration the two equations are to be applied to determine the cake characteristics and filtering times.

When both the pressure and flow rates are varied the same principle applies. In practice the variable pressures are controlled relatively easily by using centrifugal pumps and less so with diaphragm or other pumps.

For such operations, therefore, the time required for a cumulative quantity of filtrate can also be determined using the basic filtration Eq. (14.3) which is re-written as:

\[
\Delta P = \frac{Q_v}{A} \left( \frac{\mu C_m V}{A} + \frac{\mu R_m}{A} \right)
\]

Equation (14.29) can be re-written as:

\[
V = \frac{A^2}{\mu C_m} \left( \frac{\Delta P}{Q_v} - \frac{\mu R_m}{A} \right)
\]

To solve Eq. (14.30), \( \Delta P \) is determined from the characteristics of the pump [5]. For example, the characteristics of a centrifugal pump operating at 1500 rpm is given in Fig. 14.19. In Eq. (14.30), \( V \) is the cumulative volume. To determine the time required for filtration it can be seen that:

\[
t = \int \frac{dV}{Q_v}
\]

In practice the integration is usually recommended by plotting \( V \) against \( 1/Q_v \) and finding the area under the curve between the limits \( V = 0 \) to \( V = V \).

Example 14.3 illustrates the use of the method to determine the time of filtration.
Example 14.3

A slurry containing 900 kg solids per cubic meter of slurry was filtered in a plate and frame press. The total filtering area was 50 m$^2$. Filtering at constant pressure with a centrifugal pump produced a cake having a resistance of $1.1 \times 10^{11}$ m/kg when a medium of resistance $5 \times 10^{10}$ m$^{-1}$ was against the plates. Determine the cumulative volume of filtrate and time for filtration.

Data:
- Viscosity of filtrate (water) = 0.001 Pa.s
- Cake moisture = 15%
- Filtrate density = 1000 kg/m$^3$
- Solid density = 2800 kg/m$^3$

Use Fig. 14.18 for pump characteristics.

Solution

Step 1

Eq. (14.30) may be used to determine $V$ in terms of $(\Delta P/Q_v)$.

From a moisture of 15%,

$$m = \frac{100}{(100-15)} = 1.176$$

![Characteristic curve of centrifugal pump at 1500 rpm.](image)

For a feed concentration of 900 kg of solid in 1 m$^3$ of slurry:

$$\text{Volume of solids} = \frac{900}{2800} = 0.321 \text{ m}^3$$
Volume of water \(= 1 - 0.321 = 0.679 \text{ m}^3\) and

Mass of water in the feed \(= 0.679 \times 1000 = 679 \text{ kg}\)

Therefore, the feed fraction of solids, \(m_F = \frac{900}{(900+679)} = 0.570\)

Then  \(C_m = \frac{1000 \times 0.57}{(1-(1.176 \times 0.57))} = 1729 \text{ kg/m}^3\)

Step 2.
Substituting the values in the Eq. (14.30):

\[
V = \frac{50^2}{0.001 \times 1.1 \times 10^{11} \times 1729} \left[ \frac{\Delta P}{Q_v} - \frac{0.001 \times 5.0 \times 10^{10}}{50} \right]
\]

\[
= 1.31 \times 10^{-4} \left[ \frac{\Delta P}{Q_v} - 1.0 \times 10^6 \right]
\]

\(\frac{\Delta P}{Q}\) is determined from the pump characteristics curve and substituted in Eq. (14.30) to determine \(V\).

Step 3
To determine time \(t\), integrate Eq. (14.31) with limits of \(V = 0\) and \(V = \) the value from Step 2. The integration may be done graphically by plotting \(V\) against \(1/Q\).

14.2.4. Compressibility of Deposited Cakes
Some filter cakes tend to be soft and compress under the high differential pressures as applied during filtration. Compression involves a decrease in porosity and permeability of the cake.

Where tests are carried out at low pressure and a plant design is required at high pressure, the relationship between \(\Delta P\) and \(\alpha\) is required. The relationship can be obtained experimentally using a compression-permeability cell or obtained from filtration rate data using an empirical expression such as:

\[
\alpha = \alpha_0 \Delta P^n
\]

where \(\alpha_0\) = the specific cake resistance at unit pressure and
\(n\) = a compressibility index or coefficient.

For incompressible cakes, \(n = 0\) while for most cakes, \(n = 0.2 - 0.8\) but can be greater than 1 for highly compressible cakes.
The constants are determined from plots of \( \frac{t}{V} \) versus \( V \) at different pressures. The slopes of the plots are given by \( K_1/\Delta P \) so that the slope \( \times \Delta P = K_1 \) which is proportional to \( \alpha \). Thus a plot of \( \log(\text{slope } \times \Delta P) \) versus \( \log(\Delta P) \) will describe the \( \alpha \) versus pressure relationship.

**Example 14.4**

From a set of filtering tests at five constant pressures, the following results were obtained. Determine the compressibility index of the cake. Is the cake compressible?

<table>
<thead>
<tr>
<th>( \Delta P ) (kPa)</th>
<th>Q( _{VL} ) (L/min)</th>
<th>V( _L ) (L)</th>
<th>Q( _{VL} ) (L/min)</th>
<th>V( _L ) (L)</th>
<th>Q( _{VL} ) (L/min)</th>
<th>V( _L ) (L)</th>
<th>Q( _{VL} ) (L/min)</th>
<th>V( _L ) (L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.03</td>
<td>1</td>
<td>0.03</td>
<td>2</td>
<td>0.015</td>
<td>4</td>
<td>0.012</td>
<td>6</td>
</tr>
<tr>
<td>100</td>
<td>0.022</td>
<td>2</td>
<td>0.036</td>
<td>2</td>
<td>0.023</td>
<td>4</td>
<td>0.016</td>
<td>6</td>
</tr>
<tr>
<td>150</td>
<td>0.015</td>
<td>4</td>
<td>0.022</td>
<td>4</td>
<td>0.021</td>
<td>4</td>
<td>0.013</td>
<td>4</td>
</tr>
<tr>
<td>200</td>
<td>0.012</td>
<td>6</td>
<td>0.016</td>
<td>6</td>
<td>0.021</td>
<td>6</td>
<td>0.010</td>
<td>6</td>
</tr>
<tr>
<td>300</td>
<td>0.01</td>
<td>8</td>
<td>0.013</td>
<td>8</td>
<td>0.016</td>
<td>8</td>
<td>0.016</td>
<td>8</td>
</tr>
</tbody>
</table>

Q\( _{VL} \) – Filtration rate; V\( _L \) – Filtrate volume.

**Solution**

**Step 1**

For convenience Eq. (14.32) may be written as:

\[
\frac{\alpha}{\alpha_o} = \Delta P^n \quad \text{and}
\]

\[
\log(\alpha) - \log(\alpha_o) = n \log(\Delta P)
\]

Plot reciprocal of filtering rate (\( t/V \)) against volume of filtrate collected (\( V \)) as shown in Fig. 14.20.

**Step 2**

The slopes and intercepts of linear regressions of each set of pressure data are given below:

<table>
<thead>
<tr>
<th>( \Delta P ) (kPa)</th>
<th>Slope, m/s²</th>
<th>Intercept, m/s²</th>
<th>Slope ( \times \Delta P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>567195.1</td>
<td>1559.2</td>
<td>28359756097</td>
</tr>
<tr>
<td>100</td>
<td>558729.6</td>
<td>901.0</td>
<td>55872963097</td>
</tr>
<tr>
<td>150</td>
<td>475029.5</td>
<td>789.7</td>
<td>71254430501</td>
</tr>
<tr>
<td>200</td>
<td>365459.9</td>
<td>701.1</td>
<td>73091976517</td>
</tr>
<tr>
<td>300</td>
<td>363321.6</td>
<td>284.3</td>
<td>1.08996E+11</td>
</tr>
</tbody>
</table>
Step 3
Column 4 in the above table is proportional to the specific cake resistance, $\alpha$.
Plot Column (4) against column (1) on log-log axes. The slope of this line is the value of $n$. In this case the slope is 0.71, therefore the cake is compressible.
14.2.5. Filtration through Compressible Deposits
We have seen that the flow of fluid through continuous capillaries in a bed is given by Darcy’s Law. Re-writing it for convenience we have:

\[ v_e = \frac{\Delta P d_e^2}{K L \mu} \]  

(14.33)

where

- \( v_e \) = interstitial or average pore velocity,
- \( d_e \) = the mean diameter of the pores,
- \( L \) = thickness of the bed,
- \( \mu \) = viscosity of fluid,
- \( \Delta P \) = the applied differential pressure and
- \( K \) = a proportionality constant.

The average pore velocity, \( v_e \), is related to the average velocity over the whole cross-sectional area of the bed, \( v \), as:

\[ v = \varepsilon v_e \]  

(14.34)

Thus Eq. (14.33) transposes to:

\[ v = \frac{\Delta P \varepsilon d_e^2}{K L \mu} \]  

(14.35)

The diameter of pores in the cake are never uniform and may even vary within a single capillary. Kozeny [16.17] therefore considered the hydraulic diameter being defined as:

\[ d_e = \frac{\varepsilon}{S(1-\varepsilon)} \]  

(14.36)

where

- \( S \) = the wetted perimeter or specific surface of the pore, \( m^2/m^3 \).

Substituting the value of \( d_e \) into Eq. (14.35):

\[ v = \frac{\varepsilon^3 \Delta P}{K S^2 \mu (1-\varepsilon)^3 L} \]  

(14.37)

This is known as the Kozeny-Carman equation and is applicable to compressed cakes as long as the pores are continuous and not blocked by compression due to packing and particle characteristics like flat laminar particles.

Because of frictional losses arising from the flow of filtrate through the cake, there will be a fluid pressure gradient across the cake. The actual compressive pressure will depend on the structure of the cake and the nature of the contacts between the particles, but it can be expressed as a function of the difference between the pressure at the surface of the cake \( P \) and that at a depth \( L \) in the cake. The packing characteristics of particles in the cake will thus change with depth and the absolute values of \( L \) and \( \varepsilon \) also changes and the method of
computing \( v \) becomes complicated. The porosity decreases, and hence the cake resistance increases, from the free surface to the filter medium interface. The simplest method to determine \( v \) is to determine the mean or average specific resistance of the bed between pressures 0 to \( \Delta P_C \). The mean specific resistance is given by:

\[
\frac{\Delta P_C}{\alpha_{\text{AVE}}} = \int_{\alpha}^{\Delta P_C} \frac{d\Delta P}{\alpha} \quad (14.38)
\]

where

\( \alpha_{\text{AVE}} \) = average cake resistance,

\( \Delta P_C \) = pressure drop across the whole cake = \( \Delta P - \Delta P_m \),

(\( \Delta P_m \) = pressure drop across the medium),

Further analytical approaches have been made by later workers [18,19] who considered the flow through an element \( dL \) of cake at a distance \( L \) from the surface of the cake (Fig. 14.22). Other workers [20,21] have attempted incremental analysis of the element \( dL \). The basic steps involved in the analysis of single element is described in the following section. As computer simulation of incremental analyses is more or less an advance on single element analysis, the reader is directed to consult the original papers.

Since the cake resistance varies with depth in the cake, the filtration equation is expressed in differential form as:

\[
v = \frac{\varepsilon^3}{KS^2 \mu (1-\varepsilon)^2} \left[ \frac{dP_L}{dL} \right] \quad (14.39)
\]

where

\( dV_L \) = Filtrate volume from the element \( dL \) and

\( dP_L \) = fluid pressure drop across element \( dL \).

It is assumed that the mass of deposited cake, \( M_C \) is an independent parameter, given by:

\[
dM_C = A(1-\varepsilon)\rho_s dL \quad (14.40)
\]

Substituting the value of \( dL \) from Eq. (14.40) in Eq. (14.39), we get:

\[
v = \frac{A \varepsilon^3 \rho_s dP_L}{KS^2 \mu (1-\varepsilon) dM_C} \quad (14.41)
\]

Note: that:

1. \( K \) is known as the Kozeny constant and usually taken as equal to 5,

2. the specific resistance of the cake element, \( \alpha = \frac{K(1-\varepsilon)S^2}{\varepsilon^3 \rho_s} \)

Substituting the expression for \( \alpha \), \( v = V/At \) and \( dM_C = C_m dV \) and re-arranging we have:

\[
\frac{dP_L}{\alpha} = \frac{\mu C_m V dV}{A^2 t} \quad (14.42)
\]
Fig. 14.22. Incremental element dL through a compressible cake.

Using the specific cake resistance function in Eq. (14.32) and integrating gives:

\[ \int \frac{dP_c}{\alpha \Delta P^n} = \frac{\mu C_m}{A^2} \int \frac{V}{t} dV \]  

(14.43)

Assuming a negligible media resistance to simplify the integration gives:

\[ V^2 = \frac{2 \Delta P^{(n-1)} t}{\alpha \mu C_m (1-n)} \]  

(14.44)

The compressibility factor \( n \) is assumed to be reasonably constant over a small range of pressure and has a value between 0.01 to 0.90. Tiller [22] has estimated there is an error of less than 5% in assuming negligible media resistance provided \( (\alpha M_C/A R_M) \) is greater than 20.

The average specific cake resistance across the compressible cake is equal to the integral of the change in resistance with pressure across the incremental sections of cake [5,19] as expressed in Eq. (14.38). Using the cake-pressure relationship in Eq. (14.32) and integrating gives:

\[ \alpha_{AVE} = \alpha_c (1-n) \Delta P_c^n \]  

(14.45)

14.2.6. Optimum Operation of Filters

The commercial operation of any continuous filter involves:

1. filtration, for time \( t_F \)
2. washing the cake on the medium, for time $t_w$
3. dislodging the cake off the medium, for time $t_D$.

The time involved in each of these processes, contribute to the filtering cycle. For a rotary filter bed, the time of operation is divided between sectors roughly in the proportion shown in Fig. 14.7. The total cycle time $t_c$ is the sum of the time spent in each section plus any downtime. That is, for a single cycle of operation, if the total time of the filtration operation is $t_f$ and $t_d$ the non-filtering time, then:

$$t_c = (t_f + t_w) + t_d = t_f + t_d$$  \hspace{1cm} (14.46)$$

The time involved in the filtering operation is given by the standard filtering Eq. (14.22), which may be written as:

$$\frac{K_f}{AP} V^2 + \frac{K_{f1}}{AP} V = t_f$$  \hspace{1cm} (14.47)$$

where $V$ is the cumulative volume of filtrate.

If cake washing is carried out at the same pressure as filtration, then the washing rate can be estimated as being some function of the filtration rate or a function of the filtrate volume. Thus $t_w$ and $t_f$ will be similar functions of $V$. The filter production can then be expressed as:

$$Q_M = \frac{C_m V}{(t_w + t_f + t_D)} = \frac{C_m V}{(f_1(V) + f_2(V) + t_D)}$$  \hspace{1cm} (14.48)$$

The optimum filter capacity is then obtained by differentiating Eq. (14.48) with respect to volume and equating to zero. If cake washing is not employed then the filtration capacity is optimum when the filtrate volume is optimum as expressed by the simplified equation:

$$V = \frac{\sqrt{\frac{AP t_D}{K_f}}} {K_{f1}} \text{ or } t_D = \frac{K_{f1} V^2}{\Delta P}$$  \hspace{1cm} (14.49)$$

For cases where the filter medium resistance is negligible then from Eq. (14.47) the filtration time is:

$$t_f = \frac{K_f}{\Delta P} V^2$$  \hspace{1cm} (14.50)$$

Thus optimum capacity will occur when the down time is equal to the filtration time. In practice, for maximum overall filtration rate, the filtration time must be slightly greater than the down time to allow for the resistance of the filter cloth.

**14.3. Capacity of Continuous Vacuum Filters**

In metallurgical practice the drum, disc and leaf filters are most commonly used and as the underlying principles of operation are the same, we will consider the capacity of these
continuous filter types. The capacity is based on the volume rate of filtrate obtained during the filtering operation.

In a continuous drum, disc or leaf filter only a part of the drum, disc or leaf is immersed in the pulp that has to be filtered. Thus the time of filtration \( t_F \), will depend on the time for which the fraction of the drum is submerged and the time taken for complete rotation, \( t_C \), the cycle time. We have seen that for constant pressure filters:

\[
\frac{dt}{dV} = \frac{K_1}{\Delta P} V + \frac{K_2}{\Delta P}
\]  

(14.51)

The time of filtration can be evaluated by considering the volume of filtrate obtained during the operation. Thus if \( V \) is the volume of filtrate obtained then the time of filtration would be given by Eq. (14.47). This is a quadratic equation in \( V \) which can be solved for the filtering time \( t_F \) during which a section of the drum is immersed in the slurry. If the medium resistance is small, then \( K_2 \) may be neglected and the optimum filtrate volume per cycle is given by Eq. (14.50).

This is a rapid method of estimating the volume of filtrate and therefore capacity in a given filtration time. From the filtrate volume, the mass of dry cake deposited in time \( t_F \) is calculated from Eq. (14.16) and the solids capacity of the filter calculated as \( M_c/t_C \).

Fig. 14.23 shows that while considering the actual pressure on the filtering surface, the hydrostatic head of the slurry in which the filtering drum, disc or leaf filter is immersed has been neglected. Taking this into account, the total differential pressure, \( \Delta P_T \), on the drum surface would be:

\[
\Delta P_T = \Delta P_C + \rho_p g H
\]

(14.52)

where  
\( H \) = the distance, (m) below the slurry level at any point P,
\( \rho_p \) = the density of the pulp,
\( g \) = the acceleration due to gravity and
\( \Delta P_C \) = pressure at the drum surface

Also, if the slurry is not stirred adequately, a density difference will built up with the bottom of the trough having a denser slurry. With a variation of slurry concentration, a variation in slurry thickness would result and the mass deposited per unit area of drum, including possibly the cake characteristics, will not be uniform. Rushton and Hameed [23] suggests that this error can be accounted for by multiplying the total cake resistivity by a factor ranging between 0.9 and 1.0.

The capacity of a rotary drum filter will depend on the time that the drum surface is exposed to the slurry. The deposit time or filtration time will depend on the drum speed and the depth of submergence of the drum in the slurry. Thus if \( t_F \) is the time that any point on the drum surface is submerged during a cycle (the filtration time), \( \theta \) the angle of submergence as indicated in Fig. 14.23 and \( t_C \) the time for a complete rotation of the drum, then:

\[
t_F = \text{Fraction of cycle submerged } x \ t_C \quad \text{or} \\
\frac{\theta_D}{360} \frac{t_C}{t_C} = \frac{\theta_B}{2\pi} \frac{t_C}{t_C} \quad \text{or} \quad \frac{\theta_D}{360} \frac{t_C}{\omega}
\]

(14.53)
Fig. 14.23. Submergence of drum filter in slurry tank where $\theta$ is the angle of submergence.

where $\theta_D$ = angle of submergence in degrees, 
$\theta_R$ = angle of submergence in radians, and
$\omega$ = rotational speed of the drum in revolutions/s or rpm.

Osborne [24] expressed the mass of dry cake per unit area per unit time, after neglecting the resistance due to the medium, as:

$$Q_M = \frac{A - \alpha R}{\theta_D}$$

Rushton and Hameed [23] included the resistance of the medium and the filter cake and derived the quadratic equation which gave the flow of the filtrate during total filtering time and also the dry solid yield per unit area per filter cycle, as:

$$Q_M = \left[ \frac{2\Delta P^{1-n} C_m \phi}{\mu t_c \alpha_o} \right]^{0.3}$$

where $Q_M$ = capacity, kg/m$^2$/s,
n = compressibility factor,
$\phi$ = fraction of cycle under filtration
$= \theta_R/2\pi = t_f/t_c$,
t$C_m$ = cycle time, one revolution, s,
$\mu$ = viscosity of fluid,
$C_m$ = mass of dry cake per unit volume of filtrate,
$\alpha_o$ = specific cake resistance at unit pressure,
$A$ = filter area.
The filter capacity is then obtained by dividing Eq. (14.55) by \( t_c \) to give kg/m\(^2\)/s if the capacity is calculated using the total drum surface area. If the capacity of the filter, \( Q_M \) as kg/s, is calculated using the filtration area, \( \phi A \), then Eq. (14.55) should be divided by the filtration time, \( t_F \).

Equation (14.55) serves as a satisfactory model for the performance of rotary vacuum filters.

### 14.4. Washing of Deposited Cake

The cake formed on the surface of a filtering medium is usually washed to:

1. Removing the adhering fluid from the surface,
2. Removing the entrapped fluid in the pore space between the particles,
3. Removing the solute.

Washing and displacement of retained fluid within a porous cake is a complex phenomenon. The law governing the washing mechanism has been identified as Darcy’s law of fluid flow through a porous body, but in dewatering the washing mechanism involves two forms, namely:

1. Displacement of bulk fluid by wash water,
2. Diffusion of fluid held in capillaries within the medium.

In excess of 90% of the contained fluid is usually removed by displacement with water as the wash liquid. The displacement curve tends to be asymptotic with time. In practice, displacement is never complete and a fraction of liquid is retained. Lowering the viscosity of the wash water helps to reduce the residual saturation level. Squeezing the cake by application of force also helps in reducing the residual water content. The fraction of fluid that remains is often referred to as connate water.

The ratio of the volume of wash liquor to the volume of filtrate remaining in the cake in the saturated state is known as the wash ratio.

Several early workers [19,25-31] have attempted to establish mathematical models to describe the phenomenon of washing. Of these the displacement model is relatively well established.

#### 14.4.1. Displacement Model of Washing

The process of washing involves the flow of wash water through the cake and the medium, driving the slurry ahead and out of the filtering medium. Thus the same equations as filtration at constant pressure filtration of an incompressible cake in a drum or disc filter applies. In most cases the resistance due to the medium is comparatively very small and may be neglected. For an applied differential pressure, therefore, the volume of wash liquor per unit area of the filter, \( V_w \), would also be given by Eq. (14.21) which is written as:
The filtrate volume remaining in the saturated cake, $V_M$, will be

\[
\frac{V_M}{A} = \left( \frac{2\Delta P \tau_F}{\mu C_m \alpha} \right)^{0.5} k \tag{14.57}
\]

Dividing Eq. (14.56) by (14.57) gives:

\[
n = \frac{\text{Volume of wash water}}{\text{Volume of filtrate remaining}} = \frac{V_w}{V_M} = \frac{t_w}{2k \tau_F} \tag{14.58}
\]

where $\tau_F, \tau_W$ = times for filtration and washing respectively and

$k$ = a constant which is generally determined experimentally for specific slurries.

Example 14.5 illustrates the application of the method.

Example 14.5

A rotary drum vacuum operating at constant pressure of 85 kPa was required to wash a cake formed by filtering a slurry containing 5.0% solids by volume. The filtering area was 0.20 m$^2$ when 0.01 m$^3$ of feed was filtered. The resistance of the cake and cloth were $1.1 \times 10^{12}$ m/kg and $3.8 \times 10^{10}$ m$^{-1}$ respectively. The cake porosity was determined as 40%. The densities of the solid and water are 2650 and 1000 kg/m$^3$ respectively, and the viscosity of water $10^3$ Pa.s. The cake is then washed for 60 seconds at the same pressure. Determine:

1. the washing ratio, and
2. the rate of washing.

Solution

Step 1: Calculate the feed concentration.

Vol. of solids in feed suspension = $(5/100) \times 0.01 \text{ m}^3 = 5 \times 10^{-4} \text{ m}^3$

Hence water in the feed suspension = $(0.01-0.0005) \text{ m}^3 = 9.5 \times 10^{-3} \text{ m}^3$

Mass of solids in the feed = $5 \times 10^{-4} \times 2650 = 1.325 \text{ kg}$

Mass of water in the feed = $9.5 \times 10^{-3} \times 1000 = 9.5 \text{ kg}$

Solids mass fraction in the feed, $S = 1.325/(1.325+9.5) = 0.1224$

Step 2: Calculate the cake properties.

Since porosity of cake = 40%.

Volume of liquid in the pore volume = $[0.40/0.60] \times 10^{-4} = 3.33 \times 10^{-4} \text{ m}^3$,
(assuming a saturated cake)

Mass of water in the cake = $3.33 \times 10^{-4} \times 1000 = 0.333$ kg

Hence the cake volume = $(5.00 + 3.33) \times 10^{-4} = 8.33 \times 10^{-4}$ m$^3$

This can also be calculated from $V_{\text{Cake}} = V_S/(1-\epsilon) = 5.0 \times 10^{-4}/(1-0.4) = 8.33 \times 10^{-4}$ m$^3$

Cake thickness = \frac{\text{Cake volume}}{\text{area}} = \frac{8.33 \times 10^{-4}}{0.2} = 0.004165$ m

Cake moisture, $m = (0.333 + 1.325)/1.325 = 1.2513$

Step 3: Calculate the filtrate volume.

$V = \text{water in feed} - \text{water in cake} = (95.0 - 3.33) \times 10^{-4} = 9.167 \times 10^{-3}$ m$^3$

Step 4: Calculate the corrected solids concentration, $C_m$.

\[ C_m = \frac{\rho m_r}{(1-m_m)} = \frac{1000 \times 0.1224}{1-(1.2513 \times 0.1224)} = 144.537 \text{ kg/m}^3 \]

Step 5: Calculate the wash rate and wash ratio.

Substituting values in Eq. (14.18):

\[ Q_w = \frac{\text{d}V}{\text{d}t} = \frac{85000 \times 0.2}{10^{-3} \left[ \frac{1.1 \times 10^{12} \times 144.5 \times 9.167 \times 10^{-3}}{0.2} + 3.8 \times 10^{16} \right]} = 2.32 \times 10^{-6}$ m$^3$/s

Therefore for a wash of 60 seconds, assuming the wash rate is the same as the filtration rate, at the same pressure:

$V_w = 60 \times 2.32 \times 10^{-6} = 1.392 \times 10^{-4}$ m$^3$

and $V_M = 3.33 \times 10^{-4}$ m$^3$

Hence the wash ratio = \frac{1.392 \times 10^{-4}}{3.33 \times 10^{-4}} = 0.418

---

14.4.2. Diffusion Model of Washing

The diffusion model of washing is applicable after the displacement of slurry from the cake has been achieved and slurry together with any solute remaining entrapped has to be removed. The phenomenon can be visualised by a simplified conceptual illustration as in Fig. 14.24 where in (A) the filtrate (dark) is located in the capillaries and the pores, in (B) the filtrate has been displaced by the wash fluid, but some remains behind in the capillaries, in (C) further removal of filtrate from capillaries has taken place by diffusion leaving a very small amount
that cannot be displaced even after prolonged washing. The phenomenon is complex especially when removal of solutes are involved, e.g., washing of TiO₂ cake for the removal of ferrous sulphate from the cake [32].

Filtrate removal depends on:

1. pore size, and particle shape and
2. diffusion of the solute from the capillaries into the wash stream.

The diffusion model has difficulties especially in cases of thin cakes where cracking and by-passing of wash liquor is difficult to avoid. To understand the complex phenomenon, the concept of a dispersion parameter was introduced and defined as [33,34]:

\[
D_n = \frac{v_l L}{D} \tag{14.59}
\]

where \( v_l \) = average interstitial velocity,
\( L \) = depth of bed, and
\( D \) = axial dispersion coefficient.

\( D_n \) is a function of pore diameter, pore shape factor and molecular diffusion. Using computer simulations, and the concepts of dispersion and molecular diffusion, Purchas and Wakeman [30] worked on the effect of axial distribution and diffusion during washing and corrected the value of the dispersion parameter \( D_n \) as:

\[
D_n(\text{corrected}) = 0.49 + 1.348 \ln [D_n]_{\text{calculated}} \tag{14.60}
\]

Fig. 14.24. Cake washing by diffusion; A - Filtrate (dark) in capillaries; B - Filtrate displaced by wash water - some filtrate in capillaries; C - Filtrate displaced by diffusion from capillaries.
The corrected values of $D_n$ were recalculated and used to determine the dispersion number. Wakeman and Attwood [35] published a series of plots relating the ratio of concentration of solute to the initial solute concentration against wash ratios for dispersion coefficients ranging from 0.01 to 100. Typical plots for $D$ equals 0.1, 50 and 100 are illustrated in Fig. 14.25. Thus, for a desired ratio of filtrate concentration to original concentration of fluid in the pores the wash ratio can be determined directly from the graphs for calculated values of $D_n$ (corrected) using Eq. (14.60).

14.4.3. Washing Efficiency
Choudhury and Dahlstrom [26], using the mass balance at the face of the filtering media, determined the efficiency of filtering in terms of a wash efficiency number, $E$. Thus the mass fraction, $m_C$, of the original solute remaining in the cake after filtration was given by the expression:

$$m_C = \left[1 - \frac{E}{100}\right]^n$$

where $m_C$ = mass fraction of original solute remaining in the cake, $E$ = washing efficiency equal to the % solute removed by a wash ratio on 1.0, $n$ = wash ratio.

Eq. (14.61) can be written as:

$$E = \left[1 - m_C^{1/n}\right] \times 100$$

According to Dahlstrom [2,12], the efficiency of washing is generally of the order of 70% though it ranges from 45% to about 85%.

Fig. 14.25, Concentration ratio versus wash ratio [35].
14.5. Drying of Deposited Cake
After the filtration and washing processes, the cake is generally saturated with the washing fluid. To use the cake for subsequent operation it is generally dried. In continuous drum filters the drying process usually commences during the last portion of the filter-cycle. On batch pressure filters a drying period is allowed just prior to the cake being removed from the filter press.

The drying operation is executed by:

1. blowing or drawing hot or cold air, steam or gas through the cake and/or,
2. squeezing the cake.

Blowing air (gas) is the most common method of drying. The immediate effect of air blowing through a cake is to displace a major amount of fluid contained in the pores of the cake. With time, breakthrough occurs. The breakthrough or threshold pressure is the point where the first drops of the wetting fluid emerge from the cake which corresponds to the point where the first non-wetting fluid enters the inlet face of the cake. The air then passes through without significantly reducing the moisture content. At this stage, the absorbed and adsorbed moisture, (which is the wetting phase), is held on the particles and within the fine pores mainly by capillary forces. The non-wetting phase, that is air or gas, largely permeates through. The moisture removal at this stage is very slow and by diffusion only. In fact a small portion of the moisture is retained in the cake and cannot be removed even at high pressures.

If pressure is applied some of the liquid held in the pores will be expelled and removed but with further application of pressure further desaturation does not take place. A typical relation between saturation and applied pressure is illustrated in Fig. 14.26. The figure shows the limiting saturation that could be obtained at the highest pressure applied was about 18%.

The threshold pressure depends precisely on the packing of the particles in the cake and hence is difficult to determine accurately. The modified threshold pressure, as indicated in Fig. 14.26 is less prone to variation.

The permeability of each individual component, that is wash-water and air, while permeating through the porous cake will follow Darcy's law (Eq. (14.7)), which is re-written as:

\[
\frac{dV_a}{dt} = K_a A \frac{\Delta P}{\mu_w L} \quad \text{and} \quad \frac{dV_w}{dt} = K_w A \frac{\Delta P}{\mu_w L}
\]  

(14.63)

where the suffixes \(w\) (or \(L\)) and \(a\) represent wash water (or liquid) and air respectively. When both components are simultaneously flowing through the system, as occurs during drying, the effective permeability of each component will depend mostly on the pore size distribution. However, the relative permeability of each component can be assessed by dividing each permeability with a common factor \(K\). The factor, \(K\), is the permeability of a single component completely saturating the cake and the two components fully saturated in the fluid. That is:

\[
\text{Relative permeability of water, } K_{RW} = \frac{K_w}{K} \quad \text{and} \quad \text{Relative permeability of air, } K_{Ra} = \frac{K_a}{K}
\]  

(14.64)  

(14.65)
Equations relating the relative permeabilities to a reduced cake saturation was expressed by Lloyd and Dodds [36] and Wyllie and Gardner [37] as the basis of a model for cake drying. Typical relative permeabilities of each component with saturation are illustrated in Fig. 14.27. The curve shows that at about 25% water saturation the air permeability is 100% and the water permeability is nil. That is, the residual saturation of cake is about 25%.

Wakeman [38] attempted a solution of the relative permeability model to predict the residual saturation at a limiting pressure by considering the capillary pressure mainly responsible for holding the wash liquid in the pores. The capillary pressures were related to pore size distribution, the capillary numbers and the reduced saturation of the cake, $S_R$, expressed as a function of the threshold pressure:

$$S_R = \frac{S-S_w}{1-S} = \left(\frac{P_B}{P_{CAP}}\right)^\lambda$$

where $S$ = cake saturation, fraction of voids in cake filled with liquid,
$S_w$ = the irreducible saturation, the saturation at high pressure across the cake or the minimum residual saturation,
$P_B$ = breakthrough or threshold pressure, in absolute pressure,
$P_{CAP}$ = capillary pressure (absolute pressure) and
$\lambda$ = an exponent and an index of the pore size distribution.

Wakeman expressed the relationships between relative permeability and reduced saturation for liquid and gas as:

$$K_{RL} = S_R^{(2+3\lambda)/\lambda}$$

and
Fig. 14.27. Permeabilities of the cake to air and wash water environment.

\[ K_{Ra} = (1 - S_R)^3 \left(1 - S_R^{(2+\lambda)/\lambda}\right) \] (14.68)

To determine the reduced saturation, the modified threshold pressure in Eq. (14.66) has to be determined by drawing a tangent at the point of inflexion in the pressure-saturation curve as illustrated in Fig. 14.26. The breakthrough pressure is expressed by the following relationship for randomly deposited cakes of sand and glass beads.

\[ P_B = \frac{4.6(1-\varepsilon)\gamma}{\varepsilon \, d} \] (14.69)

where

- \( \gamma \) = surface tension of the liquid,
- \( \varepsilon \) = porosity of the cake, and
- \( d \) = mean particle size.

The correlation for the actual breakthrough pressure by Carman [39] suggested a value of 6.0 for the constant in Eq. (14.69) for packed spheres.

As the residual saturation was assumed to be due to capillaries in the cake, Wakeman defined a capillary number, \( N_c \), in terms of the diameter of the pores, porosity of the cake, depth (thickness) of the cake and the pressure differential such that:

\[ N_c = \frac{d^2 \varepsilon^3 (\rho_{liq} g L + \Delta P)}{(1-\varepsilon)^2 L \gamma} \] (14.70)

The capillary number was a function of the reduced saturation and their ratio was constant. That is:
Using Eqs. (14.69)-(14.71) and empirical correlations, the minimum residual saturation was determined for coarse materials such as quartz and fine coal as:

\[ S_\infty = 0.155 \left( 1 + 0.031 \gamma_{0.49} \right) \text{ for } \gamma \geq 10^{-4} \]  

(14.72)

Substituting the value of \( \gamma \) in Eq. (14.71), the residual saturation becomes:

\[ S_\infty = \frac{P_n}{\sigma} \left[ \frac{(1 - \varepsilon) L y}{(\rho g L) + \Delta P} \right] \]  

(14.73)

The following example illustrates the method of calculating the pressure differentials and determining the residual saturation.

**Example 14.6**

On filtering a slurry in a rotary drum filter a uniform cake of 8 mm thickness was formed. The cake on examination had the following properties:

- Mean particle diameter = 3.03 microns
- Density of liquid = 1000 kg/m\(^3\)
- Density of solid = 2700 kg/m\(^3\)
- Cake porosity = 35%.
- Pressure drop across cake = 86 kPa
- Surface tension of filtrate = 0.072 N/m.

Determine: 1. the threshold pressure, 2. the Capillary number and 3. residual saturation.

**Solution**

Step 1: Calculate the threshold pressure.

Threshold pressure is given by Eq. (14.69). Substituting the values into the equation gives:

\[ P_n = \frac{4.6 \times (1 - 0.35) \times 0.072}{0.35 \times 3.03 \times 10^{-6}} = 203.0 \text{kPa} \]

Step 2: Calculate the capillary number.

The capillary number is given by Eq. (14.70) as:
\[ N_c = \frac{0.35^3 \times (3.03 \times 10^{-6})^2 \left(1000 \times 9.81 \times 0.008 \right) + 86 \times 10^3}{(1-0.35)^2 \times 0.008 \times 0.072} = 0.000139 \]

Step 3: Calculate the residual saturation.
From Eq. (14.72):

\[ S_\infty = 0.155 \left(1 + 0.031 \times (1.39 \times 10^{-4})^{0.49}\right) = 0.528 \]

That is, 52.8% water will be retained after drying.

Wakeman [38] suggested a graphical solution for determining the reduced saturation by introducing the concept of dimensionless time, \( t^* \), dimensionless pressure, \( \Delta P^* \), and the average dimensionless airflow rate, \( \nu^* \), defining them as:

\[ t^* = \frac{K P_B t}{\mu L^2 \varepsilon (1-S_\infty)} \quad (14.74) \]

\[ \Delta P^* = \frac{P}{P_B} \quad (14.75) \]

\[ \nu^* = \frac{\nu \mu L}{K P_B} \quad (14.76) \]

where \( K \) = permeability of bed, (cake plus medium), \( \mu \) = viscosity of air or water at the given temperature, \( \nu \) = velocity of air or water (volumetric flux density of fluid relative to solid, \( \text{m}^3/\text{m}^2\text{s} \)).

The relationship between \( t^* \) and ultimate residual saturation and dimensionless air flow at various differential pressures were determined. Typical plots are illustrated in Figs. 14.28 and 14.29. In order to plot the relationships, the pressure differentials were calculated using the expression:

\[ \Delta P^* = \begin{bmatrix} P_{s,\text{INLET}} \\ P_{s,\text{OUTLET}} \end{bmatrix} - \begin{bmatrix} P_{s,\text{INLET}} \\ P_{s,\text{OUTLET}} \end{bmatrix} \quad (14.77) \]

By the use of Fig. 14.28 the ultimate residual saturation can be estimated at \( t^* \) and at a given operating pressure. Fig. 14.29 indicates the air volume required to achieve the saturation at the calculated dimensionless time. The method of calculating the air flow rate per unit area of filter surface is illustrated in example 14.7.
The basis for these charts is for an inlet dimensionless pressure equal to 100. The air flowrate therefore has to be corrected for the actual drying air pressure using the factor [19]:

$$\frac{100 - \Delta P_a}{P_{\text{outlet}}} \left( \frac{p_{\infty, \text{outlet}}^2 - (p_{\infty, \text{air}}^2)}{(100 - \Delta P_a)^2 - 10^4} \right)$$  \hspace{1cm} (14.78)

**Example 14.7**

During the filter cake drying operation described in example 14.6, the inlet air pressure was 5 atmospheres and the air temperature 18° C. The air-pressure below the cake was 1 atmosphere (absolute) or 101.325 kPa. Estimate the flowrate of air per square meter of a drum filter of 2 m in diameter and 4 m in length. A sixth of the drum surface is exposed for drying.
Fig. 14.29. Mean dimensionless air flow rate versus dimensionless dewatering time and dimensionless pressure [30].

Data: Time of air flow = 120 s
Specific cake resistance = \(1.10 \times 10^{11}\) m/kg
Depth of cake = 0.008 m
Porosity of cake = 35%
Density of solid = 2700 kg/m³
Density of water = 1000 kg/m³
The viscosity of air at 18°C = \(1.82 \times 10^{-5}\) Pa.s

Solution

Step 1: Calculate the dimensionless differential air pressure.

The threshold pressure determined in example 14.6 is 203.0 kPa. Hence from Eqs. (14.75) and (14.77):

\[
p_{\text{o, inlet}}^e = \frac{5 \times 101.325 \times 10^3}{203.0 \times 10^3} = 2.496 \quad \text{and} \quad p_{\text{o, outlet}}^e = \frac{1 \times 101.325 \times 10^3}{203.0 \times 10^3} = 0.499
\]
\[
\Delta P^o_s = P^o_{\text{inlet}} - P^o_{\text{outer}} = 2.496 - 0.499 = 2.0
\]

From Eq. (14.78) the pressure correction factor is:
\[
\frac{100 - \Delta P^o_s}{P^o_{\text{outer}}} \left( \frac{(P^o_{\text{inlet}})^2 - (P^o_{\text{outer}})^2}{(100 - \Delta P^o_s)^2 - 10^4} \right) = \frac{100 - 2.0 \left( \frac{0.499^2 - 2.496^2}{(100 - 2.0)^2 - 10000} \right)}{2.97}
\]

Step 2: Calculate the bed permeability.
To determine the permeability, \( K \), of the bed, use Darcy’s Eq. (14.6):
\[
K = \frac{1}{\alpha(1-\varepsilon)\rho_s}
\]

Substituting values:
\[
K = \frac{1}{1.1 \times 10^{-11} \times (1-0.35)} = 5.18 \times 10^{-15} \text{ m}^2
\]

Step 3: Calculate the dimensionless time.
The dimensionless time is calculated using Eq. (14.74).
\[
t^o = \frac{K P_B t}{\mu L^2 \varepsilon (1-S_w)}
\]

Substituting values we have:
\[
t^o = \frac{5.18 \times 10^{-15} \times 203.0 \times 10^3 \times 120}{0.001 \times 0.008^2 \times (1-0.528) \times 0.35} = 11.93
\]

Step 4: Calculate the flowrate per unit drum area.
From Fig. 14.29 the corresponding value of \( v^o_a \) is 1.5 and applying the pressure correction factor:
\[
v^o_a = 1.5 \times 2.97 = 4.455
\]

Hence by substituting into Eq. (14.76):
\[
v_a = \frac{4.455 \times 5.18 \times 10^{-15} \times 203.0 \times 10^3}{1.82 \times 10^{-5} \times 8 \times 10^{-3}} = 0.03217 \text{ m/s}
\]

The drum surface area offered for drying is:
\[
= \frac{2\pi(2/2)\times 4}{6} = 4.1888 \text{ m}^2
\]
Therefore the flowrate of air per square meter = \( \frac{0.03217}{4.1888} = 0.00768 \text{ m/m}^2 \text{ s} \)

### 14.6. Optimum Thickness of Cake

The optimum thickness of filter cake, \( L_{\text{OPT}} \), depends primarily on its solid bulk density \( (\rho_B) \), the concentration of the feed slurry, and the optimum filtrate flow rate, \( V_{\text{OPT}}/t \). The optimum thickness is given by:

\[
L_{\text{OPT}} = \frac{V_{\text{OPT}} C_m}{\rho_B A} \tag{14.79}
\]

Shirato and Tiller [31] gives the following operational conditions normally practiced for continuous operation of filters having diameters between 1.8 and 3.7 m that will provide the optimum cake thickness.

- Filtration Time up to 7.5 mins,
- Submergence 25% to 75% (normal about 40%),
- Rotating speed 0.1-3.0 rpm.

### 14.7. Filtering Media

Filtering mediums are commercially available with pore size and pore size distributions marked by manufacturers. Thus for the separation of a particular particle size in a slurry, the appropriate medium can be chosen to suit the size distribution of solids in the slurry. The medium should be non-reactive and preferably non-wetting to the slurry.

The media generally used in industrial practice are woven fabric, woven synthetic material and non-woven synthetic material. The woven fabrics are made of plain cloth, twill or satin. Both twill and satin are much stronger than plain cloth. They are woven in different patterns with a variety of weft and warp combinations which offer different permeabilities. The pore sizes range between 30 and 5000 µm [19]. The surfaces are sometimes flattened to reduce the pore size.

Satin finish cloths have a polished surface that releases the cake more easily than plain cotton. All cotton material are attacked by alkaline and acidic slurries, hence they are best used under neutral conditions. Cotton and cellulose based weaves shrink or stretch under load and sometimes swell. As a result, the structure of pores and pore sizes are affected which can adversely affect the size of particles in the filtrate.

The woven synthetic material are usually polypropylene, polyesters and various polyamides. The strength of the synthetic fibres are greater than cotton fibres. Rushton et al [19] quotes \( 9.0 \times 10^5 \text{ N/m} \) for warp and \( 1.60 \times 10^7 \) for weft fibres. These materials can withstand acidic and alkaline conditions. Synthetic media of wide ranging permeabilities are available. The manufacturers usually indicate air-permeability. The air permeability for nylon cloths is less than for polypropylene.

The non-oven synthetic fibre filter cloths are made from randomly assembled synthetic fibres and pressed together. The fibres are interlocked producing cloth of high permeabilities, and adequate strength for use in batch pressure filters [41,42].
14.8. Filtering Aids

Pulps containing fine kaolinitic or bentonitic clays form cakes that inhibit filtration and soon becomes impermeable as the process proceeds. Filtering such slurries and other gelataneous slurries are therefore difficult to filter. These slurries are therefore “spiked” with material that helps to keep open the porosity and permeability of the cake. It is essential that the added material be non-reactive. These filter aids are added in the form of powders of known size distribution.

The normal filtering aids are:

1. Diatomaceous Earth, like Kieselguhr, Fuller’s Earth, Celite,
2. Cellulosic material and asbestos fibre, saw dust,

In addition, the hard alumino-silicate minerals like harbolite (perlite) with a SiO$_2$ content about 75% and 12.5% Al$_2$O$_3$, and celite (75-85% SiO$_2$ and 4-12% Al$_2$O$_3$) are also commonly used.

The filtering aids are obtained in the form of crushed and screened powders of the minerals in several grades of pore size, pore size distribution and specific resistances. Sometimes the minerals are calcined when their pore size changes. In celite for instance, the pore size increases by about 40-60% and are uniform and their silica content increases to about 90%.

The natural minerals have a pore size of about 1.5 μm while the calcined mineral pore size is about 2.5 μm.

In batch processes they are added in powder form to slurries as they enter the filtering chamber. The thickness of the layer is regulated so that the medium forms a coat 1-3 mm thick. This roughly translates to about 0.5 to 1.0 kg/m$^2$ [19]. In the continuous filtering processes, the filtering surface is first coated with the filtering aid material. This is usually achieved by suspending the filter aid in the form of a slurry and pumping to the trough below the filter. The filtering surface is then immersed in the trough. Vacuum is applied and the filter surface thus coated. The quantity of the filter-aid deposited depends on the time of immersion, particle size, shape and viscosity of the pulp. The depth of the layer can be about 15 cm.

Filter aids mixed with the cake are usually discarded after filtration is completed. However, when the cake is also of interest, the filter aid can be chemically treated or the aid material removed physically.

14.9. Filtration in Mineral Processing Circuits

Filters and thickeners are usually integrated in series in the process plant. For example, placed in series with cyclone overflows and underflows and sometime following the filtration circuits, e.g. in coal washeries, lead-zinc extraction plants, copper-lead-zinc circuits. A typical layout from a flotation circuit is shown in Fig. 14.30. Several variations are seen in practice. As the filtration rates are relatively slower than most other unit operations constituting a processing plant, several filtering units are placed in parallel to meet the production target.
Fig. 14.30. Typical set-up of a filtration circuit.

14.10. Problems

14.1
A siliceous filter feed slurry contained 120 kg solid/m$^3$. It was filtered in a plate and frame press at a constant pressure of $1.5 \times 10^5$ Pa through a medium of resistance $2.5 \times 10^{10}$ m$^{-1}$ and cake resistance of $3.62 \times 10^{11}$ m/kg. The press dismantling and reassembly time was 2 minutes. The densities of the liquid and the solids were 1000 kg/m$^3$ and 2640 kg/m$^3$ respectively and the filtrate viscosity was 0.001 Pa.s. A filtration test indicated that the ratio of wet to dry filter masses was 1.32. The filter was required to produce 40.5 kg/hour dry solids. Estimate the area of the filter surface required to meet the required yield.

14.2
A rotary drum filter 0.5 m long x 0.45 m diameter was continuously fed with a nickel sulphide slurry containing 0.15 kg of the mineral per kg of water. The filter revolved at the rate of one revolution in 270 seconds. The filtering surface was submerged in the slurry to a level of 18%. A pressure difference of 50 kPa was applied and 350 kg of filtrate obtained per hour. The following laboratory determinations were made:
1. porosity of the deposited dry solids = 33%
2. specific gravity of the mineral = 4.5
3. cake moisture = 15%.

Estimate the thickness of the deposit.

14.3
A rectangular pressure filter with a plate size of 0.3 m x 0.25 filtered a mineral suspension at the rate of $2 \times 10^{-4}$ m$^3$/s when a pressure differential of $1.5 \times 10^5$ Pa was applied.

After filtration, 0.5 minutes were required to dismantle, 1.0 minutes to remove the cake and 0.5 minutes to reassemble the filter. The feed slurry contained 0.4 kg of solids per kg water.

Estimate:

1. the number of frames used, and
2. thickness of the frames.

Data:
- Porosity of cake = 8%,
- Density of solid, $\rho_S$ = 4010g/m$^3$,
- Cake moisture = 12%,
- Cake compressibility index = 0.05,
- Viscosity of water = 0.001 Pa s,
- $\alpha_0$ = $1 \times 10^{10}$ m/kg

14.4
A drum filter has 33% of its surface (15.7 m$^2$) submerged in a mineral slurry. The solid content of the slurry was 25% by mass. The densities of the dry mineral and the filtrate were 3020 and 1000 kg/m$^3$ respectively. The drum filter revolved at 0.35 revs./min with a pressure of 35 kPa applied. The filter cloth resistance was $1.2 \times 10^{-10}$ m$^{-1}$.

The drum speed was increased and a 4 mm thick cake was formed. The specific resistance of the cake deposit was determined to be $4.8 \times 10^{-10}$ m/kg and the porosity 40%. If the cake saturation is 50% and the filtrate viscosity is 0.001 Pa s determine:

1. the increase in the filtering rate at the increased drum speed,
2. thickness of the deposit at the initial drum speed,
3. the maximum possible increase in filtering rate.

14.5
A Dorr Oliver disc filter has discs of diameter equal to 1.57 m. The filter is fed continuously by a silicious slurry at the rate of 0.4 m$^3$ of slurry/min. The slurry contained 120 kg of solids/m$^3$ water. For 3 mins the filter surface is submerged 33% in the slurry and a pressure of 0.70 x $10^5$ Pa is applied for filtration. The average moisture content of the filter cake is 45%.

Neglecting resistance of the filter medium, estimate:

1. the filter area required,
2. the number of filtering segments,
3. the flow rate of the filtrate.
Additional Data:
- Resistance of deposit = $1.88 \times 10^{10}$ m/kg
- Compressibility coefficient = 0.28
- Density of solids = 2.75 kg/m$^3$
- Density of water = 1000 kg/m$^3$
- Viscosity of water = 0.001 Pa s

14.6
A titania (TiO$_2$) plant produced sludge with a mean particle size was 75 microns. The sludge contained 120 kg of solids/m$^3$ of slurry. Filtration tests on the slurry showed the following results for the production of 1 litre of filtrate:

<table>
<thead>
<tr>
<th>Time (mins)</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure differential (kPa)</td>
<td>17.0</td>
<td>45.0</td>
<td>80.0</td>
<td>128.0</td>
<td>198.0</td>
</tr>
</tbody>
</table>

The filter test area was 0.03 m$^2$ and the filter cake porosity was 20% on average with a moisture content of 40%. The viscosity of the slurry was 0.00089 Pa s and the density of solid and water were 4300 and 1000 kg/m$^3$ respectively. Estimate:

1. the resistance of the cake,
2. the compression coefficient of the cake.

Assume that the media resistance is negligible.

14.7
The porosity of a bed of cake was estimated as 35% and the mean particle size was 75 microns. It had to be washed and dried with a fluid which had a surface tension of 0.052 N/m. Determine the minimum pressure required for the drying process.

14.8
A rotary drum filter was loaded uniformly with 0.035 m thick cake with a 45% porosity and specific resistance of $1.1 \times 10^{10}$ m/kg. The density of the solid was 2600 kg/m$^3$ and the mean particle size was $5.0 \times 10^{-6}$ m. The fluid in the cake had a surface tension of 0.05 N/m and a density of 1000 kg/m$^3$. If the drying pressure was 103 kPa, estimate:

1. the capillary number,
2. the residual saturation.

14.9
Filtration tests were carried out with a plate and frame filter press under the following conditions:

- Solid density = 2710 kg/m$^3$
- Liquid viscosity at 25°C = 0.001 Pa s
feed concentration = 10 kg solid/m$^3$ of slurry
Filter dimensions = plate and frame press, 10 frames
                        dimensions 430 x 430 x 30 mm

From the filtration data, calculate the specific cake resistance and the medium resistance for the test.

<table>
<thead>
<tr>
<th>Pressure (kPa)</th>
<th>time (s)</th>
<th>Filtrate vol. (m$^3$)</th>
<th>time (s)</th>
<th>Filtrate vol. (m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>180</td>
<td>0</td>
<td>0</td>
<td>2736</td>
<td>0.14</td>
</tr>
<tr>
<td>180</td>
<td>305</td>
<td>0.02</td>
<td>3229</td>
<td>0.16</td>
</tr>
<tr>
<td>180</td>
<td>662</td>
<td>0.04</td>
<td>3719</td>
<td>0.18</td>
</tr>
<tr>
<td>180</td>
<td>1017</td>
<td>0.06</td>
<td>4227</td>
<td>0.2</td>
</tr>
<tr>
<td>180</td>
<td>1412</td>
<td>0.08</td>
<td>4755</td>
<td>0.22</td>
</tr>
<tr>
<td>180</td>
<td>1809</td>
<td>0.1</td>
<td>5299</td>
<td>0.24</td>
</tr>
<tr>
<td>180</td>
<td>2271</td>
<td>0.12</td>
<td>5875</td>
<td>0.26</td>
</tr>
</tbody>
</table>

Assume that the cake is incompressible.

14.10
A vacuum filter leaf tests operating at a form pressure (vacuum) of 47 kPa produces a cake resistance and media resistance as given below.

If a horizontal belt vacuum filter uses the same filter cloth and filters the same slurry and has the following dimensions, calculate the filter throughput in dry solids per hour.

**Filter leaf test**
- cake resistance = $1.29 \times 10^{11}$ m/kg
- medium resistance = $0.1645 \times 10^{11}$ m$^{-1}$
- solid concentration = 50% (mass)
- cake moisture = 15% (mass)
- liquid viscosity = 0.001 Pa s
- solid density = 2600 kg/m$^3$
- liquid density = 1000 kg/m$^3$

**Belt filter**
- feed box dimensions = 900 mm x 1100 mm
- belt width = 1 m$^2$
- belt speed = 7.5 m/min

Note: the last filtrate is extracted from the cake just as the cake leaves the feed box.
REFERENCES


[34] W.R. Sherman, AIChE Journal, 10 No. 6 (1964) 855.


[38] R.J. Wakeman, Filtration and Separation, November/December, 16 No. 6 (1979) 655.


